Using Imagination in the Math Classroom

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What does your mathematics classroom look like? Does it look like the math classroom you experienced through childhood? Were you bored? Are your students bored? Are they engaged in the material presented? Do they enjoy learning? Do they look forward to your class? Do they participate in mathematical thinking and inquiry? Simply put, do they like mathematics? Reflect on these questions while you transport yourself into one of my recent tenth grade math classes.

I called the class to attention and began. "Once upon a time there was a little boy named Carl who lived in Brunswick, Germany. The year was 1785. Little Carl was a very smart boy even though he came from a poor family. He taught himself to read and write—as well as to add, subtract, multiply, and divide—by the age of three. At the age of seven, he attended school in a one-room schoolhouse with students both older and younger than himself. Even though he had older classmates, he was the smartest in the class. The problem was that he was also the poorest behaved.

You see, Carl learned very little at school because he was already so smart. Some even say that he knew more that the teacher. I paused slightly, knowing what was coming.

"It sounds like me, Mr. Wilke," quipped Tim.

"Funny guy, Tim. Funny guy," I responded laughing with the whole class. I continued, "Anyway, because Carl was so bored, he did things that made the teacher very angry. He was also the first to answer questions the teacher posed to the class. This annoyed the teacher even more. On one particular occasion during a math class, where Carl seemed to be crawling on the ceiling while answering every question, the teacher could not take it anymore. Finally she said, "Carl I am sick and tired of you. Go into the corner. I do not want to hear from you for the rest of the day. Oh ... go add the numbers from one to one hundred." So Carl grabbed his things and plopped himself in the corner eager to meet the challenge.

> "Now, if you really think about this, Carl could actually be there all day. Carl did not have one of these," I said, holding up my graphing calculator.
>
> "Is this a true story?" asked Megan.
>
> I responded, "That is a secret. I could tell you but…"
>
> "Then I'd have to kill yah," the rest of the class yelled in chorus.

"I want you to imagine you are Carl. You need to solve the same problem that his teacher posed. There are a few rules however. First, you may not use a calculator. Secondly, you can work together if you wish. And lastly, you only have thirty seconds to work on it." I waited for the response.

"What?" "You're kidding!" the students cried in disbelief.

I nodded. "After thirty seconds in the corner, Carl stood up and said 'I did it! I did it.' Well, can you imagine the teacher's reaction? So, pretend you are Carl; and yes, I am kidding, you have ten minutes to find out how Carl did it in thirty seconds."

Immediately the desks moved together, the talking commenced, and they started to work, determined to solve the mysterious problem.

You may have heard this story before; but students, most likely, have not. If you have not, it is, for the most part, true. After giving my students time to work on the problem, I reconvened the class, and we discussed how Carl went about solving it. Carl is, of course, Johann Carl Friedrich Gauss, the second most famous mathematician ever to have lived.[2]

After a discussion of the problem and their solutions, I continued by showing the connection between it and the formula to sum a series taught in high school mathematics curricula. The formula is: $S_n = \frac{n}{2}(a + t_n)$ where S_n is the sum of the series, n is the number of terms in the series, a is the first term in the series, and t_n is the last term in the series.

Carl discovered that the series 1 + 2 + 3 +. . . 98 + 99 + 100 has a pattern. The first term + the last term = 101, the second term + the second last term = 101, etc. Now, how many 101's will there be? 50 pairs! Therefore, 50 x 101 = 5050. Applying this to our formula, $n = 100$, $a = 1$ and $tn = 100$ giving $S_{100} = \frac{100}{2}(1+100) = 50 \times 10$.

Before showing this formula to my students, I posed another question allowing them to apply "Carl's logic" to a similar problem. My students were then encouraged to apply this same principle to every series! For example, given the series 9 + 11 + 13 + . . . + 41 that has 17 terms, the sum is $S_{17} = \frac{17}{2}(9+41) = 425$. Awesome!

Why the story? Is it a waste of valuable class time? I would like to strongly argue that it is not a waste of time. In fact, I would like to argue that taking the time to engage students in this manner is vitally necessary!

I started by asking a series of questions. What does your mathematics classroom look like? Does it look like the math classroom you experienced through childhood? Are your students bored as you were as a child? Are they engaged in the material presented? Do they enjoy learning? Do they look forward to your class? Do they participate in mathematical thinking and inquiry? Simply put, do they like mathematics? The answers to these questions are of course varied and complex. However, I hope you see that my students were engaged in this lesson. They did enjoy learning! They did participate in mathematical reasoning and thinking!

So what makes this class different from the norm? Can we use

[2]Isaac Newton is, of course, the most famous mathematician.

this example to establish criteria that we can then use to engage students and help them think mathematically? I believe we can. Students must be encouraged to think for themselves by tapping into their imaginations. Only when mathematical concepts are presented in an imaginative way will students fully benefit from their experience. Both teachers and students must engage themselves in this creative realm of thinking, where imagination plays a vital part in learning and teaching. As Egan (2005) states

> Imagination is too often seen as something peripheral to the core of education, something taken care of by allowing students time to 'express themselves' in 'the arts,' while the proper work of educating goes on in the sciences and math… [But] imagination is the center of education; it is…crucial to any subject, mathematics and science no less than history and literature. Imagination can be the main workhorse of effective learning [and teaching] if we yoke it to education's central tasks. (p. xiii)

By allowing my students to imaginatively enter Carl's world they became interested and involved. Students were intrigued. They wanted to know more. They were not bored. "This concept of teaching implies the need for teacher's use of their own imagination capacities while interacting with students to engage them in truly enjoyable and relevant learning" (Jagla, 1994, p. 4). I was able to turn a very theoretical math concept, summing a series of numbers, into a meaningful, enjoyable, and relevant topic.

What were the characteristics of this particular class that made it so effective in an imaginative way? Firstly, I allowed my students to take part in the narrative. Students love to hear stories, but they also like to reenact them. A "boring" concept was made to come alive by engaging students in the story of "Carl," and what is especially fascinating to them is that the story was actually true! Story telling can be used in any subject at any grade level, including tenth grade mathematics. As Jagla (1994) observes, "Storytelling is a delightfully imaginative activity for all ages…the telling of stories is a wonderful way to provide context and make connections for students at any level" (p.132). With a little effort, students were able to put themselves in Carl's classroom and picture themselves in his predicament. The story included themes of hero and villain, mystery, humor, and excitement. My students actually became part of the narrative! And most of them solved the problem!

I believe students will remember this concept and how to use this formula because it was given a personal dimension with the introduction of Carl. The mathematical concept was reinforced by the story. It would have been very easy to stand in front of the class in a traditional manner and give them the formula with three or four examples. Would this have been as effective? Would students have been engaged, or would they have been bored, if I had taught it in this way? Stories serve two purposes: they are effective in communicating information and they orient the hearer's feelings about the information. The story of Carl communicated a mathematical concept, but it also instilled positive feelings toward the material that I was going to be testing them on the following week.

Secondly, this was an effective classroom experience because students were able to discover the required learning outcomes on their own. Through experimentation, most students were able to solve the problem 1+2+3...+100. Some were able to apply it to another example, and some even came up with a generalized formula. Of course, this technique employs an old idea that has been around for a long time: allowing students to discover material on their own adds up to a rewarding educational experience. Thinking skills are important to develop. With practice in problems like the one that Carl solved, students are able to improve their problem solving skills. It would have been very easy for me simply to show them the concept; but students are much more likely to own the material, understand it, remember it, and apply it to new and different situations if they are given a chance to think. Herbert Spencer puts it this way: "Children should be led to make their own investigations, and to draw their own inferences. They should be told as little as possible, and induced to discover as much as possible" (Kazamias, 1966, p. 75). Encouraging the use of a students' imagination is directly related to critical thinking and problem solving skills. Jagla agrees: "imagination is thinking of the various possibilities of a certain situation…you have to know scientifically what will work—you also have to go after what has never been done before, you don't even know if it could work" (1994, p. 30). My students had never seen a problem like this before! By using their imagination, they were able to experiment with different new ideas that led them to the solution.

Thirdly, Carl's problem appealed to my students' sense of mystery and intrigue. My students, like Carl, were anxious and excited to solve this problem on their own. They wanted to be the "naughty boy" who was able to solve the problem the teacher posed as punishment. How could Carl do this in thirty seconds? Impossible! No, it was possible! Well then, prove it! My students were engaged and became engrossed by the problem. It is an awesome sight to see students become completely immersed in a problem. And they solved the mystery! Math can actually be interesting! As Egan (2005) states, "All the subjects of the curriculum have mysteries attached to them, and part of our job in making curriculum content known to students is to give them an image of richer and deeper understanding that is there to draw their minds into the adventure of learning" (p.6).

Lastly, another way to engage students in a topic is to use humor. People love to laugh. When students enjoy themselves they are more eager to participate, they are more likely to retain and understand the material, and they are more likely to use the concepts covered. Although it is not evident in my narrative, humor was part of Carl's story. In relating the story of Carl, my interactions with my students involved humor. The general presentation of the story can easily be made funny, thus increasing the enjoyment of the experience. Humor can alleviate monotony and boredom. Egan (2005) explains that "[It] can also assist in the struggle against sclerosis of the imagination as students go through their schooling—helping to fight against rigid conventional uses of rules and showing students rich dimensions of knowledge and encouraging flexibility of mind" (p. 4).

Are these the only techniques at our disposal to engage students in learning? Of course not! If our chosen technique to relay information to our students allows them to embrace the material, then it is worthwhile. Narratives, self-discovery, a sense of mystery, and the use of humor were four powerful tools used in this particular situation. What other imaginative techniques could I have employed in this lesson? Two other strategies that I often employ involve role-playing and the use of mental imagery.

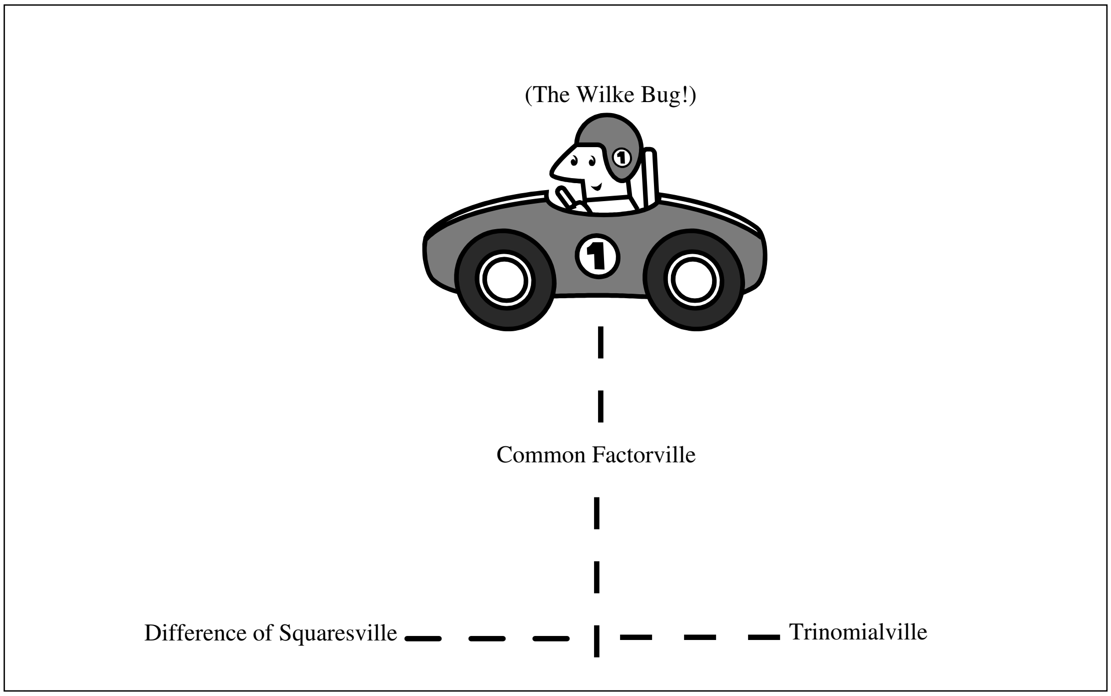
Let me offer another example. One of the most important concepts taught in high school mathematics is factoring. The skill of factoring is taught in every grade from nine to twelve. Less complicated factoring is introduced in the elementary school grades. To be successful in each of these courses, it is vital that students understand how to factor, why to factor, when to factor, and what to factor. For that reason, factoring is a huge "production" in my classroom. If there is one thing my students remember about math when they graduate it is the Wilke Bug.

The Wilke Bug is used as an analogy for factoring. In grade nine, the fundamentals of factoring are learned. Students begin by learning common factoring. Given any polynomial, the first step is always to look for a common factor; for example, $3x^2+6x$ can be factored into the form $3x(x+2)$ because the factor $3x$ is shared between the two terms. After common factoring, there are two other main types of factoring: difference of squares and trinomials. Difference of squares has two terms, such as y^2-16. This can be factored into $(y + 4)(y – 4)$ since its expansion is its corresponding multiplication question. Trinomial factoring has three terms—for example, x^2+5x+6. This can be factored into $(x + 2)(x + 3)$ for the same reason. (Just as +2 and +3 multiplied is +6; and added is +5). So, where and when does the Wilke Bug roll into town?

The procedure for factoring is compared to the process of driving from town to town using a map to guide the steps. The road first leads to the town of Common Factorville. It is always the first stop! If a common factor exists between the terms of the polynomial, it must be removed! I explain to my students that the journey does not end at Factorville. "Keep driving" is a phrase in common use in my classroom. However, after Factorville, the road forks and students must then choose which path to take: two terms leads to the town of Difference of Squaresville, while three terms leads to Trinomialville. Understanding this concept is important because some factoring questions involve both types; for example, $2x^2–2$ is first factored using common factoring $2(x^2–1)$, before one continues to Difference of Squaresville" $2(x + 1)(x – 1)$.

So why bother with such an elaborate metaphor to explain a mathematical operation? Wouldn't it have been more straightforward just to show the students the three methods and then assign them practice questions? Maybe! But, for some reason students love the whole Wilke Bug experience! Past students walk by me in the hallway and ask me "How's the Wilke Bug?" Some students want me to get a license plate with the letters WILKBG on my car! Each year, I have a Wilke Bug contest in my math classes. The person that designs the best Wilke Bug receives bonus marks! (See the attached picture for last year's winning entry). In higher grades I introduce students to other types of factoring, and I add more paths to the fork in the road. I believe that this type of activity contributes to an enjoyable, exciting, and unique learning atmosphere. Students appreciate that there's a serious side to

Students use a factoring map to drive the Wilke Bug through the process of factoring. I draw the factoring map as follows:



having fun in the class—they are actually learning one of the most important concepts in high school mathematics.

The use of metaphors in the classroom is an effective tool that teachers have at their disposal in any grade or subject. Egan (2005) lists metaphor as one of the cognitive tools used to benefit student learning. The Wilke Bug is an example that plays upon a student's (and teacher's) imagination. "Metaphor is the capacity, or cognitive tool, that enables people to see one thing in terms of another" (Egan, 2005, p. 13). In this example, students use a simple map to guide them through the journey of factoring. Egan (2005) argues that, "The use of appropriate metaphors can stimulate the imagination and creativity in all subject areas" (p. 13).

Another cognitive tool that I frequently employ when I teach factoring is the use of role-playing. After the whole factoring routine is explained, my class practices questions using the Wilke Bug. I set up two chairs in the front of the classroom and ask for a volunteer. Together, we drive the Wilke Bug around the classroom using our map to guide us to the different towns. I start off by calling shotgun. The use of this type of role-playing is an extremely effective teaching technique, and it can be employed in any subject and grade level. Role-playing engages students in a physical manner and helps students to learn and retain information. Students are captivated when material is presented in a different, often outrageous manner. As Egan (2005) states, "there are endless ways to shift the context so that the routine classroom becomes a place where students never quite know what to expect… the imagination can transform the classroom" (p.105). In my classroom I sometimes feel that I am more of an actor than a teacher. Acting out the Wilke Bug routine is part of the math show!

Another important tool for learning used throughout these classes is mental imagery. I ask students to picture in their minds the crazy Mr. Wilke driving the Wilke Bug around the classroom when they are at work on factoring questions. This helps them to remember the routine. These images provoke positive and humorous feelings, while contributing to their understanding of the subject material. In mathematics, it is extremely important to visually picture ideas and strategies to various problems. Talking through questions can sometimes help struggling students.

When it comes to evaluating students on factoring, I ask them to write a factoring letter. They are to write a letter to anyone they wish explaining the factoring process, so that the reader will be able to do factoring. The letter allows the student to go through factoring in a manner that works for them. Writing down the routine allows them to sort through difficulties and put the procedure in their own words. In other words it helps reinforce the idea.

Carl's series and the factoring Wilke Bug are two examples of the use of imagination in teaching mathematics. As educators it is our responsibility to engage students in meaningful and exciting lessons that aid in learning course material. The lessons are imaginative in that they do just that. There are many techniques to present material imaginatively, and it is the teacher's task to develop various activities that challenge students to think in creative and innovative ways. The lesson about Carl employs storytelling, discovery learning, mystery, and humor to achieve learning outcomes effectively. The Wilke Bug makes use of role-playing, metaphors, and imagery. These are a few of the imaginative learning tools at our disposal. Be creative and have fun!

References

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