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# Why Not Philosophy? Problematizing the Philosophy of Mathematics in a Time of Curriculum Reform

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This article argues that, as teachers struggle to implement curriculum reform in mathematics, an explicit discussion of philosophy of mathematics is missing from the conversation. Building on the work of Ernest (1988, 1991, 1994, 1998, 1999, 2004), Lerman (1990, 1998, 1999), the National Council of Teachers of Mathematics (1989, 1991, 2000), Davis and Hersh (1981), Hersh (1997), Lakatos (1945/1976), Kitcher (1984), and others, the author draws parallels between social constructivism and a humanism philosophy of mathematics. While practicing mathematicians may be entrenched in a traditional, Platonic philosophy of mathematics, and mathematics education researchers have embraced the fallibilist, humanist philosophy of mathematics (Sfard, 1998), the teachers of school mathematics are caught somewhere in the middle. Mathematics teachers too often hold true to the traditional view of mathematics as an absolute truth independent of human subjectivity. At the same time, they are pushed to teach mathematics as a social construction, an activity that makes sense only through its usefulness. Given these dichotomous views of mathematics, without an explicit conversation about and exploration of the philosophy of mathematics, reform in the teaching and learning of mathematics may be certain to fail.

The teaching and learning of mathematics is going through tremendous changes. The National Council of Teachers of Mathematics' (NCTM, 2000) Principles and Standards for School Mathematics calls for reforms to both curriculum and classroom instruction. Constructivist learning, student-centered classrooms, worthwhile tasks, and reflective teaching are all a part of NCTM's vision of school mathematics in the 21st century. Along with calls for changes in how mathematics is taught, there are numerous calls for changes in who engages in higher level mathematics courses. NCTM's Equity Principle calls for high expectations, challenging curriculum, and high-quality instructional practices for all students. In addition, recent publications from the National Research Council (NRC, 2001, 2005) have, in many ways, redefined the teaching and learning of mathematics. These documents call for a move away from the teaching of isolated skills and procedures towards a more problem solving, sense-making instructional mode. This changing vision of school mathematics—student-centered pedagogy, constructivist learning in our classrooms, focus on the problem-solving aspects of mathematics, and mathematics success for all— cannot come about without a radical change in instructional practices and an equally radical change in teachers' views of mathematics teaching and learning, as well as the discipline of mathematics itself.

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As state curricula, assessment practices, and teaching expectations are revamped, a discernable theoretical framework is essential to the reform process (Brown, 1998). This theoretical framework must include a re-examination of teachers' views of mathematics as a subject of learning. What are teachers' beliefs about mathematics as a field of knowledge? Do teachers believe in mathematics as a problem-solving discipline with an emphasis on reasoning and critical thinking, or as a discipline of procedures and rules? Do teachers believe mathematics should be accessible for all students or is mathematics only meant for the privileged few?

Recent studies examining teacher beliefs and mathematical reform have primarily focused on teachers' views of mathematics instruction (see e.g., Bibby, 1999; Cooney, Shealy, & Arvold, 1998; Hart, 2002; Mewborn, 2002; Sztajn, 2003). Further research is required to understand how teachers view not only mathematics teaching and learning, but mathematics itself. Unlike many previous studies, this research should examine teachers' philosophies, not simply their beliefs, regarding mathematics. Philosophy and beliefs, although similar, are not identical. Beswick (2007) asserted that there is no agreed upon definition of the term beliefs, but that it can refer to "anything that an individual regards as true" (p. 96). Pajares (1992) affirmed the importance of researching teacher beliefs, although he acknowledged that "defining beliefs is at best a game of player's choice" (p. 309). Not only is any definition of beliefs tenuous, but distinguishing beliefs from knowledge is also a difficult process (Pajares, 1992). I argue that a study of philosophy moves beyond the tenuousness of beliefs, in that philosophy is a creative process. "Philosophy is not a simple art of forming, inventing, or fabricating concepts, because concepts are not necessarily forms, discoveries, or products. More rigorously, philosophy is the discipline that involves creating concepts" (Deleuze & Guattari, 1991/1994, p. 5).

## Why philosophy?

Current calls for reform in mathematics education are not without controversy (Schoenfeld, 2004). This controversy, and the reluctance towards change, may well be rooted in philosophical considerations (Davis & Mitchell, 2008). Webster's Dictionary (2003) defines philosophy as "the critical study of the basic principles and concepts of a particular branch of knowledge, especially with a view to improving or reconstituting them" (p. 1455). A study examining philosophy, therefore, seeks to better understand those basic principles and concepts that teachers' hold regarding the field of mathematics. Philosophy, not just philosophy of mathematics teaching and learning, but the philosophy of mathematics, is rarely examined explicitly: "Is it possible that teachers' conceptions of mathematics need to undergo significant revisions before the teaching of mathematics can be revised?" (Davis & Mitchell, p. 146). That is a question not yet answered by the current research on teacher change and mathematics education.

Researchers seldom ask teachers to explore their philosophies of the mathematics they teach. But such a study is in keeping with the writings of Davis and Hersch (1981) and Kitcher (1984) who sought to problematize the concept of mathematics. If we are to change the nature of mathematics teaching and learning, we have to look beyond the traditional view of mathematics as a fixed subject of absolute truths, what Ernest (1991) and Lerman (1990) termed an absolutist view. Constructivist teaching and inquiry-based learning demand a new view of mathematics, the fallibilist view that envisions "mathematical knowledge [as a] library of accumulated experience, to be drawn upon and used by those who have access to it" (Lerman, p. 56) and "focuses attention on the context and meaning of mathematics for the individual, and on problem-solving processes" (Lerman, p. 56).

Sfard (1998) argued that mathematicians are entrenched in an absolutist view of mathematics while researchers in mathematics education are deeply immersed in the fallibilist view:

> On the one hand, there is the paradigm of mathematics itself where there are simple, unquestionable criteria for distinguishing right from wrong and correct from false. On the other hand, there is the paradigm of social sciences where there is no absolute truth any longer; where the idea of objectivity is replaced with the concept of intersubjectivity, and where the question about correctness is replaced by the concern for usefulness. (p. 491)

The teachers of school mathematics are caught between these two opposing groups, yet are rarely asked to explore their philosophies of mathematics. The very existence of philosophies of mathematics is often unknown to them. Yet the question, what is mathematics, is as important to the work of K–12 mathematics teachers as it is to the mathematics education researcher and the mathematician. In the following sections, I will outline the recent explorations that researchers and mathematicians have undertaken in the areas of philosophy of mathematics and mathematics education. Unfortunately, few researchers have engaged teachers of mathematics in this important discussion.

## Changing Views of Education and Mathematics

An investigation of philosophy of mathematics is rooted in three areas. Postmodern views of mathematics, 20th century explorations in the philosophy of mathematics, and social constructivism have contributed to discussions regarding the philosophy of mathematics. I begin by describing the emergence of social constructivism, which in many ways is the driving force behind mathematical reform in the United States and other nations (Forman, 2003).

### *Social Constructivism*

Forman and others (e.g., Restivo & Bauchspies, 2006; Toumasis, 1997) have argued that NCTM's Professional Standards for Teaching Mathematics (1991) and the later Principles and Standards for School Mathematics (2000) clearly build upon a social constructivist model of learning. But Ernest

(1991, 1994, 1998, 1999) argues that social constructivism is more than just a learning theory applicable to the teaching and learning of mathematics. According to Ernest, social constructivism is a philosophy of mathematics that views mathematics as a social construction. Social constructivism focuses on the community of the mathematics classroom and the communication that takes place there (Noddings, 1990), and grew out of Vygotsky's (1978) work in social learning theory. It has been further developed in mathematics teaching and learning through the work of Confrey (1990), Lerman (1990, 1998, 1999), and Damarin (1999). This theory is in keeping with NCTM's (2000) emphasis on the social interplay in mathematics instruction:

> Students' understanding of mathematical ideas can be built throughout their school years if they actively engage in tasks and experiences designed to deepen and connect their knowledge. Learning with understanding can be further enhanced by classroom interactions, as students propose mathematical ideas and conjectures, learn to evaluate their own thinking and that of others, and develop mathematical reasoning skills. Classroom discourse and social interaction can be used to promote the recognition of connections among ideas and the reorganization of knowledge. (p. 21)

Overall, social constructivists advocate that educators form a view of mathematical learning as something people do rather than as something people gain (Forman, 2003).

It is upon this foundation that Ernest (1998) built his theory of social constructivism as a philosophy of mathematics. He argued that the teaching and learning of mathematics is indelibly linked to the philosophy of mathematics:

> Thus the role of the philosophy of mathematics is to reflect on, and give an account of, the nature of mathematics. From a philosophical perspective, the nature of mathematical knowledge is perhaps the central feature which the philosophy of mathematics needs to account for and reflect on. (p. 50)

Without that link, Ernest argued, we cannot truly understand the aims of mathematics education. Ernest (2004) emphasized the need for researchers, educators, and curriculum planners to ask "what is the purpose of teaching and learning mathematics?" (p. 1). But, in order to answer that, both mathematics and its role and purpose in society must be explored.

Dossey (1992) also placed an emphasis on the philosophy of mathematics: "Perceptions of the nature and role of mathematics held by our society have a major influence on the development of school mathematics curriculum, instruction, and research" (p. 39). Yet, in the educational sphere, there is a lack of conversation about and exploration of philosophy that "has serious ramifications for both the practice and teaching of mathematics" (Dossey, p. 39). Without a direct focus on philosophy, the consequences of differing views of mathematics are not being explored.

Ernest (1991, 1998) described two dichotomist philosophical views of mathematics—the absolutist and the fallibilist. The Platonist and formalist philosophies both stem from an absolutist view of mathematics as a divine gift or a consistent, formalized language without error or contradiction. Both of these schools of thought believe mathematics to be infallible, due either to its existence beyond humanity, waiting to be discovered (the Platonist school), or to its creation as a logical, closed set of rules and procedures (the formalist school). The fallibilist philosophy, what Hersh (1997) termed a philosophy of humanism, views mathematics as a human construction and, therefore, fallible and corrigible. One important implication of the fallibilist philosophy of mathematics is that if mathematics is a human construct then so must be the learning of mathematics. In the fallibist philosophy, mathematics is no longer knowledge that is simply memorized in a rote fashion. It is societal knowledge that must be interpreted in a manner that holds meaning for the individual. The constructivist approach to learning, therefore, aligns well with the fallibilist philosophy of mathematics.

Ernest (1991, 1998) characterized a cycle of subjective and objective knowledge to support his view of the social constructivist foundations of mathematical knowledge. In this cycle, new knowledge begins as subjective knowledge, the mathematical thoughts of an individual. This new thought becomes objective knowledge, knowledge that may appear to exist independent of humanity, through a social vetting process. This objective knowledge then enters the public domain where individuals test, reformulate, and refine the knowledge. The individuals then internalize and interpret the objective knowledge, once again transforming it to subjective knowledge. The social process of learning mathematics is intricately linked to society's ideas of what is and is not mathematics. Thus, Ernest was able to connect a learning theory, social constructivism, with a philosophy of mathematics.

*A Postmodern View of Mathematics*

During the past 50 years, there has been a growing discussion of the historical and philosophical foundations of mathematics. What was once seen as objective is now viewed by some as a historical and social construction, changing and malleable, as subjective as any social creation. Aligned with these changing views of mathematics are new ideas about mathematics instruction. The absolutist view of mathematics is associated with a behaviorist approach, utilizing drill and practice of discrete skills, individual activity, and an emphasis on procedures. The fallibilist view of mathematics aligns itself with pedagogy consistent with constructivist theories, utilizing problem-based learning, real world application, collaborative learning, and an emphasis on process (Threlfall, 1996). Although there have been numerous calls to change and adapt the teaching of mathematics through the embracing of a constructivist epistemology, little has been done to challenge teachers' conceptions of mathematics. The push towards student-centered instructional practices and the current challenges to traditional views of mathematics teaching have been brought together through Ernest's work over the past 20 years: "Teaching reforms cannot take place unless teachers' deeply held beliefs about mathematics and its teaching and learning change" (Ernest, 1988).

Ernest's (2004) more recent work advocates a postmodern view of mathematics. He seeks to break down the influence of what he terms the "narratives of certainty" that have resulted in "popular understandings of mathematics as an unquestionable certain body of knowledge" (Ernest, 2004, p. 16). Certainly, this understanding still predominates in mathematics classrooms today (see e.g., Bishop, 2002; Brown, Jones, & Bibby, 2004; Davison & Mitchell, 2008; Handel & Herrington, 2003). However, Ernest draws upon postmodern philosophers such as Lyotard, Wittgenstein, Foucault, Lacan, and Derrida, to challenge traditional views of mathematics and mathematics education. He embraces the postmodern view because it rejects the certainty of Cartesian thought and places mathematics in the social realm, a human activity influenced by time and place. Others have joined Ernest in exploring mathematics and mathematics instruction through the postmodern perspective (see e.g. Brown, 1994; Walkerdine, 1994; Walshaw, 2004). Neyland (2004) calls for a postmodern perspective in mathematics education to "address mathematics as something that is enchanting, worthy of our esteem, and evocative of wonder" (p. 69). In so doing, Neyland hopes for a movement away from mathematics instruction emphasizing procedural compliance and onto a more ethical relationship between teacher and student, one that stresses not just enchantment in mathematics education but complexity as well.

Walshaw (2004) ties sociocultural theories of learning to postmodern ideas of knowledge and power, drawing, as Ernest does, on the writings of Foucault and Lacan: "Knowledge, in postmodern thinking, is not neutral or politically innocent" (p. 4). For example, issues of equity in mathematics can be seen in ways other than who can and cannot do mathematics. Indeed, societal issues of power and reproduction must be considered. A postmodern analysis forces a questioning of mathematics as value-free, objective, and apolitical (Walshaw, 2002). Why are the privileged mathematical experiences of the few held up as the needed (but never attained) mathematical experiences of all? Furthering a postmodern view of mathematics, Fleener (2004) draws on Deleuze and Guattari's idea of the rhizome1 in order to question the role of mathematics as lending order to our world: "By pursuing the bumps and irregularities, rather than ignoring them or 'smoothing them out,' introducing complexity, challenging status quo, and questioning assumptions, the smoothness of mathematics is disrupted" (p. 209). The traditional view of mathematics has ignored the bumps and irregularities, forcing a vision of mathematics as smooth, neat, and orderly.

Another postmodern view is that our representations of mathematics cannot be divorced from the language we use to describe those representations:

> Any act of mathematics can be seen as an act of construction where I simultaneously construct in language mathematics notions and the world around me. Meaning is produced as I get to know my relationships to these things. This process is the source of

> the post-structuralist notion of the human subject being constructed in language. (Brown, 1994, p. 156)

Brown used Derrida's ideas on deconstructing language to examine how the social necessity of mathematical learning means that mathematics is always, in some way, constructed. And, in examining new mathematical ideas, the learners cannot help but bring their entire mathematical and personal history to the process.

Brown's (1994) postmodern view of mathematics strengthens Ernest's (1998) own contention of the philosophical basis of social constructivism. Mathematical ideas begin as social constructions but "become so embedded within the fabric of our culture that it is hard for us to see them as anything other than givens" (Brown, p. 154). Thus, the establishment of mathematical meta-narratives2 camouflages the social/cultural roots of mathematical knowledge. As a result, mathematics continues to be viewed primarily as something discovered, not constructed. Siegel and Borasi (1994) described the pervasive cultural myths that continue to represent mathematics as the discipline of certainty. In order to confront this idealized certainty, they state a need for an inquiry epistemology that "challenges popular myths about the truth of mathematical results and the way in which they are achieved, and suggests, instead, that: mathematical knowledge is fallible… (and) mathematical knowledge is a social process that occurs within a community of practice" (p. 205). This demystifying process is necessary, argued Siegel and Borasi, if teachers are to engage students in doing mathematics, not simply memorizing rote procedures and discrete skills.

*New Ideas in the Philosophy of Mathematics*

> A world of ideas exists, created by human beings, existing in their shared consciousness. These ideas have objective properties, in the same sense that material objects have objective properties. The construction of proof and counterexample is the method of discovering the properties of these ideas. This branch of knowledge is called mathematics. (Hersh, 1997, p. 19)

A reawakening of the philosophy of mathematics occurred during the last part of the 20th century (Hersh, 1997). In their landmark book The Mathematical Experience, Davis and Hersh (1981) explored ideas of mathematics as a human invention, a fallibilist construct. Davis and Hersh described several schools of philosophical thought regarding mathematics—including Platonism and formalism. In the Platonist view, mathematics "has evolved precisely as a symbolic counterpart of the universe. It is no wonder, then, that mathematics works; that is exactly its reason for existence. The universe has imposed mathematics upon humanity" (p. 68). The Platonist not only accepts, but embraces, God's place in mathematics. For what is mathematics but God's gift to us mortals? (Plato, trans. 1956). The Platonist, forever linking God and mathematics, sees the perfection of mathematics. If there are errors made in our mathematical discoveries (and, of course, they are discoveries not inventions because they come from a higher power), then the errors are ours as flawed humanity, not inherent to the mathematics. And because mathematics is this higher knowledge it follows that some will succeed at mathematics while many others fail. Mathematics, in the Platonic view, becomes a proving ground, a place where those who are specially blessed can understand mathematics' truths (and perhaps even discover further truths) while the vast numbers are left behind.

Euclid's Elements was (and still is) the bible of belief for mathematical Platonists (Hersh, 1997). As Davis and Hersh (1981) pointed out, "the appearance a century and a half ago of non-Euclidean geometries was accompanied by considerable shock and disbelief" (p. 217). The creation of non-Euclidean geometries—systems in which Euclid's fifth postulate (commonly known as the parallel postulate) no longer held true—momentarily shook the very foundations of mathematical knowledge.

> The loss of certainty in geometry was philosophically intolerable, because it implied the loss of all certainty in human knowledge. Geometry had served, from the time of Plato, as the supreme exemplar of the possibility of certainty in human knowledge. (Davis & Hersh, p. 331)

A result of the uncertainty brought on by the formation of non-Euclidean geometries was the development of formalism. In formalism, mathematics is the science of rigorous proofs, a language for other sciences (Davis & Hersh, 1981). "The formalist says mathematics isn't about anything, it just is" (Hersh, 1997, p. 212). In the early part of the 20th century, Frege, Russell, and Hilbert, among others, each attempted to formalize all of mathematics through the use of the symbols of logic and set theory. Russell and Whitehead's "unreadable masterpiece" (Davis & Hersh, p. 138), Principia Mathematica, attempted the complete logical formalization of mathematics. But the attempts to complete the logical formalization of mathematics were doomed to failure as demonstrated by Gödel's Incompleteness Theorem that proved any formal system of mathematics would remain incomplete, not provable within its own system (Goldstein, 2005).

Proofs and Refutations: The Logic of Mathematical Discovery, a beautifully written exploration of the philosophy of mathematics penned by Imre Lakatos (1976), offered an alternative philosophy of mathematics to those of Platonism and formalism, termed the humanist philosophy. In Proofs and Refutations, Lakatos used the history of mathematics, as well as the structure of an inquiry-based mathematics classroom, to explore ideas about proof. Through a lively Socratic discussion between a fictional teacher and students, Euler's formula ($V-E+F=2$) is dissected, investigated, built upon, improved, and, finally, made nearly unrecognizable. Lakatos used the classroom dialogue to challenge accepted ideas about proof. He forced the reader to question if proofs are ever complete or if mathematicians simply agree to ignore the non-examples, which Lakatos's students termed monsters, that contradict the proof. Through this analogy, Lakatos demonstrated that in mathematics there are many monsters, most of which are ignored, as though the mathematical community has made a tacit agreement to turn away from that which makes it uncomfortable.

Ernest built much of his philosophy of mathematics and mathematics education on the writings of Lakatos. Like Lakatos, Ernest (1998) saw mathematics as indubitably tied to its creator—humankind: "Both the creation and justification of mathematical knowledge, including the scrutiny of mathematical warrants and proofs, are bound to their human and historical context" (p. 44). Hersh (1997), in his book, What is Mathematics, Really?, included both Lakatos and Ernest on his list of "mavericks"—thinkers who see mathematics as a human activity and, in so doing, influenced the philosophy of mathematics. Others are included as well: philosophers Charles Sanders Peirce (Siegel & Borasi, 1994) and Ludwig Wittgenstein (Ernest, 1991, 1998b), psychologists Jean Piaget and Lev Vygotsky (Confrey, 1990, and Lerman, 1994), and mathematicians George Polya and Philip Kitcher.

Polya's (1945/1973) classic, How to Solve It: A New Aspect of Mathematical Method, revived the study of the methods and rules of problem solving—called heuristics—in mathematics. Although he eschewed philosophy, Polya saw mathematics as a human endeavor. He described the messiness of the mathematician's work:

> Mathematics in the making resembles any other human knowledge in the making. You have to guess a mathematical theorem before you prove it; you have to guess the idea of the proof before you carry through the details. You have to combine observations and follow analogies; you have to try and try again. (Polya, 1954/1998, pp. 99)

Both Polya and Lakatos led mathematicians into new areas that questioned the very basis of mathematical knowledge. Their combined impact on the philosophy of mathematics was as important as the development of non-Euclidean geometries and Gödel's Incompleteness Theorem (Davis & Hersh, 1981). By defining mathematics as a social construct, they opened up the field to new interpretations. Polya's heuristic emphasized the accessibility of problem solving. Lakatos, by using dialogue to trace the evolving knowledge of mathematics—the proofs and refutations—stressed the social aspects of mathematical learning as well as the fallibility of mathematical knowledge, and defined mathematics as quasi-empirical. No longer was mathematics a subject for the elite. Ernest (1998) credited Lakatos with a synthesis of epistemology, history, and methodology in his philosophy of mathematics—a synthesis that influenced the sociological, psychological, and educational practices of mathematics.

Ernest (1998) and Hersh (1997) also referred to Kitcher as a maverick, in that he stressed the importance of both the history of mathematics and the philosophy of mathematics. Kitcher (1983/1998) underscored the concept of change in mathematics: "Why do mathematicians propound different statements at different times? Why do certain questions wax and wane in importance? Why are standards and styles of proof modified?" (p. 217). His conclusion was that mathematics changes in practice, not just in theory. Kitcher identified five components of mathematical practice—language, metamathematical views, accepted questions, accepted statements, and accepted reasoning—that are developmentally compatible: As one component changes, others must change as well (Hersh, 1997). Kitcher's five components emphasized the social aspect of mathematics as a community activity with agreed upon norms and practices. Kitcher's view of mathematics mirrors Ernest's cycle of subjective knowledge → objective knowledge → subjective knowledge and Lakatos' idea of proofs and refutations in that each generation simultaneously critiques, internalizes, and builds upon the mathematics of the previous generation (Hersh, 1997).

## Conclusion

Few studies have addressed the issue of teachers' philosophies of mathematics. Too often, those that have relied on surveys and questionnaires to define the complexity that is a teacher's philosophy (see e.g., Ambrose, 2004; Szydlik, Szydlik, & Benson, 2003; Wilkins & Brand, 2004). Although studies conducted by Lerman (1990), Wiersma and Weinstein (2001), and Lloyd (2005) briefly examined their participants' expressed perceptions of mathematics, none of these studies specifically examined the results of an exploration of philosophy. What remains to be investigated is what happens when teachers are presented with non-traditional views of mathematics and explore philosophical writings about mathematics. At a university in Greece, Toumasis (1993) developed a course for preservice secondary school mathematics teachers that centered on readings about the history and philosophy of Western mathematics, as well as "discussion and an exchange of views" (p. 248). The purpose of the course was to develop a reflective mathematics teacher because:

> To be a mathematics teacher requires that one know what mathematics is. This means knowing what its history, its social context and its philosophical problems and issues are. . . . The goal is to humanize mathematics, to teach tolerance and understanding of the ideas and opinions of others, and thus to learn something of our own heritage of ideas, how we came to think the way we do (p. 255).

According to Toumasis, teacher preparation programs continue to shortchange mathematics teachers by focusing only on coursework in higher level mathematics, e.g., Linear Algebra, Discrete Mathematics, and Analysis. Knowledge of mathematics, especially if one is to teach mathematics, must include a reflexive study of mathematics.

Toumasis (1997) argued that the philosophical and epistemological beliefs about the nature of mathematics are intrinsically bound with the pedagogy of mathematics. In his examination of the philosophical underpinnings of NCTM's Curriculum and Evaluation Standards for School Mathematics (1989), Toumasis identified a clear fallibilist point of view; mathematics is "a dialogue between people tackling mathematical problems" (p. 320). Yet in current attempts to reform mathematics based on both the 1989 Standards and the later Principles and Standards for School Mathematics (NCTM, 2000), an investigation of philosophy is rarely undertaken.

The teaching and learning of mathematics is a politically charged arena. Strong feelings exist in the debate on how "best" to teach mathematics in K–12 schools, feelings that are linked to varying perceptions about the nature of mathematics (Dossey, 1992). Is mathematics an abstract body of ideal knowledge, existing independently of human activity, or is it a human-construct, fallible and ever-changing? These perceptions of mathematics then drive beliefs about the appropriateness of instructional practices in mathematics. Is mathematics a body of knowledge that must be memorized and unquestionably mastered, or do we engage the learners of mathematics in personal sense-making, in constructing their own mathematical knowledge? That we are still in the midst of "math wars" is indisputable (Schoenfeld, 2004). What it means to teach mathematics and the very nature of mathematics is at the center of these wars:

> Traditionalists or back-to-basics proponents argue that the aim of mathematics education should be mastery of a set body of mathematical knowledge and skills. The philosophical complement to this version of the teaching and learning of mathematics is mathematical absolutism. Reform-oriented mathematics educators, on the other hand, tend to see understanding as a primary aim of school mathematics. Constructivism is often the philosophical foundation for those espousing this version of mathematics education. (Stemhagen, 2008, p. 63)

I agree with Schoenfeld (2004), Greer and Mukhopadhyay (2003), and others (e.g., Davison & Mitchell, 2008) that the math wars are based on philosophical differences. It has therefore been my intent to inject philosophy into the discussion of mathematics educational reform and research. Research is needed that focuses on what we teach as mathematics and, more importantly, how teachers view the mathematics that they teach. Is mathematics transcendental and pure, something that exists outside of humanity, or is it a social activity, a social construction whose rules and procedures are defined by humanity (Restivo & Bauchspies, 2006)? An extensive review of the literature found no studies that led teachers to explore their philosophies of mathematics. Yet Restivo and Bauchspies recognized the need to push teachers' understanding of mathematics beyond the debate of mathematics as a social construction. To understand mathematics (and thus to teach mathematics) is to understand the social, cultural, and historical worlds of mathematics (Restivo & Bauchspies). Should we not then explore mathematics in a philosophical sense, its "basic principles and concepts…with a view to improving or reconstituting them" (Webster's Dictionary, 2003, p. 1455)? Change in classroom practices may not be possible without first "improving or reconstituting" teachers' philosophies of mathematics.

Many studies have addressed the need to engage both preservice and inservice teachers in constructivist learning in order to change their instructional practices (see, e.g., Hart, 2002; Mewborn, 2003; Thompson, 1992). Yet little has been done to engage teachers in a philosophical discussion of mathematics: "Teachers, as well, should be encouraged to develop professionally through philosophical discourse with their peers" (Davison & Mitchell, 2008, p. 151). Philosophy and mathematics have a long-standing connection, going back to the ancient Greeks (Davis & Hersh, 1981). Mathematics teachers are seldom asked to explore philosophy beyond an introductory Philosophy of Education course. If one is going to teach mathematics, one should ask "Why?" What is the purpose of teaching mathematics? What is the purpose of mathematics in society at large? Should not mathematics' purpose be tied to how we then teach it? These questions come back to teachers' perception of mathematics, and more specifically, their philosophies of mathematics.

I contend that discussions of philosophy, particularly philosophy of mathematics, should be brought to the forefront of mathematics education reform. If teachers are never asked to explore the philosophical basis of their perceptions of mathematics, then they will continue to resist change, to teach the way they were taught. The growing philosophical investigations of mathematics (see, e.g., Davis & Hersh, 1981; Hersh, 1997; Restivo, Van Bendegen, & Fischer, 1993; Tymoczko, 1998) in the past 30 years have not often been addressed in mathematics education research. We seem afraid to raise issues of philosophy as we implement curriculum reform and study teacher change. But philosophy too often lies hidden, an unspoken obstacle in the attempt to change mathematics education (Ernest, 2004). Researchers can bring the hidden obstacle to light, should engage both policymakers and educators in a conversation about philosophy, not with the intent of enforcing the "right" philosophy but with the acknowledgement that, without a continued dialogue about philosophy, the curriculum reform they research may continue to fall short.

I end this article by revisiting a definition of philosophy of mathematics: "The philosophy of mathematics is basically concerned with systematic reflection about the nature of mathematics, its methodological problems, its relations to reality, and its applicability" (Rav, 1993, p. 81). If our goal in mathematics education reform is to make mathematics more accessible and more applicable to real-world learning, we should then help guide today's teachers of mathematics to delve into this realm of systematic reflection and to ask themselves, "What is mathematics?"

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[1] Building from the botanical definition of rhizome, Deleuze and Guattari (1980/1987) used the analogy of the rhizome to represent the chaotic, non-linear, postmodern world. Like the tubers of a canna or the burrows of a mole, rhizomes lead us in many directions simultaneously. Deleuze and Guattari described the rhizome as having no beginning or end; it is always in the middle.

[2] A meta-narrative, wrote Kincheloe and Steinberg (1996), "analyzes the body of ideas and insights of social theories that attempt to understand a complex diversity of phenomena and their interrelations" (p. 171). It is, in other words, a story about a story; a meta-narrative seeks to provide a unified certainty of knowledge and experience, removed from its historic or personalized significance.

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