Mentoring is proposed as an effective strategy for helping novice teachers develop professional knowledge (Odell and Huling, 2000). However, a recent analysis of mentor-novice conversations in the United States about teaching reveals that they may not be important to the development of teachers' reform-minded practice (Wang, Strong & Odell, 2004). While this might be the case involving student teachers or novice teachers, it might not necessarily reflect the nature of reflective conversations between more experienced mentees and their colleague-mentors, especially if the team is teaching and learning in a shared context. This case study explores the challenges and opportunities that peer mentoring (mentoring between two teachers with some degree of experience) create for learning to teach mathematics in the context of urban diverse school context.

Several theoretical assumptions guide the analysis of this case. First, mentoring in the case is seen as situated learning to teach practice, as literature on learning to teach assumes, much of what teachers need to know about learning must be learned in their teaching situation (Feimen-Nemser & Remillard, 1996) and pedagogical knowledge is situated in and grows out of that context (Putnam & Borko, 2000).

The second assumption is that learning to teach is a constructivist process in which novices build their own knowledge as they learn to teach. In addition, learning about teaching from focused, collaborative reflection on authentic experiences is another important assumption of this case.

The last important assumption is related to the content knowledge of both the mentor and mentee. In order to effectively mentor towards mathematics teaching reform, a teacher must develop content knowledge and conceptual understanding of underlying mathematical ideas (Ball, 1998). Thus, his or her mentor should have content knowledge and experience with key concepts and connections among the mathematics they teach (Manouchehri, 1997).

In short, teachers' knowledge and beliefs about effective teaching do not form in a vacuum. They form in and with teaching behaviors (Jensen, 1993). Collaboration with and assistance from a more experienced teacher and/or other experts may help the mentee to internalize teaching skills and knowledge including content knowledge (Feimen-Nemser & Remillard, 1996).

Methodology of the Study
Participants and context
The two teachers involved in this peer mentoring relationship have been collaborating intermittently over the past three years. They both work in the same professional development school attached to a state university. The school is in an at-risk, urban setting, where the majority of students speak English as a second language. The mentor teacher in this case is a nationally certified teacher with twenty-five years of experience in teaching fourth and fifth grade mathematics and has received many hours of additional training in mathematics and mathematical pedagogy as part of the school district's nationally-funded reform efforts, and have been designated as a "teacher leader" in mathematics and science for the school district.

The "novice" teacher, referred to as "Pascal" here, is a teacher in his third year of teaching. Prior to having his own classroom, he was part of an 18-month teacher-training program at the same school. He is novice only in the respect that he recently volunteered to be reassigned from teaching second grade to teaching fourth grade within a week, as a consequence of the school district's reorganization of teaching staff.

Pascal wanted to try a more open-ended approach to teaching mathematics that he had used in his second grade class. He had guided his students to investigate big ideas in mathematics and had supported his students to use actively their own experiences and a variety of manipulative materials in developing their conceptual understanding. However, the previous teacher in his fourth grade class had been using a mathematics program that focused on a computation-based instruction. Pascal is unfamiliar with content and strategies for mathematics instruction in the
fourth grade and uncertain about how best to proceed with his mathematics instruction in a way that he was committed to teaching. His problems in this transition reflect some of the challenges in mathematics instruction across the United States. Although state and local objectives for mathematics are based on the national standards, methods of mathematics instruction and the texts used to support them vary widely across the school district and even within the school.

In addition, not all the teachers in the school supported such instruction, nor is it encouraged by the school because of the district's emphasis on student achievement as measured by some local standardized tests that focus largely on computational skills. Therefore, Pascal asked me for help with his math instruction in the new classroom as I was also committed to reform-based mathematics practices that ask students to spend time exploring problems in depth, to find more than one solution to problems that they work on, and to develop alternative strategies and approaches to problem solving. With our shared beliefs of teaching, we started our mentor-mentee relationship. We met weekly to talk about ideas and challenges he was experiencing with mathematics instruction.

Data and analysis
I kept field notes of my observations of teacher, students, and interactions in Pascal's classroom, our subsequent conversations, and our reflections around the progress of his lessons and his thinking about the mathematics involved. I also used student work related to Pascal's lessons. As I started to look at these data and my own reflections over this period, I began to see certain recurring issues important to effective mathematics instruction in Pascal's concerns and in our conversations and challenges for the mentor/mentee conversations.

The following sections will describe mathematics lessons taught by Pascal and two follow-up conversations, followed by an analysis of how the content knowledge in mathematics of both the mentor and mentee influences the mentoring process and classroom instruction. It concludes with a description of the ways in which peer-mentoring conversations affect mentee and mentor understandings about how to teach mathematics effectively, the challenges and opportunities for learning to teach mathematics associated with these events, and further conclusions and implications for mentoring in mathematics.

The Case Description
For the first lesson in this unit, Pascal asked his students to think of things that are packaged in rectangular arrays, e.g. a six-pack of juice cans. As students came up with ideas for other things that came in arrays, Pascal recorded their ideas on a class chart, and began to introduce the vocabulary associated with arrays (dimensions, columns, rows). At this point, a discussion occurred among students about what the proper order was to list the dimensions of each array. For an array with 2 rows of 3, for example, as shown in Figure 1 below, would the dimensions be written 2 X 3 or 3 X 2? Did it make any difference?

Another example from the discussion is whether the array with 2 rows of 3 as shown in figure 2 was the same as an array with 3 rows of 2? Does the direction of the array make a difference?

Pascal realized during the lesson that he himself did not have a clear idea of what the acceptable order was for listing dimensions. He continued the lesson with the students volunteering their ideas about which number should go first, and why. The lesson ended with Pascal assuring the students that he would ask other teachers in the building about the recording dimensions of arrays.

Following this lesson, Pascal and I met to talk about the dimension-recording challenge. The examples in the teach-
er's guide indicated that the number for the rows went first and the columns second, but the teacher's guide failed to explain why that was so or if indeed the dimensions were always recorded in a certain order.

I asked Pascal what kinds of thinking his students had about the way in which they should be recorded so that I could have an idea of how his students were thinking and how Pascal was reacting to their ideas. I was also interested in how comfortable Pascal and his students were with the struggle for building their own understandings. In addition, I wanted to model how to think about the beginning point for an investigation into a mathematical concept based on the existing ideas and experiences of students.

Pascal shared with me that the class was pretty evenly split between those who thought that rows should go first, and those that thought the columns should. However, their reasoning simply stemmed from which method they perceived as being "easier" to see or record. I pointed out that since much of the learning in elementary mathematics forms the basis of their understandings of more complex mathematical ideas later on, we should think about what bigger ideas might be related to this simple question. Together, Pascal and I began to wonder about the dimensions question, and how it might relate to other mathematical ideas.

We came up with the idea that the established procedure might have something to do with how one lists the coordinates for points on a grid or graph. For example, with the X, or horizontal coordinate, coming first followed by the Y, or vertical coordinate. This idea, however, led to some further confusion because we observed that when students count the rows of an array, they count them vertically, from top to bottom or vice versa. When dealing with columns, they count them horizontally from left to right. These observations seemed in some ways to contradict our coordinate theory so we decide to seek further support from other professionals.

We proposed this issue to the mathematics specialist at our school and she had not yet thoughtfully considered the dimension-recording question before either. She looked at several other texts and they all indicated that the number of rows went first while the number of columns second without explaining why that procedure was important. She offered to contact one of the mathematics professors at the university for a more complete answer.

The professor told us the correct procedure again and his reasoning reflected the ideas of Pascal's fourth grade students. The rows should go first because it is easier for the students to see the groups of items that way.

Both Pascal and I were somewhat dissatisfied with this result to our inquiry. We had an idea that there was a reason for this convention related to more sophisticated mathematical ideas. However, we were a little disappointed that neither the information in the text nor the advice from the experts addressed the issue. As is often the case in instructional inquiry, our question is still at the back our consciousness, waiting for some future lesson or discussion to shed light on the challenge.

Before his next lesson, Pascal came to talk to me about his concerns about facilitating discussion of arrays. His instructional goals were to use the arrays for different numbers to reinforce the area model of multiplication so that his students could learn how to find factors of a numbers.

He also planned to introduce prime numbers as those with only one pair of factors. The lesson started with the task for groups of students to make all possible arrays for a certain number using cubes and graph paper. Each group completed an array poster for two different numbers, posting each around the room as they finished. Once all the posters were displayed, Pascal began to ask the students to observe the posters and share any thoughts they had about them, in order to discuss ideas about arrays. Pascal was unsure how best to proceed during the discussion, so he asked me to particularly observe his questioning during the discussion.

As I entered the room, the class was already working on posters of arrays for a given number. Concerned that each group would make several arrays for their number, Pascal had directed the students to choose only even numbers for this activity. Each group completed two posters and posted each around the room as they finished.

Once all the posters were displayed, Pascal began to ask the students to observe the posters and share any thoughts they had about them. The students seemed a little confused about what they might be expected to observe. So Pascal tried asking some narrow questions to help get them started. "Which number had the most arrays? The fewest?
Why do you think that happened?” The children began to share how they were seeing some patterns in the relationship between the number of arrays and the number of factors for a given number. However, the discussion began to falter. Pascal looked over to where I was sitting and shrugged his shoulders.

I began to ask the class some questions about the shapes of their arrays. The students noticed that some arrays were rectangles and some were squares. This led into a discussion comparing the two array shapes and looking at how the dimensions of the square arrays were different from the rectangular dimensions. The students then identified the numbers that had square arrays, charted those numbers and the dimensions of their square array, looked for patterns on the chart, and used the list to predict other square numbers. Even though their array posters had been limited to even numbers, the students were able to predict which odd numbers would also have square arrays.

Pascal later shared that he could not think of any further questions that might help his students toward the objectives he had for this lesson. I asked him why he had limited the numbers for this activity to even numbers only as I assumed that he had underestimated the possibilities of the array activity in terms of introducing square numbers and prime numbers in addition to providing visual models for multiplication facts and defining factors. Pascal told me that he was trying to make the activity easier and more manageable for the students. He thought that even numbers would ensure that all groups would have at least two arrays on their posters.

We then began to discuss the effect that this choice had on his lesson. In particular, how making things “easier” and more efficient might have also limited students’ discoveries. For example, the class discussion might have been enriched by students’ comparisons of the arrays for odd and even numbers. In the lesson discussion of square numbers, several students initially assumed that only even numbers could be square because only even squares were represented in the array posters. In addition, no investigation of prime numbers (except for 2) was possible using arrays for only even numbers.

In debriefing the lesson, Pascal began to realize that having students include odd numbers for their posters would have enabled a richer discussion and allowed the students to make some important discoveries. He was able to see from my conversation how the discussion with his class might have progressed with a greater variety of array examples to work with.

Building on the talk, we discussed the kind of vocabulary the students were using to describe the arrays: factor, multiple, product, height, width, rows, and columns. As we talked about these terms, we realized that only some of the comments from the students had used these terms correctly. Pascal recognized that future lessons would need to clarify and solidify students’ understandings of these terms.

The Case Analysis

These mathematics and mentor-novice discussions illustrate several challenges and learning opportunities for novice, mentor, and student learners as we strive to move our teaching toward reform-based instruction. In particular, they suggested three pairs of challenges and opportunities.

The first lesson and follow-up mentor-mentee conversations showed that a deeper understanding of mathematics of both mentor and mentee would greatly affect the nature and direction of the conversations. It became apparent in this case that neither the mentor nor the mentee was confident in the level of their consideration of the dimension-recording question. This situation actually pushed us to explore further the meaning of this mathematics idea and its connection to other mathematics ideas that students need to learn.

However, the contexts of elementary teachers who have to teach all the subjects in limited periods of time left both mentee and mentors two questions that might not necessarily result in the same answers from both parties of mentoring relationship: “How much content knowledge is enough to teach effectively?” and “How much content knowledge is enough to mentor effectively in a content area?”

As Pascal and I discussed this issue, he was becoming impatient with trying to reason the correct procedure and would preferred that I just told him, if I could, what he should say to his class. In contrast, I was thinking about how we might enter the process of inquiry so Pascal might get a feel
for how to build his own content and pedagogical knowledge.

These events showed that good mentoring practice relies on the mentor's proper judgment of what the mentee was thinking regarding content knowledge and relevant pedagogy. However, to develop such a judgment is also a challenge for the mentor. Thus, instead of following prescribed behaviors of mentoring and offering preset advice, the mentor needs to learn how to ask questions for the mentee in order to develop such a judgment.

During our discussion about Pascal's second lesson, it first occurred to me that he had underestimated the possibilities of the array activity in terms of introducing square numbers, prime numbers, and factors. However, further inquiry onto his thinking showed that Pascal was trying to make things easier for his students, which did not always result in the most effective instruction. With this understanding as a base, I was able to build my suggestions on my understanding of his own knowledge of what and how his students are learning and Pascal will be able to develop useful ideas for pushing students to move further in their learning about mathematics with my suggestions.

Another important idea to be considered is how not exactly knowing the "right answer" might actually have enhanced the learning experience for Pascal's students. For example, in the first lesson, Pascal did not feel he was able to tell them the correct procedure during the lesson. This situation actually gave the students the opportunity to consider the question for themselves. Although he did eventually share with them the information he had gathered about the established procedure, the process of asking students to come up with an explanation and defend it might have been more beneficial to building their own understanding than simply explaining the procedure.

Conclusion and Implications

This case suggested that effective teaching, learning to teach, and mentoring share a similar feature. All three situations require the use of strategies that allow the learner to establish his/her own reflective practices for building further understanding. In our case, Pascal's students began to build some ideas about number related to their observations of arrays. Pascal began to build some understanding about how to use questioning to facilitate student learning and how to reflect on his own practice and mathematical content knowledge to investigate instructional challenges. I was able to develop my ideas about the importance of facilitating and modeling reflective practice in mentoring toward reform-based practice.

This case also provides an example of a peer mentoring relationship that encouraged inquiry and reflective practice. Pascal and I both gained insight into the importance of content area knowledge in mathematics instruction, and we also saw the value of proceeding with our own instructional inquiry.

The implications of this study is that the design of teacher mentoring in mathematics education should include considerations of student learning, mentee learning and mentor learning in the context of teacher content knowledge and strategies for mentoring toward reform-based mathematics instruction.

REFERENCES


