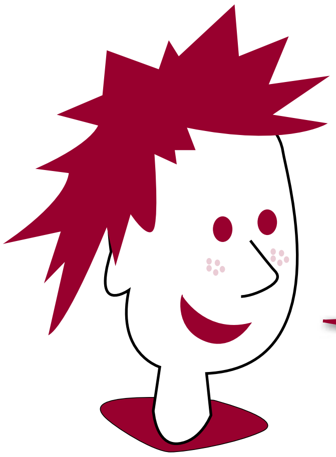

Using a Concept Cartoon to Gain Insight Into Children's Calculation Strategies

MATTHEW SEXTON, ANN GERVASONI and **ROBYN BRANDENBURG** use a concept cartoon as an effective way to encourage students to communicate their preferred strategies for solving addition calculations.

Introduction

Life in the twenty-first century requires an unprecedented level of mathematical knowledge, visualisation and skill for full participation in community life, and for access to opportunities in education and employment. It is therefore concerning that some Australian students under-perform in mathematics despite consistent attempts to improve mathematics learning and teaching. For example, Gervasoni, Hadden and Turkenburg (2007) found that 30% of students beginning their final year of primary school in regional Victoria have underdeveloped arithmetic reasoning strategies, a key indicator of mathematical competence. This highlights the importance of children developing reasoning strategies for calculating as opposed to using rote procedures (algorithms) or counting-based strategies.

Curriculum reform aimed at improving students' reasoning strategies for calculating was the focus of a research project in which we participated along with two school communities in regional Victoria. The research involved trialling a curriculum in which the teaching of algorithms for Grade 3 and Grade 4 students was withheld in favour of emphasising mental computation, with students' reasoning strategies recorded on empty number lines to enable monitoring of strategy choice (Gervasoni, Brandenburg, Turkenburg, & Hadden, 2009). Another focus of the research was assisting

teachers to gain insight into how their students approached calculations.

This article explores one aspect of the research in which *concept cartoons* (Sexton, 2008) were introduced as an innovative way for gaining insight into children's strategies for addition calculations in a situation that begs for the use of mental strategies ($24+99=n$). We examine some of the responses of the 101 Grade 3 and Grade 4 students who participated in the research to find out about their calculation strategies, and then consider the implications for subsequent learning and teaching.

Concept cartoons

A concept cartoon is a learning and teaching tool used primarily in science education to explore scientific concepts. However, we believe they also have great potential in mathematics education. The cartoons share some common traits with those used in comic strips, but rather than being designed to arouse hilarity, they aim to present students with the opportunity to interpret and to understand concepts (Naylor & Keogh, 1999).

Concept cartoons involve the pictorial representation of characters in settings familiar to students along with the use of written language or speech bubbles (Naylor & Keogh, 1999). The familiar settings and characters give relevance to the ideas that are being presented. It is important that alternative conceptions, statements or questions pertaining to a central idea are presented within the cartoon (Kabapinar, 2005; Naylor & Keogh, 1999). In most cases, alternative viewpoints are presented through the use of a group of characters engaging in a dialogue through the use of speech bubbles with minimal use of written language. Due to the characters' dialogue, students have the freedom to make judgements that agree or disagree with the views expressed by the characters without feeling threatened by needing to express their own opinions publicly (Kinchin, 2004). Concept cartoons are primarily intended to act as a teaching and learning tool but they can also be used to assess student cognition (Naylor & Keogh, 1999). In some cases they have been used to assess the affective domain (Kinchin, 2004; Sexton, 2008). Indeed, Sexton (2008) explored the use of concept cartoons in mathematics and found that they were a successful tool for gaining insight into children's and teachers' perceptions of effective mathematics learning environments. In June 2008 the students were shown the Concept Cartoon in Figure 1. The cartoon depicts four characters and dialogue that explains each character's strategy for solving $24 + 99 = n$. This addition calculation was selected because it is used in the Early Numeracy Interview (Clarke et al., 2002), and because we had noted previously that many children sought to perform the calculation using an algorithm, even though it lends itself to being solved using mental reasoning strategies. Overall, we sought greater insight about this phenomenon. In creating the concept cartoons, the strategies

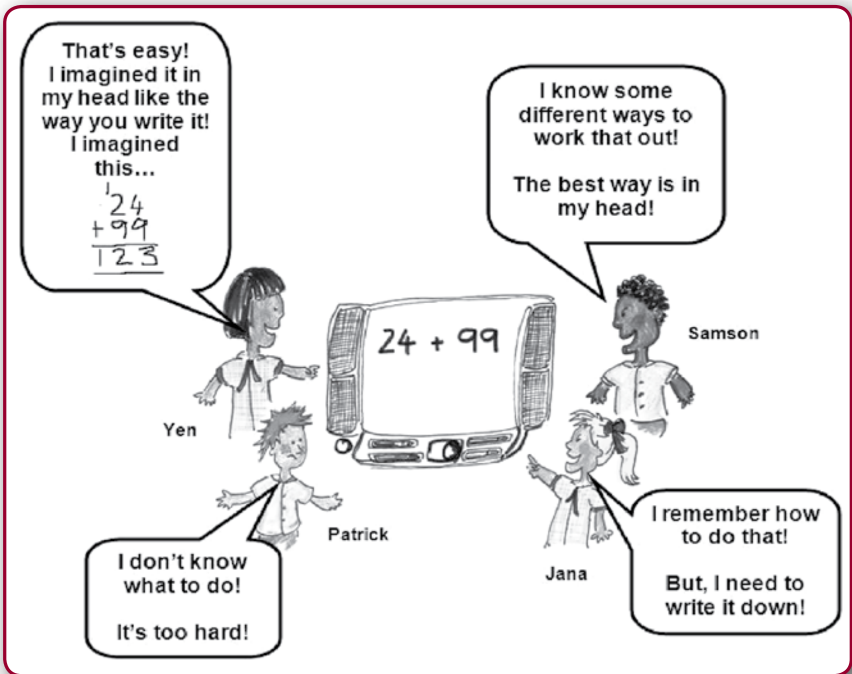

Figure 1. Concept cartoon for gaining insight about children's mental calculation strategies.

used by each character were chosen to enable teachers to determine whether children:

1. perceive the calculation as too hard (Patrick);
2. can solve the calculation in different ways, but recognise that a mental strategy is most efficient (Samson);
3. would solve the calculation using a visualised addition algorithm (Yen), or;
4. can remember a strategy, but would need to write to perform the calculation (Jana).

Once shown the concept cartoon, the students were asked to identify the name of the character that best matched their personal strategy choice for this calculation and provide a reason for choosing this character. Finally, the students were required to explain how they would calculate $24 + 99 = n$. These explanations provided further insight about children's approaches, and enabled students to describe alternative strategies to the ones used by the four characters. Some illustrative examples follow.

Kate chose Jana (who remembers how to do it but needs to write it down) because "I like to write it down on paper because it is easier." However, when explaining how she would perform the calculation, Kate explained, "I would turn 99 into 100 and then plus 24 and take away 1 from the 100 and make it 123." This is a mental reasoning strategy such as Samson used. Tania also chose Jana, but explained that to work it out she "would get a friend to help me and grab a calculator." This strategy suggests a lack of confidence in her ability to perform the calculation.

Jake chose Yen (who imagined the addition algorithm) because "I know the sum," but he "would make sure that it is right with blocks." This reliance on materials is problematic when the addends become so large, and suggests that Jake is not using reasoning-based strategies. Eva also chose Yen but explained that to solve the story she "counted with my fingers. I go to the tens and then I do my ones and I trade." This use of fingers suggests that Eva is following a procedure and using counting-based strategies to calculate.

Jason chose both Jana and Samson explaining, "I find it easier to write it down or I could do it in my head." Jason explained he would actually "add the 9 and 4 then the 90 and 20. Next I would put them together." Chris would also do it like Samson, but would "add on 1 to 99 to give me 100 and (then add) 23. That gives me 123."

Matt chose Patrick because "it is hard," but wished that he "could be like Samson because he is so good at working things out." Let's hope that both Matt and his teacher believe that this wish can come true.

Table 1

The number of students who chose each character ($n = 101$)

Class June 2008	Yen	Samson	Patrick	Jana	Samson & Yen	Patrick & Samson	Jana & Samson	Missing	Total
A	3	2	5	6					16
B	0	9	2	6	1			1	19
C	5	10	0	4			1	1	21
D	2	4	5	6		1		1	19
E	6	3	1	5				1	16
Total	16	28	13	27	1	1	1	4	101

A summary of the students' character choices is provided in Table 1.

The characters chosen by the students lead to some important reflections. We could assume that most students would recognise that this calculation ($24 + 99 = n$) might easily be performed mentally, just as Samson did. In contrast, fewer than one third of the students did so. Further, about one sixth of the students selected Yen who imagined solving the problem using an algorithm; for these students, imagining the steps of a learnt procedure was more useful than recognising the potential of using compensation to add 23 to 100. Another third

of the students chose Jana who remembered a way to solve the problem but would need to write it down. We suspect that many in this group might also use the standard addition algorithm. About one sixth of the students selected Patrick who did not know what to do because the problem was too hard.

The fact that about two-thirds of Grade 3 and Grade 4 students find this problem either too hard to solve, or would be likely to solve it using an algorithm suggests that we need to provide students with more opportunities to develop mental reasoning strategies. We argue that an aim of mathematics education is for students to learn a range of strategies for performing mental calculations, and to make wise choices about the best strategy to use for a given calculation.

Teaching implications

Mental computation and arithmetic reasoning strategies have been the focus of many studies (e.g., Clarke, Cheeseman, Gervasoni, et al., 2002; Fuson, 1992; Gervasoni, 2006; Steffe, Cobb, & von Glasersfeld, 1988). The data presented here suggest that approximately half of the students would use an algorithm (either writing it down or performing the steps mentally) to solve this type of problem. However, we argue that this type of problem is best solved mentally. These results support the conclusions of earlier research, suggesting that learning algorithms can reduce students' capacity to draw upon number sense to solve calculations that are best performed mentally (e.g., Narode, Board, & Davenport,1993). Our data also suggest that not all children have mental strategies available or select calculation strategies according to how they best fit the demands of a task (Griffin, Case, & Siegler, 1994). Further, Narode, Board, and Davenport (1993) suggest that introducing algorithms too early in schooling is detrimental to students' developing arithmetic reasoning strategies. Clarke, Clarke and Horne (2006) also argue that the early introduction of written methods impacts negatively on students' development of number sense and mental calculation strategies. Despite this, it is quite common for Australian teachers to introduce Grade 2 and Year 3 students to algorithms for addition and subtraction and this may have impacted on our students' strategy choices for solving $24 + 99 = n$.

We propose that a more effective approach is to delay the introduction of algorithms until grade 5 and instead focus on assisting students to construct a range of powerful mental calculation strategies. One approach that teachers in our research used was to encourage students to record their calculation strategies on empty number lines, so that monitoring and reflection on strategy choice could occur. This approach is widely used in the Netherlands (Beishuizen & Anghileri, 1998). For example, some children in a project class were asked to calculate mentally the answer to $26 + 17 = n$, and then represent their thinking on an empty number line. An illustrative example from Leroy, a student in a project class is shown in Figure 2.

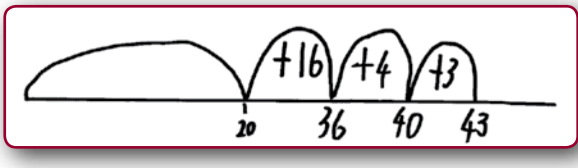

Figure 2. Leroy's empty number line representation of his strategy for calculating 26+17=43.

Leroy's strategy involved some flexible number splitting and re-grouping. His explanation using the empty number line representation provided a good opportunity for discussion and reflection on his strategy choice, comparison with the strategies other students used, and consideration of which students' strategies involved the fewest steps or were the most elegant. Interestingly, Leroy initially found it simpler to add 16 to 20, than to add 10 to 26. Monitoring strategy selection by written recording on empty number lines and through reflection on strategy choice in class discussion is important for stimulating progress towards higher-level strategies (Beishuizen & Anghileri, 1998).

Conclusion

The use of this concept cartoon with Grade 3 and Grade 4 students demonstrates that it was successful in providing teachers with insight about a range of strategies that students use to perform an addition calculation that begs to be solved using mental reasoning strategies. Our findings highlight that few students actually recognise that using mental strategies is the most powerful and efficient way to solve a calculation such as 24 + 99 = n and instead rely on written methods or the visualisation of written methods. The concept cartoon also provides a useful context for discussing the advantages and disadvantages of the strategies used by the various characters in the cartoon. This type of reflection leads students towards using more powerful strategies for similar calculations in the future. Our challenge for you is to use this concept cartoon to find out how your students approach this calculation. Perhaps they have also become over reliant on written methods. Please let us know!

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