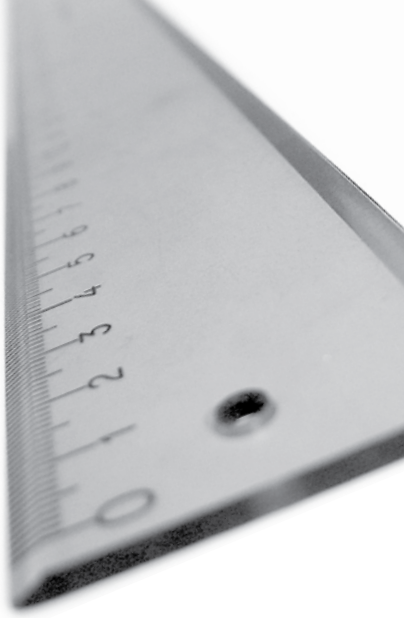
Assessing Children's Understanding of Length Measurement:

A focus on Three Key Concepts



"I need to teach length to my students but I don't really know where to start. I don't really know what's important for them to understand and what I should focus my teaching on."

HEIDI BUSH presents three different tasks that can be used to assess children's understanding of the length concepts of transitivity, use of identical units and iteration.

This comment was made by an experienced and highly effective teacher; it illustrates an issue faced by many teachers when approaching mathematics, and in this case, length measurement. What are the important concepts for students to understand? How can I develop this understanding and how can students demonstrate their understanding?

Three important measurement concepts for students to understand are transitive reasoning, use of identical units, and iteration. In any teaching and learning process it is important to acknowledge students' existing knowledge and focus teaching on moving students on from this point. Assessment tasks that provide insight into students' current understanding of or misconceptions about these concepts is, therefore, vitally important.

The tasks presented here were developed to assess students' understanding of transitive reasoning, identical units and iteration. The Early Numeracy Interview (Department of Education, Employment and Training, 2001) is acknowledged as an excellent tool for assessing students' mathematical understanding and the tasks presented here are structured and scripted to align with this clinical interview process. The insights gained from these tasks can be used to inform future teaching and learning.

Transitive reasoning

Transitive reasoning, or transitivity, is a concept based on comparison. For example, if three lines (A, B and C) are different lengths and A is longer than B which is longer than C, then we can also say that A is longer than C (Wilson & Osborne, 1992, p. 102). Understanding this idea can be challenging for young children and is a skill that develops over time. The following task may be used to determine children's understanding of transitive reasoning.

The Straws and the Barrier

(Modified from Wilson & Osborne, 1992, p. 102)

Materials required

- 1 piece of A4 card
- 20 cm straw glued in a vertical position on the left hand side of the card
- 10 cm straw glued in a vertical position on the right hand side of the card
- 15 cm straw
- 1 piece of A4 card folded in half widthways

Activity

Take the card with the straws glued down each side and stand the folded card between the two straws to create a barrier between them (see Figure 1).

Place the card with straws and barrier in front of the child.

Say to the child – [Point to straw on left hand side of the barrier.]

Here is a straw.

Here is another straw. [Point to the straw on the right side of the barrier.]

Which straw do you think is the longest?

[Place the loose 15 cm straw on the table in front of the child.]

Please use this straw to check.

Which straw is the longest?

Tell me how you used this straw to check.

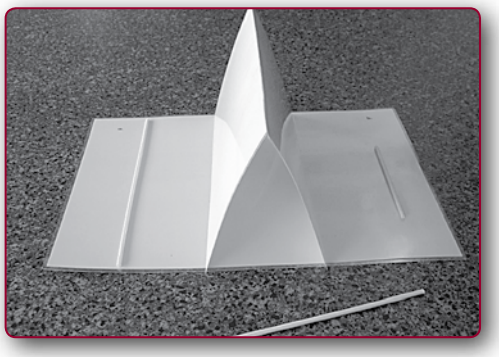


Figure 1

Asking students to check using the loose straw ensures that even if the answer was obtained by using a direct comparison of the appearance of the two straws or transitive reason with reference to the width of the card, it is possible to gain important insight into their understandings of transitivity.

Important observations

- Does the student give a reasonable estimate of which straw is longer?
- Does the student accurately use the loose straw to compare the lengths of the two straws?
- Does the student place the loose straw beside each straw in order to make a comparison?
- Does the student's explanation show evidence of transitive reasoning? (For example, if the loose straw is longer than the straw on the right but shorter than the straw on the left, the straw on the left must be longer.)
- Does the explanation include language such as "longer than," "shorter than," etc.?

Ideas for teaching and learning

Teachers need to provide opportunities for students to use objects for direct and indirect comparison in order to make judgements about length.

The above task may be replicated with larger objects that are not as easily visible or comparable. For example, draw chalk lines of different lengths and some distance apart outside. Ask the students to suggest how the lines could be compared if they cannot be moved. If student arrive at a solution involving a third object for comparison, they may be encouraged to decide what sort of object may be used. If the student is having difficulty arriving at a solution, a length of stick or string may be provided as a prompt. How might we use this to find out which line is longer?

Identical units

This concept is an important one for students to understand when measuring with either standard or non-standard units. If an accurate measure is to be gained, the units of length used to measure an object must be identical. The Straw and Mixed Paper Clips task may be used to determine students' understanding of the importance of using identical units when measuring an object.

The Straw and Mixed Paper Clips

(Modified from Department of Education, Employment and Training, 2001, p. 42).

Materials required

- 15 cm plastic drinking straw
- 8 large (5 cm) paper clips
- 5 small (3 cm) paper clips

Activity

Place the straw and collection of mixed paper clips in front of the child.

Say to the child:

Here is a straw.

Here are some paper clips.

Please measure how long the straw is with paper clips. [If the child hesitates] Use some paper clips to measure the straw.

What did you find? [If correct answer is given (5 or 3) but no units, ask "5 (or 3) what?"]

Important observations

- Does the student use identical units to measure the straw, i.e., all large or all small paperclips?
- Does the student use a mixture of large and small paperclips to measure the straw?
- Does the student accurately line the paperclips up with the beginning of the straw when measuring?
- Does the student lay the paperclips beside the straw without gaps or overlap?
- Does the student link the paperclips together, therefore creating overlap?
- Does the student give the correct unit of measure in their response, i.e., "5 paperclips" or "3 paperclips"?

Ideas for teaching and learning

Many tasks involving non-standard units require students to measure an object with a given unit; for example: "How many blocks long is your table/pencil case/foot…?" Students, however, should be provided with tasks that require them to choose an appropriate unit, or choose a unit from a collection of possibilities.

For example:

- We need to measure how long our mat is. What could we use to measure it? What else could we use? What could we use that might help us measure it more quickly?
- When asking students to measure an object with non-standard units, provide a container with mixed materials, such as small and large blocks, paperclips, straws, tiles and so on. This allows students' understanding of identical units to be assessed. Which material did the student choose? Was it suitable for the object being measured? Did the student use identical units to measure the object?
- Objects may also be included that are less suitable, such as beans or cotton balls, to assess students' understanding of appropriate units for length measurement.

Iteration

Iteration involves using a unit repeatedly in order to find a measurement. Rather than laying multiple units end to end (tiling), a single unit can be repeatedly moved. In length measurement this involves laying a unit repeatedly end to end along the length of an object, counting each iteration in order to arrive at a measurement. This concept will not always come naturally to children and explicit teaching of it as a strategy for measuring will be necessary. When the envelope and three paperclips task was used with three students from Grades 1, 2 and 5, none of the students successfully used the paperclips to iterate and find the width of the envelope. They chose to use estimation or declared the task "impossible." It is therefore

important that we provide students with early opportunities to solve such problems, and not assume that it is a concept they will grasp independently.

The Envelope and Three Paperclips task may be used to assess iteration; that is, are students aware that a single unit may be used repeatedly to measure a length?

The Envelope and 3 Paper Clips

(developed by the author)

Materials required

- B4 size envelope (25 cm in width) or a 25cm line drawn on a piece of card
- 3 large (5 cm) paperclips.

Activity

Place the envelope in front of the child in the portrait position. Place 3 large (5 cm) paperclips beside the envelope or line.

Say to the child:

Here is an envelope.

Here are some paper clips.

Please use the paperclips to measure how wide the envelope is. [If the child hesitates, e.g., "There's not enough,"] Are you able to use these paperclips to measure how wide the envelope is?

What did you find? [If correct answer is given (5) but no units, ask, "Five what?"]

Important observations

- Does the student use the paperclips to iterate? That is, do they lay the three paperclips on the envelope end to end and then move a paperclip repeatedly, laying it at the end of the other paperclips, counting the moves each time until they reach the end of the envelope?
- Does the student place the paperclips without gaps or overlap?
- Does the student correctly measure the width, including stating the unit in their measurement (5 paperclips)?

Ideas for teaching and learning

Generally when working with non-standard units, students are exposed to a large number of tiling tasks, in which a sufficient number of the unit is provided and students merely count the number of units used. Students should also be provided with tasks in which too few of a unit are provided, requiring units to be repeated through iteration. For example, rather than asking the question, "How many blocks long is your pencil case/desk/foot…?" and providing sufficient blocks to allow a simple count along the length of the object, students should be provided with too few blocks to make the length of the object. They may then be encouraged to problem solve a solution, leading to explicit instruction in iteration as a measurement strategy.

The concepts of transitive reasoning, iteration and identical units are critical for students to understand when approaching measurement tasks. It is important that we explicitly teach these concepts to students and not leave their acquisition to chance. With this in mind, it is necessary to assess students' current levels of understanding in order to plan effective learning experiences. The tasks described here may be used as a starting point to determine students' understanding of three important measurement concepts. Importantly they also represent opportunities for students to construct new understandings.

Acknowledgement

I would like to thank Ann Gervasoni for encouraging me to write this article.

References

Department of Education, Employment and Training, Vic. (2001). *Early numeracy interview booklet.* Melbourne: Author.

Wilson, P. S. & Osbourne, A. (1992). Foundational ideas in teaching about measure. In T. R. Post (Ed.), *Teaching mathematics in grades K–8: Research-based methods* (pp. 89–121). Needham Heights, MA: Allyn and Bacon.

Heidi Bush
Catholic Education Office (Tasmania)
<heidi.bush@catholic.tas.edu.au>