Deciwire: *An inexpensive alternative for constructing linear representations of decimals*

JENNY YOUNG-LOVERIDGE and **JUDITH MILLS** describe how a simple teaching aid called a Deciwire may be used to help children understand decimals.

Much has been written about the value of concrete manipulatives for supporting students' mathematics learning (e.g., Fennell & Rowan, 2001; Pape & Tchoshanov, 2001). This is particularly true for the domain of fractions and decimals. The Australian mathematics curriculum stresses the importance of understanding, fluency, problem solving and reasoning as key proficiency strands that interact with the content strands (Australian Curriculum, Assessment and Reporting Authority [ACARA], 2011; Commonwealth of Australia, 2009). The curriculum document notes that in Years 3–6, students "require active experiences that allow them to construct key mathematical ideas, but also gradually move to using models, pictures and symbols to represent these ideas" (ACARA, p. 5). Understanding of decimal place-value is seen as critical for students' mathematics learning.

To understand decimals, students need first to have a solid understand of fractions as parts of a whole. Teaching about fractions is likely to begin with familiar fractions such as one-half, one-quarter, one-third, etc. In preparation for understanding decimals, students need to understand the idea that one-tenth is one piece of a whole that has been divided into 10 equal pieces.

In a recent issue of this journal, decimats were introduced as a way of helping students understand the relative sizes of

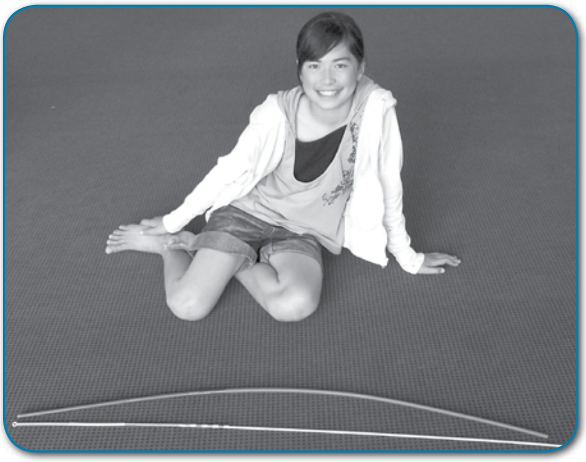

Figure 1. The decimal 0.39 represented on a bent decipipe and straightened curtain wire.

Figure 2. Components of deciwire materials for representing decimal quantities.

denominations: 0.1, 0.01, and 0.001 (Roche, 2010). Decimats consist of a rectangular-shaped piece of paper, divided into ten equal parts (two rows of five), with further subdivisions marked to enable each one-tenth part to be divided into ten one-hundredths, and the potential for each one-hundredth to be divided into ten one-thousandths. An understanding of equivalent fractions allows students to appreciate that one (whole) 'mat' is composed of ten 0.1 parts, one hundred 0.01 parts, or one thousand 0.001 parts. The decimat provides a powerful 'area' model for showing decimal quantities.

Decipipes (and Linear Arithmetic Blocks [LAB]) help students to appreciate the relative sizes of 0.1, 0.01, and 0.001, but they do so via a 'linear' representation (Helme & Stacey, 2000; Moody, 2010). Commercially available decipipes consist of plastic tubing of various lengths, cut to scale with 'pieces' of one, one-tenth, one-hundredth, together with metal washers that serve as the one-thousandth sized pieces (see Moody, 2010). To construct a particular quantity, the pieces are fed onto plastic rods ('joiners') that are one whole unit in length. Although decipipes are reasonably light and portable, they can be awkward to carry because of the length of the 'one whole' joiners (1.2 metres). Over time, the joiners can become bent, making comparisons between different quantities difficult because the pipes no longer align exactly (see Figure 1).

Deciwire was developed by Jenny as a cheaper, more easily accessible version of decipipes. Curtain wire was selected because it can be wound into circular shapes for easy storage in plastic bags, or unwound and laid out straight. Lengths of 1.2 metres were cut (consistent with commercially produced decipipes), and small threaded 'eyes' screwed into one end of the wires to prevent "pieces" from slipping off (see Figure 2). The one-tenth and one-hundredth pieces consist of brightly coloured drinking straws cut into lengths of 12 cm and 12 mm, respectively. The one-thousandth pieces consisted of either metal washers or cardboard squares hole-punched in the centre. Ten 'squares' of cardboard stacked together are 12 mm in depth, equivalent to the length of one one-hundredth 'piece' (see Figure 3). There is no one-whole 'piece', but bread tags or bulk-bin labels can be used to label bare deciwire as needed.

The advantage of straws is that they are easily replaced. Because the deciwire is

Figure 3. The decimal 0.478 constructed with deciwire materials using cardboard 'squares' to represent 0.001.

Figure 4. The decimal 0.39 represented with deciwire materials.

Figure 5. Children comparing decimals 0.8, 0.478, and 0.39 represented using deciwire materials.

inexpensive, more students can access the materials. Using different coloured straws to construct decimal quantities can helps students to appreciate the correspondence between the number of pieces of a certain size and the digit in a particular decimal place (see Figure 4). Using a variety of colours contrasts with decipipes, which use a single colour for one-tenth and one-hundredth pieces, making it more difficult to differentiate between each piece. Later a single colour may be used for the straw pieces to facilitate the transition from discrete to continuous quantity, as students develop a deeper understanding of length measurement processes.

Because the uni-dimensional nature of a linear model is easier to understand than a two-dimensional area model, we argue that a linear representation like deciwire (or decipipes) should be introduced to students prior to decimats. Providing both linear and area models of decimals gives students a basis for creating mental images when solving problems involving decimals. Deciwire (or decipipes) has the advantage that students can construct and re-construct quantities, whereas decimats cannot be easily re-constructed back to the original whole after cutting. The exchange of one unit for ten units of the next-smallest size (or vice versa) is also easier with deciwire and straw pieces than with decimats (see 0.8 – 0.39 below). A linear model may be easier for students to understand because of their familiarity with number line models (e.g, Bay, 2001).

Deciwire models are particularly useful when comparing the magnitude of decimal fractions, and can be used to examine (and dispel) students' misconceptions (Irwin, 2001; Moody, 2010; Roche, 2005). For example, some students believe that longer decimals are larger, whereas others think they are smaller. Some think putting a zero on the end of a decimal quantity makes it ten times larger, or that decimals are less than zero and are negative numbers. Others believe that a decimal is composed of two separate whole-number quantities separated by a "decorative" dot (Irwin, 2001).

Using deciwire to construct decimals shows the importance of first comparing the number of tenths to determine relative size. For example, in using deciwire and straws to put the following decimals in order of magnitude: 0.8, 0.39, 0.478, it should be clear that 0.8 is larger than the other two quantities (see Figure 5). Students who think longer decimals are larger would judge 0.478 as larger than 0.8 (perhaps reading 0.478 as "point four hundred and seventy-eight" which sounds larger than "point eight"). On the other hand, students who think shorter decimals are larger would judge 0.39 as larger than 0.478, again because they treat the decimal as a whole number. This understanding is particularly important for students at the Year 5 level, who need to "compare, order and represent decimals" (ACARA, 2011, p. 23).

Using deciwire and straws to understand decimals

Start by focusing on one half as 0.5 (a number most children know without understanding its meaning), and develop the understanding that 0.5 represents and five straw pieces comprise half a length of deciwire. Verify this with a calculator, dividing 1 by 2. Draw students' attention to the fact that the calculator display includes a zero, showing that the quantity 0.5 is between zero and one (not a negative number). According to Bobis, Mulligan and Lowrie (2009), using a calculator to check the process for constructing decimals and to reinforce place value is critical.

Explore with students what happens when you halve one half. Many of them will know that half of a half is one quarter, but not that half of 0.5 is 0.25. Ask how two and a half one-tenth pieces could be removed. Someone may suggest that one of the one-tenth pieces needs to be cut, but may not realise that, in the case of decimals, the only 'cutting' allowed is into ten equal pieces. This can be done by exchanging one of the one-tenth (0.1) pieces for ten pieces of one-hundredth (0.01) size, then halving those ten pieces and removing five of them (in addition to the two 0.1 pieces) to show 0.25 (one quarter) on the deciwire. Again, check with a calculator 0.5 divided by 2. Compare the 0.25 on the calculator with what is shown on the deciwire: two one-tenth (0.1) pieces and five one-hundredth (0.01) pieces.

Repeat the previous step by exploring with students what happens when one quarter is halved. This requires the exchange of one one-hundredth (0.01) piece for ten one-thousandth (0.001) pieces, then halving them to leave five one-thousandth pieces (0.005). Use the calculator to check that dividing 0.25 by 2 gives 0.125. Comparison of the calculator display with the deciwire model shows correspondence between the one one-tenth (0.1) piece, the two one-hundredth (0.01) pieces, and the five one-thousandth (0.001) pieces.

Using deciwire to model multiple solution strategies for decimal problems

Reform approaches to mathematics education stress the value of multiple solutions to problems as part of strengthening students' understanding of number properties. The curriculum document states that at Year 6, students should be able to "add and subtract decimals" (ACARA, p. 25). Deciwire can be used effectively to model a variety of strategies for solving problems with decimals. Four different strategies for solving 0.8 – 0.39 are described below. Start by constructing the two quantities (0.8 and 0.39) on deciwire then set them aside for later comparisons with various solution strategies.

Standard place-value partitioning

Start by constructing a second model of 0.8 on a separate deciwire. Remove three 0.1 pieces (onto a spare deciwire). To remove nine 0.01 pieces, one of the remaining five 0.1 pieces needs to be exchanged for ten 0.01 pieces, then nine of them can be removed (onto the spare deciwire with 0.3). This leaves 0.41. To check the process, compare the deciwire with 0.39 taken away, with the original 0.8 deciwire and the 0.41 left. The pieces making up the 0.41 should fit neatly between the 0.39 and the 0.8 pieces (because 0.41 is the difference between 0.8 and 0.39).

Reversibility

Start by constructing 0.39 then use addition to build it up to 0.8 (putting the additional pieces onto a spare deciwire). Adding one 0.01 piece takes 0.39 to 0.40. Adding four more 0.1 pieces takes 0.4 to 0.8. The spare deciwire should now hold 0.01 and 0.4, making a total of 0.4. Again the 0.41 on the spare deciwire can be compared with 0.8 put aside.

Rounding and compensating

Start by constructing a model of 0.8 using eight 0.1 pieces, then removing four of them to a spare deciwire. Because 0.4 has been

Figure 6. Comparison of 0.8 with 0.39, with the difference (0.41) shown between the other two quantities constructed with deciwire materials.

taken away instead of 0.39, compensation of the extra 0.01 taken needs to occur. This requires decomposition of one of the four 0.1 pieces taken away into ten 0.01 pieces, and one of these can then be moved back to join the remaining amount (0.4 + 0.01 = 0.41). Again, the answer can be compared with the 0.8 put aside (see Figure 6).

Equal additions

Start by constructing a model of 0.8 and one of 0.39. Add one 0.01 piece to each deciwire, resulting in 0.81 and 0.40. The resulting ten 0.01 pieces on the shorter line (0.40) can be exchanged for one 0.1 piece. It should then be easy to compare 0.81 with 0.4 and see that the difference is 0.41 (if necessary, the difference can be constructed on a third deciwire). This is by far the most efficient and elegant of the four possible strategies, but is not as familiar to many people. It should be noted that this strategy involves comparing two quantities, rather than operating on (i.e., adding to or subtracting from) one quantity.

Multiplication and division of decimal numbers

Deciwires can also be used to model multiplication (using repeated addition) and division (using repeated subtraction) of decimals. For example, 3 × 0.6 can be constructed using three deciwires initially, then redistributing the "pieces" from one deciwire to the other two, rounding one up to one whole.

We have found that students at the senior primary level and both pre-service and in-service teachers enjoy working with deciwire representations. They appreciate the way that the deciwire is flexible and easy to handle and to store, as well as being relatively cheap to produce. They also find that constructing the decimal quantities using deciwire helps to consolidate their understanding of decimals. As one of the children we worked with commented: "It just helps you when you are trying to do it in your head… These ones [deciwire], for some reason, are much easier than the other ones [decipipes] — it takes ages to sort them out — these take a few seconds. These ones are easier 'cause the straws are different colours so you know where it changes."

References

Anthony, G. & Walshaw, M. (2007). *Effective pedagogy in mathematics/pangarau: Best evidence synthesis iteration* [BES]. Wellington, New Zealand: Ministry of Education.

Australian Curriculum, Assessment and Reporting Authority (2011). *The Australian Curriculum: Mathematics*. Retrieved on 24 May 2011 from: http://www.australiancurriculum.edu.au/Mathematics/Curriculum/F-10

Bay, J. M. (2001). Developing number sense on the number line. *Mathematics Teaching in the Middle School, 6* (8), 448–451.

Bobis, J., Mulligan, J., & Lowire, T. (2009). *Mathematics for children: Challenging children to think mathematically* (3rd ed.). Frenchs Forest, NSW: Pearson Education Australia.

Brown, G. & Quinn, R. J. (2006). Algebra students' difficulty with fractions: An error analysis. *Australian Mathematic Teacher, 62* (4), 28–40.

Commonwealth of Australia (2009). *Shape of the Australian Curriculum: Mathematics*. Retrieved on 14 April 2011 from: http://www.acara.edu.au/verve/_resources/Australian_Curriculum_-_Maths.pdf

Ellerton, N. & Clements, K. (1994). Fractions: A weeping sore in mathematics education. *Set: Research information for teachers*, Number two, Item 10, 1–4.

Fennell, F. & Rowan, T. (2001). Representation: An important process for teaching and learning mathematics. *Teaching Children Mathematics, 7* (5), 288–292.

Helme, S. & Stacey, K. (2000). Can minimal support for teachers make a difference to students' understanding of decimals? *Mathematics Teacher Education and Development, 2*, 105–120.

Irwin, K. C. (2001). Using everyday knowledge of decimals to enhance understanding. *Journal for Research in Mathematics Education, 32* (4), 399–420.

Lamon, S. J. (2006). *Teaching fractions and ratios for understanding: Essential content knowledge and instructional strategies for teachers* (2nd ed.). Mahwah, N.J.: Erlbaum.

Lamon, S. (2007). Rational numbers and proportional reasoning: Towards a theoretical framework for research. In F. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 629–667). Reston, VA: NCTM.

Moody, B. (2010). Connecting the points: Cognitive conflict and decimal magnitude. In L. Sparrow, B. Kissane, & C. Hurst (Eds), *Shaping the future of mathematics education* (Proceedings of the annual conference of the Mathematics Education Research Group of Australasian, pp. 422–429). Fremantle, Australia: MERGA.

Norton, A. & Hackenberg, A. J. (2010). Continuing research on students' fraction schemes. In L. P. Steffe & J. Olive, *Children's fractional knowledge* (pp. 341–352). New York: Springer.

Pape, S. J. & Tochoshanov, M. A. (2001). The role of representation(s) in developing mathematical understanding. *Theory into Practice, 40* (2), 118–127.

Riddle, M. & Rodzwell, B. (2000). Fractions: What happens between kindergarten and the army? *Teaching Children Mathematics, 7* (4), 202–206.

Roche, A. (2005). Longer is larger — *Or is it? Australian Primary Mathematics Classroom, 10* (3), 11–16.

Roche, A. (2010). Decimats: Helping students make sense of decimal place value. *Australian Primary Mathematics Classroom, 15* (2), 4–10.

Steffe, L. P. & Olive, J. (2010). *Children's fractional knowledge*. New York: Springer.

Swan, P. & Marshall, L. (2010). Revisiting mathematics manipulative materials. *Australian Primary Mathematics Classroom, 15* (2) 13–19.

Thompson, C. S., & Walker, V. (1996). Connecting decimals and other mathematical content. *Teaching Children Mathematics, 2* (8), 496–502.

Thompson, P. W. (2010). Foreward. In L. P. Steffe & J. Olive, *Children's fractional knowledge* (pp. xi–xiv). New York: NY: Springer.

Usiskin, S. P. (2007). Some thoughts about fractions. *Mathematics Teaching in the Middle School*, 12 (7), 370–373.

Van Galen, F., Feijs, E., Figueiredo, N., Gravemeijer, K., van Herpen, E., & Keijzer, R. (2008). *Fractions, percentages, decimals and proportions: A learning-trajectory for grade 4, 5, and 6*. Rotterdam, The Netherlands: Sense Publishers.

Verschaffel, Greer, & Torbeyns, (2006). Numerical thinking. In A. Gutierrez & P. Boerno (Eds), *Handbook of research on the psychology of mathematics education: Past, present, and future* (pp. 51–82). Rotterdam, The Netherlands: Sense Publishers.

Wu, H. (2002). *Chapter 2: Fractions* (Draft). Retrieved 5 February 2007 from http://www.math.berkley.edu/~wu/

Zevenbergen, R., Dole, S., & Wright, R. J. (2004). *Teaching mathematics in primary schools*. Crows Nest, NSW: Allen & Unwin.

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