Including All Students in Meaningful Mathematics: The Story of Darrell

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Abstract

This article tells the story of Darrell, a lively, intelligent boy who has some learning disabilities. Although his parents are educators and are highly committed to advocating for Darrell, for most of his elementary school years, they were unable to succeed in providing Darrell a mathematics education equal to his regular education peers. Darrell’s math education included classrooms and special education settings that did not recognize his confusions, did not provide enough practice, and that focused on procedural knowledge while ignoring his visual and cognitive strengths. Because his parents recognized his strengths and worked with him at home, because a few of his teachers knew how to teach mathematics to a range of learners using NCTM standards-based mathematics curricula and because he became aware of how he learns mathematics best, Darrell began to progress towards the end of elementary school. Students with disabilities, like Darrell, all deserve the chance to participate in meaningful mathematics.

SUGGESTED CITATION:
Introduction

One of the assumptions of the Accessible Mathematics project (NSF HRD--0090070) is that all students, if given the proper support, can learn mathematics in ways that make sense to them. We are concerned about the continuing tendency to separate students with special needs from the general education class and deny them access to National Council of Teachers of Mathematics (NCTM) standards-based mathematics curricula.

Following is a true story of a co-worker’s son Darrell, a child with learning disabilities. It illustrates the unfair limitations placed on students with disabilities and why we feel so strongly about helping teachers develop inclusive mathematics communities in which all students are expected to learn with understanding. Despite Darrell’s family’s ready access to materials and their own experience as educators, their son was not offered a mathematics education equal to his peers for most of his elementary school years.

Introducing Darrell

Darrell is now an eighth grader who is curious, lively and has a variety of interests. Since he was a young child, he has been skilled in using his body, excelling at gross motor activities. He enjoyed playing outdoors, especially delighting in building structures, climbing trees, and making pulley systems to raise objects into trees. His parents encouraged his interests and did not pressure him to do small muscle activities that he did not choose on his own such as jigsaw puzzles or writing numbers and letters.

Early Schooling

For first through third grade, Darrell attended a private school that focused on project-based learning. In first grade, Darrell was the youngest child in a combination grade 1-2 class. The other students already knew their addition facts. Darrell had not been exposed to math facts before, and so early on he described math as “a foreign language from another universe.” Although the class counted and compared objects, played games, kept score, made calendars and followed recipes, the teacher did not have a scope and sequence for building computation skills and concepts in a structured way. She did not have a plan for addressing the confusions of individual students. Darrell’s family supported him at home by providing practice with board games and hands-on math activities. However, when he did not progress in learning the addition facts or addition and subtraction, his parents became concerned.

Darrell’s mother reports that in second grade, there was little mathematics instruction, and it was not well organized. Instruction centered on a month-long theme; there might be no structured math instruction during that time. When the teacher sometimes included math practice in morning circle activities that involved physical exercises such as throwing balls or marching in a circle while counting by different numbers, Darrell participated without understanding what was being taught. For example, his mother remembers that when she visited the school, the class was doing the 7’s table in a large group. “They would do 7, 14, 21, 28…I remember watching him mouthing the numbers.”

While Darrell was able to learn snowboarding, swimming, and rock climbing by watching and imitating and persistent practice, this way of learning was not helping him understand numbers. At first his
parents were patient. “We gave him time. He was unfolding slowly. We trusted that we would get to the place of things clicking.”

However, when his parents didn’t see progress by the end of second grade, they hired a private special education tutor three mornings a week during the summer. After six weeks, the tutor recommended an evaluation; she had noticed that Darrell was not able to maintain recall of number facts and vocabulary.

The local public school district, as required by law, was in charge of the evaluation. Their evaluation process took from September to November of Darrell’s third grade year. His parents received the report at the end of December. The subsequent meeting with school staff devastated the family. The school psychologist focused exclusively on Darrell’s weaknesses learning facts and procedures without acknowledging the spatial strengths that his family knew he had.

The third grade math work consisted of carrying, borrowing, measuring, and memorizing times tables. The third grade class was large, and the teacher usually presented information orally. His parents questioned the appropriateness of the school for Darrell. His mother recalled, “I knew we were in the wrong class. I didn’t know what to do. Do we pull him out in January? We invested a lot of energy into making school work and it wasn’t working.” Darrell’s mother started doing arithmetic with him at home using a visual approach because hearing about numbers was not helping him learn. She used models such as connecting cubes in strips of ten and 100 charts that he could touch as he counted on by tens and ones and later use as mental models to organize his thinking.

Figure 1: 100 Chart and Cube Strips
Tutoring: Visualizing Number

After some weeks of seeing Darrell make little progress in school but exhibit some good understanding at home, his parents took him out of school for an intensive tutoring program that emphasized visualization in both language and mathematics using activities and strategies from *On Cloud Nine* (Bell and Tuley, 2002). They hoped this visual and kinesthetic approach would help him build his computation skills. For the first time outside of the family, Darrell felt safe enough to reveal his thinking and to present his questions and confusions. He confessed that he was “faking” during math time in school. He sorted digits of numbers into “hundreds”, “ten's” and “ones” as directed by the teacher without knowing how that was useful in addition or subtraction. Using kinesthetic and visual strategies provided by the tutor, such as walking along various segments of a floor-length number line, Darrell began to understand the sequence and patterns of the tens and ones in the number system. His mother learned the activities so she could practice with him at home.

After six weeks, the family did not have the financial resources to continue paying for full-time private tutoring. Drawing on evidence from the program that twelve weeks was the recommended interval to consolidate progress, they requested that the local public system pay for six more weeks of the tutoring as in the long run it would save them money. The school system refused to pay. Darrell returned to his school full-time for the remainder of the school year. Darrell’s parents continued to supplement his school work by paying for two tutoring sessions a week until the end of the school year, and his mother continued to help him practice with the tutoring materials.

Entering the Public School

At the end of the year, Darrell’s parents enrolled him in the public school to repeat third grade. However, the third grade teacher was out on maternity leave, and the substitute teacher was not able to meet Darrell’s needs. Instead of using the school’s NCTM standards-based curriculum, *Mathland* (Brummett, Westley, McDonald, and Charles, 1995) with the rest of the class, Darrell and a student with developmental disabilities who needed a life skills curriculum were pulled out for math and taught by an aide, supervised by a first-year special education teacher. According to his mother, math instruction was a “potpourri” with little coherence. In hindsight, Darrell’s parents wished they had questioned this arrangement, but at the time they were glad that he was getting special education services and trusted the system.

Darrell’s mother continued working with him at home, using some of the same models that were part of the private tutoring sessions the previous year. She worked with him on putting 10’s together to build a number line with interlocking cubes, telling time, and learning the value of coins. She would play games with him, such as “I’m thinking of sixteen cents. What combinations of coins might there be?” She reports that by working with 100 charts and number lines, he was solidifying his understanding of number relationships and repeating patterns in the number system. However, when she asked the aide if she would use these materials in school, she was reprimanded for not trusting the school’s judgment.

Meeting Darrell’s Needs: Flexible Grouping in Fourth Grade

Darrell and his family were delighted that fourth grade turned out to be a wonderful school year for him, and a year in which he began to learn mathematics with understand-
Successful strategies for teaching students with disabilities in mathematics

- Expect all students to learn meaningful mathematics. Empower students by helping them to recognize what they know and what is difficult for them. Encourage students to be specific about what they need help with.
- Allow enough time for mathematics instruction (at least an hour a day) so that students can receive sufficient practice and spend enough time solving problems at their own pace.
- Provide additional time when possible for math routines or other practice activities, such as related problems or solving problems using multiples of 10. For example,
  - Offer regular opportunities for small groups of students to work with the teacher to learn a particular concept that is difficult for them.
  - Teach games or activities to students with special needs as a preview before they are done in class. Adapt games for different levels of difficulty. Use games and activities from earlier grades.
- Use visual representations such as arrays and number lines to make the mathematics accessible for a range of students.
- Write out directions and other information as well as presenting it orally. For example, display summaries of the mathematical work the students have done, and make resources such as a number line, 100 chart, or fraction to percent table available to students.
- Provide frequent practice to help students visualize numbers and number relationships, such as placing numbers on a number line, using Ten Frames and arrays, and recalling arrangements of dots or geometric shapes.
- Use flexible grouping; sometimes students need time alone before they enter a group; sometimes students need to be grouped with other students who have similar needs and similar strengths; sometimes an activity lends itself well to groups with a range of abilities.
students to be honest about what help they needed and taught them how to ask for help. Every student contributed to discussions during math class and got regular feedback. His mother was pleased that the special education teacher used some of the models he had learned before, such as the number line, and organized the work so he could build on what he knew, such as teaching him strategies such as, doubles and then doubles +1. Darrell gained confidence and became a star in the group. Both teachers reported that he was becoming more familiar with addition facts and that he was developing and using strategies for solving computation problems. By the middle of the year, Darrell would sometimes ask the special education teacher if he could participate in math in the general education classroom, adding, “If I need you I’ll come ask you.” Both teachers were pleased to honor his request. In the general education class, the teacher developed projects with multiple entry points and options and used multiple strategies to explain concepts so that every child could be successful. For example, she introduced an array model for multiplication. Students could build arrays to illustrate the factors for smaller or larger numbers, according to what they needed. The array model was particularly suited for Darrell’s strength as a visual learner.

**Fifth Grade: Pullout Again**

In fifth grade, Darrell moved to the middle school. The district was in the first year of implementing an NCTM standards-based curriculum, *Investigations in Number, Data, and Space* (Russell et al., 1998). In September, before special education services were organized for the year, he brought home assignments from the general education math class about patterns in number relationships and factor pairs, and he appeared to understand what he was being asked to do. After a few weeks, the work from *Investigations* stopped coming home, and Darrell’s mother called the school to find out what happened. She was told, “He’s not ready for this curriculum because he needs to be more automatic with the multiplication facts before he can do *Investigations*.” The school refused his parents’ request for part-time support from the special education department in the general education math class. Darrell was placed with all of the 22 students from six fifth grade classes who were on Individualized Educational Plans (IEPs) for a broad range of needs. They met in the cafeteria with a first year special education teacher and four aides. Neither the teacher nor the aides had received any professional development in mathematics, and at least one joked about how she didn’t understand the math curriculum.

Every two or three weeks, Darrell’s mother would send another e-mail to the special education teacher and the classroom teacher to ask for a conference. She desperately wanted Darrell to be included in his classroom’s work with *Investigations*. Although the school said they would re-evaluate Darrell’s situation in a month, weeks went by with Darrell bringing home worksheet after worksheet with hundreds of drill activities. He would say, “I hate math. I’m with all the dumb kids.”

**Inclusion at Last**

In mid-December, Darrell’s parents were finally invited to a meeting at school. His parents were surprised that the meeting included the principal, vice principal, Director of Special Education, school psychologist, classroom teacher and special education teacher. Fortunately, his mother had done

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2 Children use the doubles facts (starting with the 10 doubles facts from 0+0 to 9+9) to solve problems. Doubles +1 or near doubles includes all combinations where one addend is 1 more than the other. The strategy is to double the smaller number & add 1.

3 An array is a rectangular arrangement of quantities in rows and columns, e.g. a dozen eggs, a six-pack of juice cartons.
her homework ahead of time and was not intimidated. She had read links from the *Investigations* curriculum’s website and articles about math education. At first the classroom teacher was defensive and against his joining her mathematics class. She said, “He doesn’t have math facts automatically, and my fifth grade class has been doing the new curriculum, *Investigations*, since the beginning of the year.” Darrell’s mother said, “I know. I have been asking that he be included in the *Investigations* curriculum. I am sad that he hasn’t gotten it.” The classroom teacher concluded: “I will take him but I have 19 children. He will get one-nineteenth of my attention.”

After Darrell settled into the classroom routine, he felt comfortable and said he wanted to be there. At first he had been anxious about joining the class for math, knowing that he had missed the foundation of the first half of the year, but he soon felt it was “okay” for him, despite the fact that the teacher included many timed quizzes and worksheet drills during class and taught only some of the *Investigations* activities. Darrell especially enjoyed the group work and the practice with math games. He also enjoyed being paired with a friend during math class. The friend was someone who was more successful at school mathematics, but he admired Darrell for his artistic and athletic abilities. The boys also shared an interest in science. They worked cooperatively and respectfully together during math class.

The school system eventually did put an aide in his classroom during math time two days a week. Although the aide was not trained in the curriculum, she was present while the teacher introduced the lesson and would check in with Darrell during the period. Darrell’s mother offered to work with him at home. Fortunately, because of his parent’s professional contacts, they had access to the teacher’s guide for the *Investigations* curriculum units. Each week the classroom teacher e-mailed the math lesson plans to Darrell’s mother so she would know in advance what the class was going to be doing and could plan how to support him.

**Growing in Math Understanding**

Darrell’s mother saw how the focus of the *Investigations* fractions unit on multiple representations of fractions and relationships among fractions, decimals, and percents fit with his ability to build on what he knew. (For example, since one fifth of 100 is 20, 1/5 = 20%; 2/5 is twice as much so 2/5 = 40%.) His accurate sense of proportion and his ability to reason helped him mentally “see” the comparative sizes of fractions.

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**Expect that all children can learn, and value each child’s contribution**

Too often in the past, students’ mathematical abilities were defined by labels based on deficits instead of by strengths. Assessment designed to identify what the student knows provides the teacher with a framework for planning instruction.


When his mother introduced a game to him at home, she introduced the concepts gradually, so that he could build a solid core of knowledge and incorporate new knowledge in relationship to what he already knew. For example, when they played an ordering fractions game, she limited the fractions to halves, thirds and fourths at first, adding sixths, eighths, twelfths, and then fifths and tenths in subsequent rounds. She also continued to reinforce his understanding of the number system by playing games from earlier grade levels of the *Investigations* curriculum such as Tens Go Fish (asking for the missing number to make a total of ten), Double Compare (comparing to decide which addition problem has the larger sum), and Capture 5 (moving from one number to another on a hundreds chart by adding and subtracting tens and ones). The extra time and practice she provided using a variety of visual models and representations solidified Darrell’s knowledge of fractions of 10 and 100, providing a foundation for percents and decimals.
Applying Mathematics in Meaningful Contexts

When engaged in activities that interested him, Darrell was able to learn and use mathematics. As a sixth grader, he passed a scuba diving certification test that many adults fail the first time and for which one can receive college credit. In order to pass the diving test he had to interpret data tables and pressure gauges. For example, in order to figure out the corresponding pressure group that indicates how long to wait until your next dive, he needed to record the deepest depth he went to and the minutes he was under water. Darrell wrote a narrative description after each dive and then recorded the calculations: how deep he went, how much air he used, and how long he stayed down.

Figure 3 Diving Log Excerpt

We saw a lot of bass big too, about almost long and some were 3 and 5 feel long. First Scott showed me how to open a muzzle, open end, feed it to the fish. It was really easy. One of the box hit my finger, but I had a glove on so it didn't hurt.
Darrell was successful with the diving test because he and his parents assumed he could learn, were able to figure out what helped him learn, and developed strategies to build on his strengths. They bought him the diving course textbook and the instructional video for the course months in advance. Darrell used the book for his independent reading assignments during the school year, and he took the quiz at the end of each chapter. Knowing that he learns well from observation, he watched the video over and over. The diving class he attended to help him prepare was small, with four students and two teachers, so that Darrell got individual attention. His parents bought him a diving log with pictures that corresponded to the information required. He asked his mother to help him estimate numbers between whole thousands on the pressure gauge. (The gauge has lines with some numbers labeled as on a thermometer, and other lines in between for which the numbers need to be interpolated). He completed worksheets to practice recording the information about his dives and interpreting the conversion tables. To help him add up his cumulative underwater time, his mother used a laminated clock face picture and coached him through counting by fives. Darrell joined a diving club to be able to dive regularly. In order to pay for renting the diving gear and purchasing his

Importance of teaching mathematics for understanding instead of only procedural knowledge

As in Darrell’s case, relying on direct skill instruction in service of automaticity only emphasizes students’ memory deficits and leaves them without mental models and strategies that they can build on. On the other hand, when it is expected that the student can and will make sense of the mathematics, all students can build and retain mathematical knowledge at their own pace. Students are rewarded by their own increased understanding and skills, whether it allows them to collaborate more actively in math class, or apply them to contexts and activities that are especially meaningful to them outside of school, such as Darrell did with diving lessons.

own tank, he mowed a neighbor’s lawn. He recorded his expenses and earnings, using a calculator to find totals.

Further supporting his application of mathematical thinking was Darrell’s work with his dad, building a wooden boat together. Through this experience Darrell gained competency in measurement and estimation. His dad showed him where all the measurements were on the plan of the boat. His dad did the actual measuring, but Darrell read all of the numbers to him.

**Darrell’s Reflections on his Learning**

Darrell recognized his need to take his time, to not compare himself to other students and to see more than one computation strategy in order to find one that made sense to him. He saw his strengths in estimation of visual distances and seeing proportionally and his weakness in number work not based in a context.

The following excerpt from an interview that his mother did with him at the end of fifth grade illustrates Darrell’s growing ability to reflect on his school experience and to understand how he learns mathematics:

Mom: How did math class go this year?
Darrell: Ok, a little bit better than being in the separate math class.
Mom: What went well?
Darrell: Doing group work and playing games.
Mom: What could have gone better?
Darrell: It would have been better if the teacher were familiar with how Investigations \ worked. The teacher was good at doing math but not teaching math.
Mom: Describe a good math class.
Darrell: The class wouldn’t be too big or too small. The kids would be at my level, or a little better than me in math so I could learn from them. Tools available when I need them, for example, fraction strips, equivalency charts, calculators; no time pressure or math drills; a good teacher.
Mom: What things in math are you good at?
Darrell: Estimating length. Reading thermometers when the lines aren’t too close. Drawing humans and animals in proportion.
Mom: What things in math are hard for you?
Mom: What are ways that a good teacher could help you learn in math?
Darrell: Don’t rush me. Teach me slowly step by step. Don’t panic if I don’t know the times tables Don’t get angry. Don’t say ”You should know this by now. We shouldn’t spend time doing this.”
The teacher knows how to do math well and teach math really well. The teacher would be able to explain things in different ways and show me strategies I can use.
Mom: What are ways you have learned to help yourself?
Darrell: On tests, I need to be calm. Like on my diving test, the final exam, was a 50 question test, I got 4 wrong. I took 45 minutes longer than anyone else: the teacher knew it was ok. I thought: ”Darrell, just be calm, do one at a time.”
Don’t compare my work to others.
Go really slowly and don’t try to keep up with other kids.

**Sixth and Seventh Grades**

In sixth and seventh grades, Darrell participated from the beginning of the year in the *Connected Mathematics Program* (Lappan et al., 1998) as part of the general education class. He had math practice in a small group twice a week with a special education teacher, and his mother continued to support him at home. His teachers were positive about him, and in turn, Darrell liked them. He is able to make sense out of problems when he is given time and encour-
Importance of using multiple entry points, visual tools

Allowing for participation by students in inclusive classrooms includes addressing a range of learning styles and strengths. Whether the focus is on whole number, fractions, data, or geometry, most students need information presented visually on wall posters and written on the board or chart paper as well as hearing it described. Offering visual and kinesthetic models and representations for students to work with can provide multiple entry points so that all students, whether they solve the problem mentally, in writing, or using a chart or object to organize their counting, can work on similar problems from the curriculum and use a shared model for clarity in presenting their own solutions. Students can refer to familiar charts and models when planning together or explaining their work.

To compare fractions and solve problems in context, he often drew diagrams:

**Figure 5 Comparing Fraction Sums from Connected Mathematics Program, Bits and Pieces II p.50 #6**

Although Darrell has gaps in his mathematical understanding, such as remembering times tables and sequencing steps for procedures for adding and subtracting fractions, he also has areas of strengths in geometry, measurement, and comparing fractions, and he has interests outside of school that motivate him to learn mathematics. After varied experience with mathematics instruction, Darrell has developed insight into how he best learns mathematics, and he is aware of the difference between teachers who are prepared to work with him and those who are not.

**Implications**

Darrell’s story illustrates how students with special needs can be marginalized and seen only through their deficits. Because of their limited opportunities for professional development in mathematics, most of Darrell’s teachers were not able to identify and use his strengths. Some of the classroom teachers and many of the special education teachers who were in charge of Darrell’s mathematics instruction had not received any professional development in the NCTM standards-based curriculum the school had adopted. Special education aides who were in charge of his math instruction, had neither taken courses in math education nor had experience teaching mathematics. Except in the fourth grade, the special education teachers and the classroom teachers did not plan together to ensure that Darrell was receiving a coordinated math program. Most of his teachers emphasized automatic recall of facts to the exclusion of mathe-
matical reasoning and understanding. They expected that facts and procedures must be learned before students were able to attempt problems in context. Because of this procedural teaching, they were not aware of Darrell’s strengths in mathematics. They did not use his visual spatial sense to build his understanding, nor did they provide contexts that made sense to him. For the most part he did not have access to his classmates from whom he might learn and share strategies for problem-solving.

What Worked for Darrell

During his successful fourth grade year, on the other hand, Darrell had a special education teacher and a classroom teacher who were knowledgeable in how to teach math for understanding, who provided multiple entry points and carefully sequenced lessons, who worked together as a team, encouraged inclusion, and valued each student’s contributions. In the second half of fifth grade, and in sixth, and seventh grades, Darrell has had NCTM standards based curricular programs and his mother has been able to provide support consistent with the goals and pedagogy of the curricula.

Darrell is fortunate to have several advantages. His parents are educators who were able to support him through their knowledge of mathematics education and through resources they obtained from colleagues, and who were able to advocate for him to receive an appropriate program. In addition, he has multiple strengths, including an ability to know how he learns and when to ask for help and persistence to engage fully in his many interests. These characteristics along with his parents’ strong support sustained his self-esteem.

The need for training for special education teachers in teaching mathematics

The research about inclusion points to the importance of familiarity with content knowledge by all of the teachers involved in the child’s education. In order for special education teachers to plan for their students and prepare them to participate more fully in the general education math class, they need a deep understanding of the mathematical and pedagogical expectations of the adopted curriculum. However, special educators’ pre-service training and work experience, strong in assessment, adaptations and teaching for language development, seldom include developing strong mathematics curricular and content knowledge or exposure to NCTM standards. Thus many special educators lack the confidence, conceptual depth, and instructional experience required when using an NCTM standards-based math curriculum (Woodward and Montague, 2002; Maccini and Gagnon, 2002).

When he was finally included in the general education classroom, his classmates saw him as capable and skilled in many areas.

What happens to the children whose strengths may not be as easily apparent and who do not have family members who can be advocates and tutors? These are the questions that concern us as we work with teachers to foster inclusive mathematics communities that respect all learners.

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**Need for communication and collaboration among special education and classroom teachers**

Schools need to provide structures and support to foster collaboration so that teachers can review children's progress on a regular basis and can plan together about how to prepare and involve a range of students in developing mathematical knowledge at their own pace. The teachers can learn from each other's perspectives and knowledge, such as a special educator's experiences of useful activities from earlier grades, assessment, and remediation strategies.


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[http://www.math.msu.edu/cmp](http://www.math.msu.edu/cmp)


Notes:

Figure 2: from Name That Portion, Student Sheet 24, Russell, S. J., Tierney, C., Mokros, J. and Economopoulos, K. (2004). *Investigations in number, data, and space*. Glenview, IL: Pearson Scott Foresman. Used by permission

Figure 4: from *Connected Mathematics Bits and Pieces II* 1998 by Michigan State University Glenda Lappan, James T. Fey, William M. Fitzgerald, Susan N. Friel, & Elizabeth D. Phillips. Published by Pearson Education, Inc., publishing as Pearson Prentice Hall. Used by permission

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