The Mathematics Educator
2012 Vol. 22, No. 1, 17–38

The Roles They Play: Prospective Elementary Teachers and a Problem-Solving Task

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The transition from learner to teacher of mathematics is often a difficult one for prospective elementary teachers to negotiate. Learning to teach necessitates the opportunity to practice the discourse of teacher of mathematics. The undergraduate mathematics content classroom provides a setting for prospective teachers to practice the discourse of teacher through their interactions with each other while also learning the mathematical concepts presented in class. This qualitative study sought to examine what roles prospective teachers adopt while engaged in a cooperative problem-solving task. Discourse analysis was applied to analyze the verbal interactions between three participants in a mathematics content course. Key disruptions in the conversation revealed instances of the fluid relationship between learner and teacher of mathematics in the roles they adopted while solving an application problem: self as learner-in-teacher, collaborator as learner-in-teacher, and unlikely learner-in-teacher. The presence of this fluid relationship led to the proposal of a model of learner-in-teacher-in-learner of mathematics. This proposed model suggests that prospective teachers have the opportunity to learn how to teach in and through each other when given the opportunity to engage in dialogue with one another.

The shift from learner of mathematics to teacher of mathematics usually begins in the prospective elementary teacher's mathematics content classroom. Up to this point, the prospective elementary teacher has taken part in the mathematics community as a learner of mathematics and now hopes to take on the role as teacher of mathematics. In the mathematics content classroom, the prospective teacher is expecting to learn both mathematical concepts and how to teach them effectively. The individual in this transitory space is "learning about becoming…by participation in practices" (Lerman, 2001, p. 88).

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The process of acquiring a new identity may be complicated by past experiences with mathematics, especially if these experiences were not positive (Jones, Brown, Hanley, & McNamara, 2000). Jones, et al (2000) described the transitions between the identities of learner and teacher as a "means of reconciling the past with the present and the future" (p. 2). It is important that mathematics teacher educators understand how prospective teachers form their own identities as teachers of mathematics to develop an efficacious curriculum that supports this reconciliation.

Sfard (2003) viewed identity as a process of becoming part of a community of discourse. This is in agreement with Gee's notion of discourse as an established set of social practices, including language, gestures, beliefs and ways of acting within the society (Gee, 1989). This set of norms make up what he intentionally referred to as Discourse, with a capital D. Our ways of being are mirrored in our Discourse, which Gee referred to as our "identity kit". This identity kit comes "complete with the appropriate costume and instructions on how to act, talk, and often write, so as to take on a particular role that others will recognize" (p. 7). The roles of teacher and learner in a mathematics classroom would each have their own Discourse, with overlapping language and ways of being, but distinct in ways that others recognize which role is being played. For example, the Discourse of Teacher often differs from the Discourse of Student in regards to the intent of an inquiry. Teachers tend to pose questions that they already know the answer to, whereas students' questions usually arise from a lack of knowledge. Both teacher and student may respond to each others' questions with explanations but the reasons for asking the questions are unique to the role being played. Gee asserted that Discourse cannot be explicitly taught to the players, but must be acquired "by enculturation ("apprenticeship") into social practices through scaffolded and supported interaction with people who have already mastered the Discourse" (p. 7).

The mathematics content classroom provides a setting for prospective teachers to practice the Discourse of Teacher through their interactions with each other. However, within this setting, the prospective teacher is also using the Discourse of Student to learn the mathematical concepts presented in class. These two processes of learning often result in conflicting identities as the individual pushes to become a teacher (Gee, 1989). The ongoing process of

becoming a teacher of mathematics is imbedded in the process of learning mathematics, both of which take place within the individual. The process moves back and forth within the individual, manifesting these two identities in the discourses of the individual. The constant flux of these two identities leaves us unable to extricate one from the other (Wang, 2004). Therefore, I propose we examine this transition in movement using the learner-in-teacher-in-learner as our unit of analysis. In this manner, perhaps we can catch a glimpse of the ongoing process of becoming a teacher while preservice teachers are learning and participating in the mathematics community. The purpose of this paper is to present a glimpse into how this transition might begin in the prospective elementary teachers' mathematics classroom by listening to the voices of prospective teachers engaged in a peer problem-solving task. Analysis of the conversations will be used to answer the research question: What roles do prospective teachers assume while involved in cooperative problem-solving?

Background Information

Sociocultural Theory

The foundation of this research study is entrenched in the sociocultural theories of Vygotsky, who asserted that the process of meaning making is mediated through the use of the symbolic tools of language and other cultural artifacts (Vygotsky, 1934/1986). According to Bruner (1997), this meaning making is situated within the cultural context we find ourselves in and is facilitated by our social interactions with one another. The transferability of cultural ways of knowing takes place in the semiotic space between teacher and learner. Vygotsky described this space as the zone of proximal development in which the discourse of a more knowledgeable person supports the learner's growth in knowledge (Lerman, 2001). This zone of proximal development may emerge through the interactions between the teacher and the learner, but it may also arise through the interactions between members of collaborative learning groups (Goos, Galbraith, & Renshaw, 2002). Goos et al. analyzed transcripts of the conversations between secondary students assigned to a group problem-solving task. They noted the availability of a collaborative zone of proximal development when students with complementary abilities monitored each other's

thinking. In other words, learning can take place whenever the learner and a knower of a concept have the opportunity to interact together.

Opportunities for interactions between apprentices and ones who have mastered the skills of a craft are situated within social settings referred to as communities of practice (Greeno, 2003; Lave, 1991; Lave & Wenger, 1991; Wenger, 1999, 2000). Within these communities, the learner is able to practice the skills of the knower and gradually acquire the competencies that define the members of the community. The process of gaining these skills is enveloped in the process of becoming a member of the community (Lave, 1991; Lave & Wenger, 1991; Wenger, 1999, 2000). Lave (1991) asserted, "…without participation with others, there may be no basis for lived identity" (p. 74).

The Discourse of Mathematics

The social semiotic perspective taken by Morgan (2006) used a critical lens to describe the relationship between the learner, his or her culture, and the discourses the learner participates in. Morgan asserted that context consists of both the immediate realm of interaction and the broader culture in which the learner participates. Careful consideration of the influences of the multiple discourses a learner participates in may open "a crucial window for researchers on to the processes of teaching, learning, and doing mathematics" (Morgan, 2006, p. 219). Morgan illustrated this approach with examples of how the critical lens of social semiotics could be applied to student writing, especially in open-ended questions on high-stakes tests. However, written text, Morgan warned, provides only a partial image of the identity of the author, leaving it up to the reader to create the rest. Morgan stated that the discursive interactions between two or more people are a richer source of information concerning how individual identities are formed. Through the process of collaborating and/or jockeying for positions, participants manage to negotiate their own identities in relation to each other.

Kieran (2001) examined the discourse between pairs of adolescents assigned to work together to solve a series of problems. Drawing from the field of applied linguistics, Kieran created an interactivity flow chart to indicate the direction and the presumed intent of the utterances spoken during the event (2001,

p. 202). The flow chart then was analyzed under the umbrella of Vygotskian theory on the relationship between language and thought. This combined approach "makes explicit the integration of the two in that both talking and thinking are considered examples of communication – communication with others and communication with self" (Kieran, 2001, p. 190).

Sfard (2001) used the metaphor of learning-as-participation to describe a pedagogical model that focuses specifically on the interactions between individuals within a community of practice. The researcher working within this framework is concerned with analyzing how the artifacts of individual learning are manifested in the communications between members of a group. For instance, the use of a newly introduced mathematical term or procedure is an indication that the student is learning how to use the tool (Lerman, 2001). Sfard illustrated the application of a discursive approach to analysis through an investigation into the benefits of collaborative efforts in learning mathematics. Utilizing the same type of interactivity chart as Kieran (2001), Sfard exemplified this illustration with two contrasting examples of non-productive discourse. Her analytical approach considered the focus, or intended focus, of the discourse and the position of each participant in response to that utterance. For example, seeking to learn mathematics by questioning or challenging the thinking of others signals one's intent to become part of the mathematics community. By considering this interplay between the what, why, and for whom features of an utterance, Sfard was able to explain why the tools that people use to communicate and the meta-rules of discourse shape how we listen and learn in the classroom. Sfard claimed that "careful analyses of diverse classroom episodes can be trusted to provide a good idea of what could be done in order to make mathematical communication, and thus mathematical learning, more effective" (p. 44). Discourse analysis can also be used to explore how participants in the mathematics community co-create the identities of teacher and learner as they interact in the classroom (Sfard, 2003).

Greeno (2003) recommended that researchers study how small group conversations contribute to the formation of identities in the mathematics classroom. He detailed examples of how situated research such as focusing on the conversations of cooperative problem-solving groups may reveal how students develop their

identities as learners and knowers of mathematics. In the mathematics content classroom, prospective teachers have opportunities to engage in problem-solving experiences while working in cooperative groups. These group experiences create a space for prospective teachers to practice communicating their mathematical thinking and develop an understanding of how others learn mathematics. Within this space, there is a potential curriculum for the mathematics teacher educator to immerse with in an attempt to understand the formation of the Discourse of Teacher.

Learning to Teach

Nicol and Crespo (2003) explored how teacher educators can enable prospective teachers to learn how to teach through the critical self-examination of initial field experiences. Nicol and Crespo based their qualitative study on Wenger's theory of learning and his ideas on identity formation, stating Wenger maintained "…that learning involves the development of identity, the changing of who we are, in the context of the communities of practice that we participate in" (p. 374). Participants in the study conducted by Nicol and Crespo shared their positive and negative experiences in the classroom, discussing their personal struggles with mathematics and what they learned about how their students learn mathematics. For these prospective teachers, their identities as learners of mathematics were deeply connected to their image of themselves as teachers of mathematics by the desire to deepen their own understanding of the subject (Nicol & Crespo, 2003).

Jones, Brown, Hanley, and McNamara (2000) interviewed a group of prospective elementary teachers in order to examine their experiences as they were learning how to teach mathematics. The researchers' analysis of interview data keyed in on how these prospective teachers assimilated past and present encounters with mathematics in order to describe themselves as future teachers. For example, teachers with negative experiences with mathematics were able to reconcile the past with the future by using these experiences as models for how not to teach. Jones, et al. stated that the interactions between past, present, and future perceptions of mathematics in relation to the self play a major role in the development of identity as teacher of mathematics. Amato's (2004) work on developing a liberating mathematics curriculum

for prospective elementary teachers was based on this same interplay between past, present, and future. Amato used activities designed to build conceptual understanding of elementary school mathematics as a way to change pre-service teachers' beliefs and attitudes toward mathematics. He asserted that prospective elementary teachers needed to have meaningful experiences in mathematics to become effective teachers.

Perhaps the most compelling explanation of how individuals learn how to teach was proffered by Freire (1970/2007) in his seminal piece, *Pedagogy of the Oppressed.* Freire described how teachers who engaged in open dialogue, or praxis, with their students escape the idea that teaching is merely the unidirectional transmission of knowledge. Instead, the teacher who engages in praxis "…is no longer merely the-one-who-teaches, but one who is himself taught in dialogue with the students, who in turn while being taught also teach" (1970/2007, p. 80). Borrowing from Vygotsky's model of mind-in-society-in-mind, the idea of learning how to teach through the act of teaching can be described metaphorically as learner-in-teacher-in learner. The question arises then, how can mathematics teacher educators facilitate the transition from learner to teacher of mathematics before the prospective teacher enters the elementary classroom? What lessons can prospective teachers learn about teaching mathematics while they are learning mathematics content?

Theories of how the discourse of mathematics is learned within the classroom culture dominated the literature on the teaching and learning of mathematics examined for this study (e.g., Kieran, 2001; Morgan, 2006). Analyses of classroom discourse focused primarily on pedagogical concerns dealing with the teaching and learning of mathematics in the primary and secondary classroom. Literature on prospective teachers' experiences in the undergraduate mathematics classroom concentrated on how positive experiences with mathematics can change beliefs and attitudes toward mathematics (e.g., Amato, 2004). However, little research has been done on the apprenticeship of prospective teachers into the Discourse of teacher of mathematics. As a mathematics teacher educator, I recognize the need for communication in my classroom by inviting my students to participate in the discourse of mathematics. Besides attempting to model productive discourse during whole

class discussions, I also provide multiple opportunities for my students to engage in peer problem-solving activities. I believe that these experiences fulfill the dual purpose of learning how to teach mathematics while also learning mathematical concepts. However, as a researcher in mathematics education, I wonder if my attempts to promote communication in my classroom are sufficient to enable my students to develop their self-identity as a teacher of mathematics. The purpose of this study was to analyze the roles prospective teachers assume while engaged in problem-solving tasks and, in turn, shed light on how prospective teachers negotiate the transition from learner to teacher of mathematics within the culture of the mathematics content classroom.

Methodology

Framework

Ethnomethodology provided the framework for studying the interactions of the prospective teachers participating in this study. According to Roulston (2001, 2004), the focus of ethnomethodology has historically been on the analysis of the ordinary discourse that takes place between individuals in everyday situations. This is in contrast to the usual ethnographic approach of interviewing participants to ascertain what has taken place in the past. Ethnomethodological approaches allow the researcher to witness the interactions between group members in real-time versus relying on the memory and interpretation of participants after the fact. Roulston (2004) explained that "researchers using ethnomethodological approaches to research are keenly interested in how members' knowledge is constructed in and through talk and text" (p.140). Traditionally, researchers adhering to this methodology have focused on interactions which take place in natural setting such as the work place or the classroom. For this research inquiry, the conversations of one of the cooperative learning groups in my mathematics classroom were recorded and analyzed to investigate the roles prospective elementary teachers assume while engaged in a problem-solving task.

The Setting and the Participants

This study took place on a satellite campus of a regional university in the Midwest near the end of the spring semester of

2008. The participants in the study were all enrolled in a mathematics content course for prospective elementary teachers taught by the researcher. This three-hour credit course dealt primarily with rational number concepts. Two groups of students volunteered to participate in this study by recording the audio of the conversation shared while working collaboratively on a mathematics activity. One group, consisting of two female students in the class, tended to request help from the teacher/researcher whenever they struggled to answer a question. The second group, a triad of females, sought help from each other when they could not solve a problem. For this paper, I have chosen to discuss my analysis of the significant moments embedded in the conversations of the triad.

The group consisted of three female students who had worked together on problem-solving tasks in the past. Cindy and Brooke were both nontraditional students in their early thirties. The third student, Jenny, was in her mid-twenties at the time of this investigation.. Brooke appeared to be the least confident in the group of the three students and often voiced her frustration with mathematics during our whole-class discussions. The other two had comparable abilities in mathematics which would seemingly open a space for the emergence of a collaborative zone of proximal development as described by Goos, Gailbraith, and Renshaw (2002). The presence of this zone of proximal development may make it possible these two to support each other's thinking and learn from each other, much like the scaffolding a teacher provides for their students.

The activity involved counting and sorting M & M® candies to examine the connections between ratios, decimals, and percents. The light-hearted nature of the activity hopefully eased the tension students might have experienced about being recorded. However, the triad encountered difficulties with the contextual problems they were required to complete after sorting the candies. These disruptions in the flow of talk and how the speakers resolved misunderstandings provided pieces to the puzzle of how participants (re)negotiate self-identities and roles during the course of a conversation (Ten Have, 1999).

Methods of Data Analysis

I used a modified version of an interactivity flowchart created (See Table 1) by Kieran (2001) to create a visual representation of the mechanics of the conversation. The flow chart consisted of arrows pointing up or down, depending on the intent of the speaker. If an utterance appeared to be in response to a prior statement, then an upward pointing arrow was used to represent the utterance. If the intent appeared to be soliciting a response, then a downward pointing arrow was used. These arrows could point to self (personal channel) or to other (interpersonal channel). According to Kieran the researcher bases these classifications on the apparent intent of the speaker. I modified Kieran's flowchart so that I could apply it to triadic conversations and omitted additional classifications she had used.

In the excerpt displayed in Table 1, the participants were responding to a question in which they needed to find 30% of 86. Jenny had decided to solve the problem by multiplying 86 by 0.3 instead of using a proportion. Although Cindy recognized that Jenny's procedure would yield the same answer, she suggested to Brooke (line 187), "let's do it this way." In line 189, Jenny is speaking softly to herself as she works the problem her way; therefore the upward pointing arrow is located in her personal channel, labeled J. Brooke and Cindy are working together to solve the problem using proportions when Brooke stops Cindy on line 190 to ask her "…how did you get that?" Since this statement was directed at Cindy, the arrow appears in the column labeled $C \leftrightarrow B$ with the arrow's beginning located on the right to symbolize the statement was made by Brooke. The statement is labeled proactive, as indicated by the downward pointing arrow, since Brooke is soliciting a response from Cindy. On line 191, Jenny offers her answer up for approval. The statement is directed at both Brooke and Cindy, therefore downward pointing arrows are placed in both interpersonal channels, $B \leftrightarrow J$ and $J \leftrightarrow C$. Jenny redirects the question to Cindy (line 193) and the two engage in an exchange that excludes Brooke until line 199. Their responses (lines 194 and 195) to each other are labeled reactive as indicated by the upward point arrows in the far right column.

Table 1

Example of Flow Chart

Statement	*C*	*C* ↔ *B*	*B*	*B* ↔ *J*	*J*	*J* ↔ *C*
187 C: Let's do it this way.		↗				
188 B: Yeah.		↙				
189 J: …times 86 (*softly*)					↑	
190 B: Wait..how did you get that?		↙				
191 J: 25.8?				↙		↘
192 B: Part? What's the part?		↙				
193 J: Did you get 25.8 Cindy?						↘
194 C: Hold on. I'm not there yet.						↗
195 J: Okay. Sorry…						↗

Using Kieran's (2001) recommendation I then began to focus on the action implied in the words of the utterance. Proactive statements generally fell under the categories of seeking information in the form of help, verification, or justification. Reactive statements were categorized as helping, justifying, or simply responding with information. After characterizing the actions of each utterance, I examined both the flow of the conversation and the inferred actions of each utterance in order to focus on the nature of talk in terms of turn taking, corresponding threads, topic management, disagreements, and repair as Zhou (2006) recommended. For example, there were times when a reactive statement was made that also solicited a response, such as

when a participant responded to an unexpected solution with a request for an explanation of how the answer was obtained. These dual-coded statements usually resulted in a disruption in the progress on the task while members of the group worked to resolve the issue. Prior to the disruption, the conversation focused on verifying solutions to the problems they were working on. The participants moved to the next problem at hand as long as their solution pathways and/or solutions were the same. However, when differences in their pathways or solutions became apparent, the conversation focused on resolving those differences. Examining how the participants resolved these differences brought insight into how this group of prospective teachers negotiated the roles of learner and teacher while engaged in problem solving.

Discussion

The triad spent approximately 27 minutes on the problem-solving task. Cindy initiated the activity by asking the other students how many of each color candy they had in their individual samples and determining the total counts for each color. Throughout the conversation, Cindy played this role of leader by directing attention to the next problem on the page once issues with the previous one were resolved. The mathematics was relatively simple at first; converting ratios to decimals and percents. All three worked independently as they verified answers and questioned the reducibility of a fraction.

Self as Learner-in-Teacher-in-Learner

The first conversational disruption occurred when Jenny supplied an unexpected answer while the students were simplifying the fractions they wrote for each color of candy as part of the total and converting each fraction to decimal and percent form. The interactivity flow chart of the utterances prior to this sequence showed arrows pointing up and down in all three interpersonal channels (see Appendix). All three students were involved in the conversation as they worked in tandem, blurting out answers to one another for verification.

88 J: I got like… 15 out of a hundred

89 C: Huh?

90 J: I got like point one five which is like fifteen percent.

91 B: (Oh yeah?)
92 C: Oh…for the next column?
93 J: Well…
94 B: Where are we at?
95 J: No…if…I took 13 …divided by 86. And I got point one five one or something like that
96 C: Yess…for the decimal
97 J: Yes…
98 J: So …yeah…if you…
99 J: Oh it's just ratio as a fraction…
100 J: No that's right!
101 J: It would be 13 over 86.
102 J: I see what you're saying …
103 J: I see!
104 J: Yes, as a decimal.
105 C: Okay!
106 J: Sorry.
107 C: That's okay.
108 B: So what's the decimal?
109 J: Point one five.

Jenny's request for verification (line 88) resulted in a reactive statement from Cindy that served the dual purpose of soliciting a response (line 89). Cindy's statement was labeled with both up and down arrows on the interactivity chart (see Appendix) and signified a disruption in the flow of talk. Note that immediately following the unexpected answer given by Jenny (line 88), Brooke is excluded from the repair of the disruption. She tries to break in (lines 91 and 94), but neither Cindy nor Jenny respond to her queries. Once the issue is repaired, Jenny responds to Brooke's request by simply supplying the answer without explanation (line 109). This scenario repeated itself whenever Cindy and Jenny came up with conflicting answers. Cindy and Jenny tended to rely on each other for verification of their solutions, indicating that the two were confident in each other's ability to solve these types of problems. On the other hand, their apparent exclusion of Brooke

from the verification process seems to indicate a lack of confidence in Brooke's ability.

The other interesting story in this particular sequence is one of metacognition. Notice the string of utterances Jenny makes after Cindy made the comment "Yesss…for the decimal" (line 96). Jenny was supposed to simplify the fraction first and then record the decimal form of the quantity in the next column. She goes back and forth between the right and wrong answers, reacting to her own statements, until she finally convinces herself that she was mistakenly finding the decimal instead of the fraction form of the quantity. Jenny seeks help from self as learner and replies back to self as teacher. Through this series of utterances we see a story of self as learner-in-teacher-in-learner.

Collaborating as Learner-in-Teacher-in-Learner

According to NCTM (2000), an effective teacher of mathematics is able to "analyze what they and their students are doing and consider how those actions are affecting students' learning" (p. 18). Both Cindy and Jenny took on the identity of teacher by monitoring each other's work, as well as Brooke's. However, there were also instances in which the roles of teacher and learner merged as Cindy and Jenny supported each other's thinking. One such instance began when the triad encountered a rather long disruption. The students were attempting to solve a problem in which they had to deduct ten percent from the total number of candies (86) and then take another thirty percent off of the remaining amount. Jenny explained to the others they could eliminate an extra step by calculating 90 percent of the total instead. Brooke seemed confused by the plan, stating that she had "…no idea obviously what she does." Although Cindy initially suggested that Jenny "…do it that way and then we'll see if we come up with the same answers…," she decided to follow suit and proceeded to calculate ninety percent of 86. However, Cindy did not quite understand how to complete the problem once this issue was resolved.

274 C: Minus 86…right?

275 J: No…I didn't do it that way.

276 B: So…now you take 86 minus 77.4…So is that what you're saying?

277 J: I got 77.4…okay? That's what you have left…and you saved thirty percent of that to take home.

278 C: So we ate 8.6. Is that what you got?

279 J: No…this is what we have left (77.4)…and we're taking part of it home.

280 C: So we are taking thirty percent of the 77.4?

281 J: So what we could have eaten was 54.8. I'll show you what I did.

282 C: So we have to figure out what 30 percent of 77.4 is?

283 J: Yes…which is 23.22. So that's what you're taking home to your husband…or to your kids…or to a friend.

284 C: Thirty percent of …got it…..23.22?

285 J: That's right.

286 C: Okie dokie.

287 J: So then…so then…then it wants to know how much you could have eaten. Okay. You had 77.4 and you take 23.22 home…so what would…what could you…?

288 C: So you have to subtract it.

289 J: We subtract it.

Brooke appeared to be an outsider during most of this first sequence, but she did manage to interrupt the discussion with a question concerning the final answer. Jenny seemed about to respond to Brooke when Cindy asked for verification of the next step (line 274). Cindy wanted to subtract their previous answer from the total, which would have negated the advantage of taking ninety percent of the total instead of ten percent. During this sequence, Jenny explained the rationale behind each step in her procedure. Her approach seemed to support Cindy's thinking, enabling her to understand how to solve the problem before Jenny had finished explaining the procedures. In fact, near the end of the sequence, Cindy was explaining the steps and Jenny was confirming them (lines 284 – 289). This series of back and forth responses illustrates what Goos, Galbraith, and Renshaw (2002) referred to as mediated thinking. Within this collaborative zone of proximal development Cindy and Jenny shared, Cindy was able to

correct her own error. The scaffolding approach taken by Jenny is part of the repertoire of an effective teacher (NCTM, 2000).

Brooke re-entered the conversation soon after by repeating the same error as Cindy had on line 274. Jenny offered a quick explanation, but apparently noticed that Brooke was even more confused than before (line 307). Instead of simply supplying the answer and moving on to the next problem on the sheet, Jenny tried a more dialogical approach by asking supportive questions.

307 J: That really seems to confuse you even more.
308 B: Well, umm…
309 J: This is what you have…you're taking that home…so how much did you eat in class?
310 J: If this is your total and you took that part home…how much is left for you?
311 J: (long pause) …you know how you got there..
312 B: But if you add those…if you add all those up together it doesn't equal
313 C: (it adds up to 86)
314 J: Yes…
315 B: It doesn't add up to 86.
319 B Yeah, but 54.2 plus 77.4 doesn't add up to 86…
320 C: Because….we didn't do the ten percent. Right? We didn't do the ten percent. Right?

Cindy briefly re-entered the conversation (line 320) by offering a possible explanation for why the quantities (54.2 + 23.22 + 77.4) did not add to 86. However, Jenny began to doubt her answer (line 328). Neither she nor Brooke seemed able to explain why it might be incorrect.

328 J: Do you guys think I did it wrong?
329 B: Well…I just…no….I don't…understand
330 J: Well…if you do…just tell me what I…I may have…I may have done it wrong.
331 B: I don't know…I don't know…Well…I just don't understand.
332 J: That's the way I understood it.

333 C: If we gave away 8.6 of them…cause that would be ten percent. So we're going to save thirty percent of…77.4…Which is…twenty three point two two

334 J: MmmHmm

335 C: How many could we eat in class today? So…23.22 plus…

336 J: I see what you're saying

337 J: I'm not sure why it doesn't add up…8.6 and 77.4 should add up to 86

338 C: (8.6)

339 C: Right…so

340 J: Not the 23

341 C: Right

342 B: B…but if you had…

343 J: Cause the 23.22 is already included in your 77.4

344 B: Oh…okay…hold on

345 C: So this is what we took home…No what we gave her

346 J: Because you took thirty percent of a different total

347 C: Yeah…

348 J: That's why it' not adding up…

This time Cindy supplied the scaffolding to support Jenny's thinking. Through the scaffolding provided by their collaborative zone of proximal development, Cindy and Jenny were learning how to teach and learning how to communicate their mathematical thinking. Together they were learning the discourse of mathematics.

The Unlikely Learner-in-Teacher-Learner

Throughout this experience, Brooke seemed to remain frozen within the position of learner of mathematics. At times she was an outsider to the conversations around her despite attempts to join in the conversation. For example, while examining the interactivity chart I noted five instances where her attempts to seek help were ignored by the other two members of the group. Several other utterances she made were incomplete, cut off by one of the other

two speakers. The fact that she seemed to struggle more with the mathematics than the other two may explain why she was responsible for less than one- fourth of the total statements made during the conversation. One possible explanation for her lack of engagement in the conversation could be due to the silencing effect mathematics may have over those who do not understand its discourse (Walkerdine, 1988, 1997; as cited by Forman, 2003). Walkerdine (1985) described the effects anxiety imposes on many women in academic settings, stating that the person may come to believe "…that if they open their mouth, they will 'say the wrong thing'…" (p. 226). However, as I listened to the conversations of these three students, I began to explore the possibility that the subject labeled as 'learner' was teaching the other how to teach. What lessons was Brooke teaching to Jenny as she pushed for an understanding of why the quantities on hand did not add up as expected? Her inability to understand forced Jenny to think of another way to explain the mathematics and it also forced her to think about her mathematical thinking. As Wang (2004) stated, the "subject-in-process is intricately related to subject-in-relation because the fluidity of self is enabled by responding to the other" (p. 120). Within the culture of the mathematics classroom, the prospective teacher has the opportunity to learn how to teach through her interactions with others.

Conclusion

This study is limited by both the duration and the number of participants. Although the analysis of their conversation supports the view that prospective teachers are able to practice the Discourse of Teacher while learning mathematics content, this desired outcome may not always come to fruition. Other groups of prospective teachers may only engage in the Discourse of Student, depending on the official teacher in the classroom for explanations instead of asking/supplying explanations to each other. Mathematics educators need to encourage cooperative learning and provide opportunities for the prospective teacher to practice communicating his or her own mathematical thinking in order for the mathematics content classroom to serve as an apprenticeship into the mathematics community. The discourse of a college mathematics classroom is a place where prospective teachers can learn to talk the talk of teacher of mathematics, thus assembling

the identify kit Gee (1989) refers to in his definition of Discourse. However, further research needs to be done on ways to initiate the transition from learner to teacher of mathematics during the time prospective teachers are participating in the mathematics content classroom. For example, what types of tasks will encourage prospective teachers to share their mathematical thinking with each other? In what ways can mathematics educators foster collaboration and create an environment where participants feel safe to justify their answers to mathematical problems?

This research study was an attempt to understand how prospective teachers negotiate the transition from teacher to learner of mathematics. The socio-cultural theories of Vygotsky (1934/1986) assert that all learning takes place through the use of language within a cultural setting. Lerman (2001) suggested applying Vygotsky's mind-in-society-in-mind unit of analysis to the learning that takes place in the mathematics classroom. He proposed we view this learning within the framework of learner-in-mathematics-in-classroom-in-learner. The ongoing process of becoming a teacher of mathematics is imbedded in the process of learning mathematics, both of which take place within the individual engaging in the discourse of mathematics. The process moves back and forth within the individual, manifesting these two identities in the discourses of the subject, as illustrated in the conversation of these three pre-service teachers. The overlapping movement of these identities leaves us unable to extricate one from the other. Therefore, I propose we examine this transitory formation of identity using the learner-in-teacher-in-learner as our unit of analysis. In this manner, perhaps we can catch a glimpse of the ongoing process of becoming a teacher while prospective teachers are learning and participating in the mathematics community.

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Appendix

Interactivity Flow Chart for Triadic Communication

Line #	C	$C \leftrightarrow B$	B	$B \leftrightarrow J$	J	$J \leftrightarrow C$
88				↙		↘
89						↖↙
90						↗
91				↗		
92						↖↙
93						↗
94		↙		↘		
95						↗
96						↖
97						↗
98						↗
99					↑	↗
100					↑	↗
101					↑	↗
102						↗
103					↑	
104						↗