The Shoemaker's Knife

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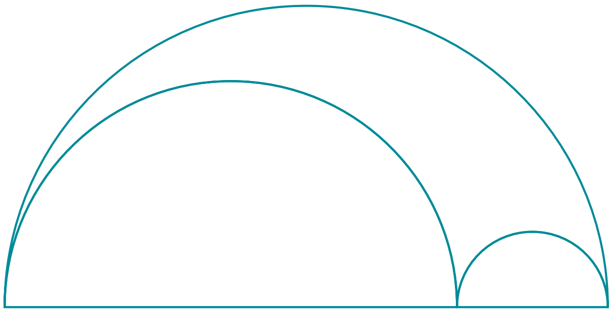
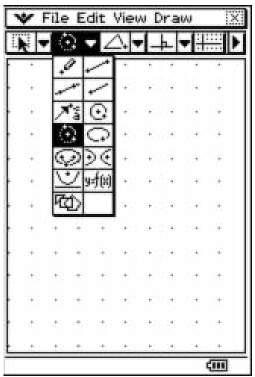
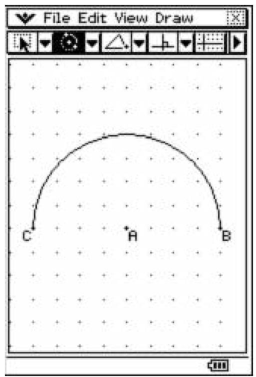
Archimedes, the famous Greek mathematician, lived from 287 BCE until approximately 212 BCE. He thought that the figure of two semi-circles on a straight line enclosed by a larger semi-circle resembled a shoemaker's knife. Archimedes called this figure an *arbelos* since arbelos is the Greek word for a shoemaker's knife. The arbelos has many interesting properties.

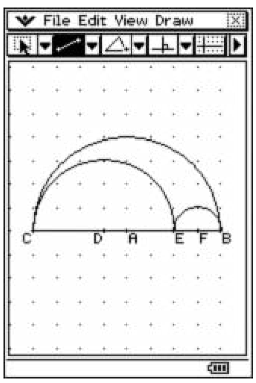


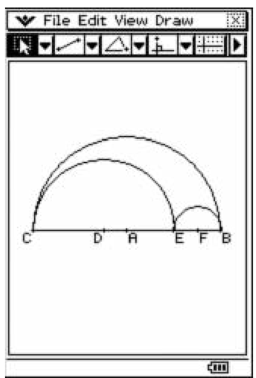
An arbelos can be drawn on a ClassPad calculator by constructing semi-circles. A semi-circle can be produced using the arc tool by tapping on the points labelled here as A, B and C.

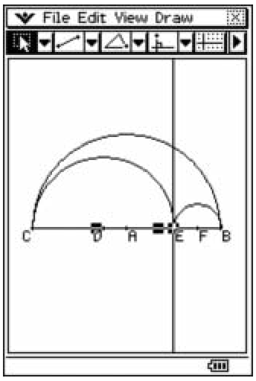


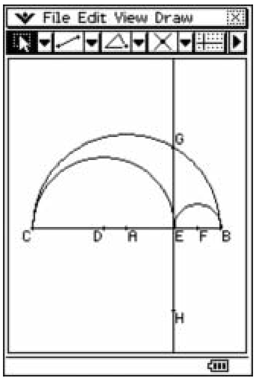


Having drawn three semi-circles in this way, a line segment may be drawn along the base to complete the figure. The Integer Grid is no longer required and may be turned off from the View menu.

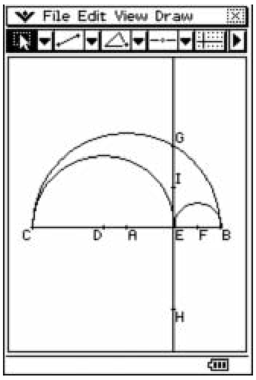


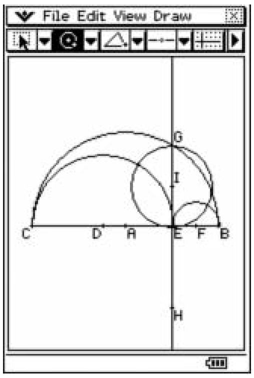




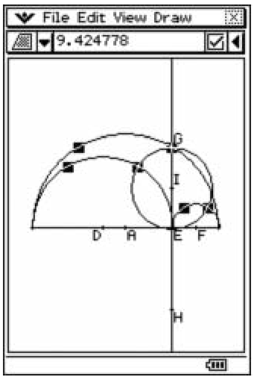


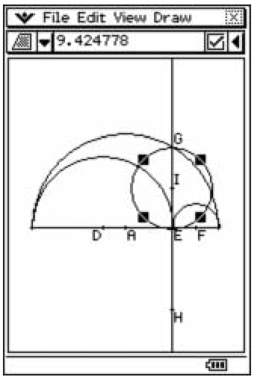
Archimedes proved that a circle could be drawn that had the same area as the arbelos. A circle like the one that Archimedes drew in the third century BCE, can be constructed on a 21st century calculator. To begin with, a perpendicular is drawn from the point where the two smaller semi-circles meet. The intersection of the perpendicular line and the large semi-circle is thus constructed



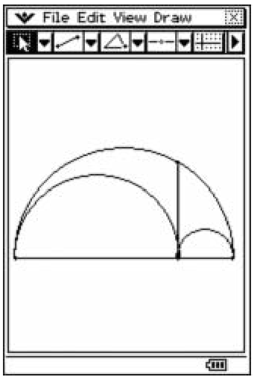


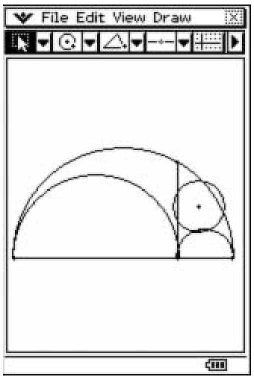
By finding the mid-point of EG (I), a circle can be constructed which has EG as the diameter.





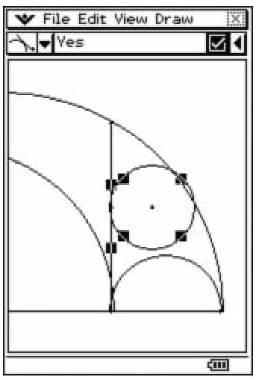
If all three semi-circles are selected the calculator can be made to display the area of the arbelos. When the circle is selected, its area can be seen to be the same as the arbelos, just as Archimedes predicted. In this construction, the arbelos and the circle each have an area of 9.424778 units squared.

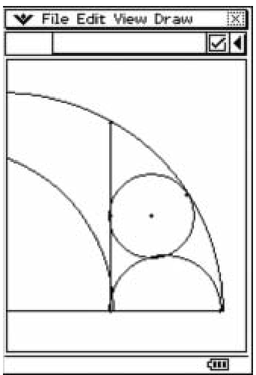




Archimedes also proved that "twin circles" could be inscribed in the arbelos on either side of the perpendicular line and tangent to it. This can also be demonstrated on the calculator. First the circle drawn previously is deleted. Then, for convenience, the perpendicular line is replaced with a line segment from E to G, and all the labels are hidden.

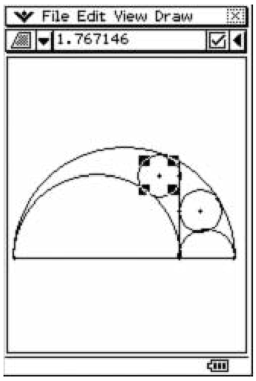
Beginning with the circle on the right of the perpendicular, a circle of approximately the correct size is drawn.

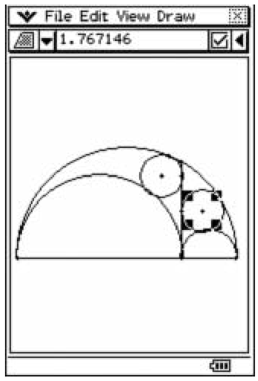




The circle can be made to just touch the perpendicular by using the tangent tool. Similarly, the circle can be made to just touch the small semi-circle and the outer semi-circle. It is helpful to Zoom In when doing these operations.

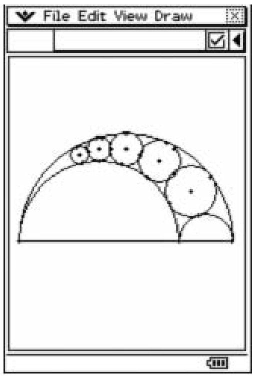
Using the same process, the other circle may be inscribed on the left of the perpendicular. When the circles are selected one at a time, the calculator displays the same area. In this particular construction, the twin circles each have an area of 1.767146 square units.

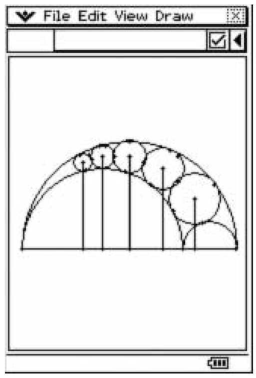




Half a millennium later, Pappus, another Greek mathematician, inscribed a succession of ever-decreasing circles inside the arbelos. His construction can also be produced on the calculator using the tangent tool.

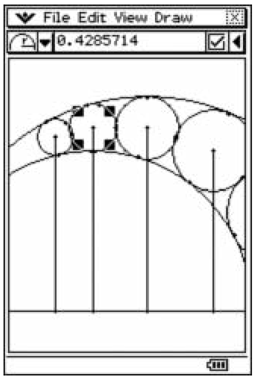
Pappus measured the heights of the centres of the circles from the base of the arbelos. He showed that the height of the *n*th circle above the base of the arbelos equaled *n* times its diameter.

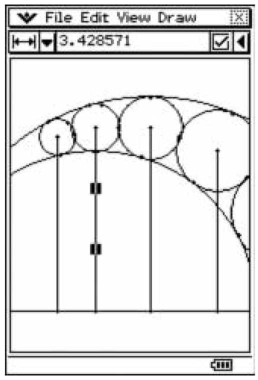


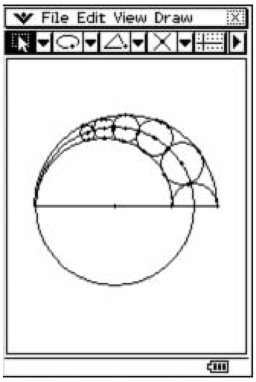


Measurements on the calculator agree with Pappus' findings. For example, the centre of the fourth circle in this construction is 3.428571 units above the base and it has a radius of 0.4275714 units.

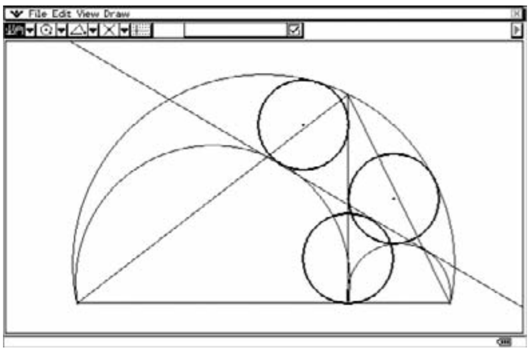
Interestingly, the centres of these circles follow an elliptical path.







After many thousands of years, the arbelos continues to fascinate mathematicians. In relatively recent times (1974), for example, it was discovered that the twin circles of Archimedes have a long lost sibling, and, as illustrated below, are in fact triplets!



References

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