Bildung as a Powerful Tool in Modern University Teaching

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Abstract

In this paper we will demonstrate how powerful "Bildung" is as a tool in modern university teaching. The concept of "Bildung" was originally introduced by the German philosopher Immanuel Kant (Kant 1787, 1798, 1804) and the Prussian lawyer and politician Wilhelm von Humboldt (Humboldt 1792, Bohlin 2008). From 1810 "Bildung" was a key concept in German university teaching where the main purposes were to give the students: 1. advanced teaching based on research, 2. ability to carry out scientific research on their own, and 3. a large amount of scientific and philosophical knowledge within all academic disciplines such that they could act with dignity as members of the learned and academic society. (Flexner 1930, Huxley 1876, Jaspers 1923, 1946).

"Bildung" was one of the most important reasons for German science to become leading in Europe in the 19th century (Petersen 1993), and in many other countries "Bildung" was introduced at the universities from around 1850. (Oersted 1850). In Denmark this happened in 1848 where the great philologist Nicolai Madvig accomplished an extensive reform of the teaching at the University of Copenhagen. The faculties were reorganized and new disciplines such as economics were introduced. Especially he established the new discipline "Philosophicum", a one year course in the most important philosophical items that was compulsory for all university students. (Petersen 1993, Slagstad 2003).

In Denmark "Bildung" is called "Dannelse", in Sweden they say "Bildning" but in English there is no word for it. Although we (with full right) might use the nouns "Education" or "Enlightenment" for "Bildung" none of these words are fully covering the concept, so we choose to say "Bildung" also in an English text.

University teaching where "Bildung" is a main ingredient requires many resources and long term studies such that the students have enough of time for intellectual reflection and contemplation. In the modern mass universities where it is a main purpose producing many candidates as quickly as possible the courses are often cut short and there is no time for deeper contemplation. In many cases there have even been set up an upper limit for the number courses the students are allowed to pass and it is obvious that "Bildung" more or less has disappeared as a tool in modern university teaching.

But university teaching without "Bildung" is a poor mass production with no perspective and without the core elements that make the students become academics. At the Department of Economics, University of Copenhagen, Denmark, the teaching of mathematics has been changed since spring 2008 such that "Bildung" is brought into the lectures as a main ingredient with remarkable results. (Olesen (17) 2008). How this works and how "Bildung" has become a powerful tool in the teaching of mathematics is the theme of this paper.

Introduction

During the last 10 years Danish high school teaching has been changed considerably. Earlier high school teaching was explicitly given to make the students suitable for university studies. They even had a lot of disciplines on an academic level such that they without too many difficulties were able to pass the gap between high school education and university studies. But since the late 1990ies and especially since 2005 Danish high school teaching has changed such that the academic training has totally disappeared. (Damberg 2006). Why this happened was actually a riddle for university teachers and many more people working in the higher educational system but it was said from the authorities in the Ministry of Education that it was necessary to modernize high school teaching. Why this modernization should imply a disastrous decrease in the level of teaching in all subjects was rather surprising and never explained.

The consequences of the new high school teaching (here and in the following called "reform teaching") were dramatic. When the students graduated from high school they didn't know anything about mathematical thinking, mathematical proofs, mathematics as a scientific discipline, scientific history, and even nothing about the most elementary mathematics such as reduction and calculation with fractions. Also within other disciplines the same pattern was seen. In physics, for example, electromagnetism, ideal gasses or simple atomic theory are no longer included in the syllabus, and they can't use a semi-logarithmic coordinate system to describe a radioactive decay. (Olesen (16) 2008).

So what do they actually learn? In mathematics they play with advanced graphical calculators but this is of no use if you don't know the basic mathematical concepts and definitions, and if you don't even know the arithmetical rules it is absolutely impossible to handle an advanced calculator correctly, so this is a bad pedagogical way of teaching. Therefore we can say that the students practically know nothing about mathematics when they are matriculated at the universities and the gap between high school level and university teaching has become very large.

It is clear that Danish university teaching is very much influenced of this dramatic change in high school education and of course it is impossible to continue traditional university teaching as if nothing has happened. In this paper we will examine this problem carefully and we will show how powerful "Bildung" can be as a new pedagogical tool in modern university teaching.

First we will look at the poor mathematical prerequisites the students have when they come to the university and then we will follow two different groups of first year students at the University of Copenhagen: 1. a group of students from The Faculty of Pharmaceutical Sciences having a traditional undergraduate course of mathematics, and 2. a group of students from The Department of Economics having a similar course of mathematics but here with a large amount of "Bildung" integrated in the lectures. The difference between the examination results of these two groups turned out to be enormous and remarkable (Olesen (18, 19) 2009), and there is no doubt that "Bildung" must be brought back into university teaching if we still want to educate candidates having a great amount of scientific knowledge and understanding on a high academic level.

Testing the Students' Mathematical Prerequisites

All first year students are matriculated at The University of Copenhagen on September 1 every year. To examine the students' mathematical prerequisites from their high school learning they were strongly urged to participate in a special test. Unfortunately the test was optional but so many students from both The Department of Economics and The Faculty of Pharmaceutical Sciences participated (62.5 per cent) such that the results from the tests could be considered being significant.

The test for the students of economics consisted of 21 different problems concerning fractions (both with numbers and algebraic terms and symbols), solutions of simple equations of first and second order, special functions (logarithms and exponentials), finding derivatives and anti-derivatives of simple functions and constructing a simple mathematical model of a given phenomenon described in words. The test for the students in pharmacy consisted of 20 problems within the same issues. (Olesen (16) 2008).

The duration of the two tests was 2 hours and the students were not allowed to use any electronic calculators, computers nor any kind of textbooks.

116 students from The Department of Economics participated in the test and if all problems were correctly solved it was possible to score 130 points. Assessing the solutions the students could obtain the following "marks": very bad (0 – 25 points), bad (26 – 50 points), middle (51 - 75 points), good (76 – 100 points), excellent (101 – 115 points), and outstanding (116 – 130 points).

The following table shows the results of the test:

Mark	Interval	Number of students	Percentage of students
Very bad	0 – 25 points	32	27.59
Bad	26 – 50 points	64	55.17
Middle	51 – 75 points	15	12.93
Good	76 – 100 points	4	3.45
Excellent	101 – 115 points	1	0.86
Outstanding	116 – 130 points	0	0.00

We notice that 96 or 82.76 per cent of the students were not able to score more than 50 points which is absolutely unacceptable. In fact they had no mathematical knowledge that was sufficient to start studying economics. And only 5 students or 4.31 per cent had solved the test better than middle.

Let us now turn our attention to the test given to the students at The Faculty of Pharmaceutical Sciences. This test was very similar to the test mentioned above (the same mathematical issues) but there were only 20 problems to be solved. This reduction was decided by leader of the study board. (Olesen (16) 2008).
86 students studying pharmacy participated in the test and if all problems were solved correctly it was possible to score 104 points. Assessing the solutions these students could also obtain the following "marks" but with slightly different intervals: very bad (0 – 25 points), bad (26 – 50 points), middle (51 - 70 points), good (71 – 85 points), excellent (86 – 95 points), and outstanding (96 – 104 points).

The following table shows the results of the test:

Mark	Interval	Number of students	Percentage of students
Very bad	0 – 25 points	45	52.33
Bad	26 – 50 points	30	34.88
Middle	51 – 70 points	10	11.63
Good	71 – 85 points	0	0.00
Excellent	86 – 95 points	1	1.16
Outstanding	96 – 104 points	0	0.00

This result was also terrible. 75 students or 87.21 per cent could not obtain more than 50 points and only one student had a score better than “middle”. Indeed this was also absolutely unacceptable and in both cases one might say that more than 80 percent (or 4 of each 5 students) actually were not qualified to start their university studies. They had such a poor knowledge of elementary mathematics and simple logical reasoning that in fact they knew nothing. This was indeed an unveiling proof of the bad high school teaching in mathematics (the “reform teaching”) given in Denmark especially after 2005. (Damberg 2006, Olesen (16) 2008).

Analyzing the students’ solutions of these two tests it was obvious that they were not able to construct a simple mathematical model, they were not able to solve very simple differential equations using integration, and they were not able to find anti-derivatives. But also we saw that most of them couldn’t handle common fractions and that was really surprising. (Olesen (16) 2008).

Confronted with the terrible results from the two tests the leading authority of high school mathematics in the Danish Ministry of Education denied that this could be a problem for the first year university teaching and he advised not to concentrate so much on the students’ mathematical capability. This really was a total defeat for the “reform teaching” but - and that was a much worse disaster - there was no intention from anybody in The Ministry of Education or among the Danish politicians to change anything or to realize that the high school teaching didn’t work at all.

The study boards at the Faculty of Pharmaceutical Sciences and at The Department of Economics reacted quite differently. At The Faculty of Pharmaceutical Sciences it was decided to teach the students in exactly the same way as usual. There were not enough resources it was said, to give more lectures or classes and the students just had to work harder. But since they were not trained doing studies on their own this certainly would turn out not being a success. Looking at the rate of failure for first year exams of mathematics at The Faculty of Pharmaceutical Sciences a dramatic development had taken place since 2004. The rate was increasing each year which is shown in the next table (Olesen (18) 2009):

Year	2004	2005	2006	2007	2008
Rate of failure	19.7	20.8	24.4	25.5	34.1

At The Department of Economics a similar development was seen with still increasing rates of failure. (Olesen (14) 2007, (15) 2008 (17) 2008). But instead of doing nothing new the lectures were enlarged with 50 per cent and the classes were structured in a very special way with a lot of group work. Furthermore 4 evening sessions each lasting 3 hours were introduced where the students under professional guidance could work concentrated with more complicated mathematical problems and with more or less advanced mathematical modeling. But the most important thing was that mathematical “Bildung” should play a central role in the lectures such that the students might experience that mathematics is not only a practical tool but rather a philosophical science with great importance for our way of thinking and describing phenomena both in the physical world and in society. So the cultural and philosophical aspects of mathematics were considered as important as the practical aspect of solving problems. (Olesen (17) 2008).

Introducing Mathematical Bildung

The main question was how to introduce mathematical “Bildung” to first year university students who had never been confronted with simple mathematical reasoning and deduction, who had never heard about the axiomatic method, who didn’t know how to solve elementary problems, and who had no practical knowledge of common mathematical terminology?

The answer of this question was starting the lectures telling the students about mathematical and scientific thoughts in a way they would never had imagined nor expected. Let us call this way of lecturing “the good story”. (Olesen (13) 2007).

Therefore the lectures started with the issue “What is mathematics and what is science”? Here the axiomatic method seen in a historical and cultural perspective as an important development of the human mind was immediately introduced and we concentrated about the following items: How did the Greeks in Antiquity create mathematics? What is an axiom? What is a proposition and what is a theorem? How do we use logics and how do we prove a proposition or a theorem only using what we consider to be true? Next the attention was turned to the most fundamental issues of epistemology and Kant’s famous thoughts of scientific understanding, mathematical reasoning, and assertions being a priori, a posteriori, analytic or synthetic. (Gowers 2008, Kant 1781, 1804, Olesen (13) 2007, Wolff 1963).

This seemed to be difficult and theoretical to a vast majority of the students but they saw, that there exists a much larger and more interesting world of thought and scientific behavior than they had ever expected and they began showing a new personal engagement which is very important for all university students. They began participating in philosophical discussions and they were eager to learn more. To cut it short: They had been motivated to university studies!

The lectures went on telling about mathematical formalism and abstract set theory starting with Euler’s diagrams or circles from 1761 (Sandifer 2007) and George Boole’s logical system (Boolean algebra) from 1854 (Gowers 2008, Wolff 1963), then Georg Cantor’s theory of sets and a little about countable sets and transfinite cardinals (Gowers 2008), and after this ending up with Bertrand Russell’s famous paradox from 1904 (Gowers 2008, Olesen (13) 2007). This was done to show the students that a logical system has certain limitations that might lead to strange and surprising paradoxes. Then it was also natural to talk about Kurt Gödel’s incompleteness theorems from 1931. (Gowers 2008, Olesen (13) 2007).

All this was also philosophical and very theoretical but it was very important for the students to know about scientific methods such that they obtained academic competences. But of course it was also very important and motivating to work with some apparently practical problems in a larger scientific context. Talking about an old given mathematical problem that was solved many years ago and later generalized to a modern theory with important applications within other disciplines was the main idea for selecting relevant topics. Here the lectures first concentrated about describing the famous problem of passing each of the 7 bridges in the city of Königsberg only once in one continuous path. The problem was solved by Leonhard Euler in 1735 (such a path didn't exist!) and later on he developed his general method so much that it became possible to build up a useful theory for net work analysis and advanced graph theory which is so important for modern computer science and economic theory. (Hopkins 2004, 2007, Olesen (13) 2007, Wolff 1963). This "good story" and many other "good stories" from the history and foundation of mathematics had a remarkable impact on the students' engagement and helped them looking at mathematics as an important part of human culture and science. In fact all these "good stories" became the core of what we might call "Mathematical Bildung", and "Mathematical Bildung" was now a pedagogical tool in the basic university teaching of the first year students at The Department of Economics.

After this philosophical and historical introduction to the lectures of mathematics it was extremely important to continue this process using "Mathematical Bildung" in the subsequent lectures where many different specific topics were taught. For example: Lecturing on "orthogonal matrices" which were introduced by the French mathematician Charles Hermite in 1854 we went further back to 1770 when Euler for the first time considered a system of linear equations in which an orthogonal matrix was used implicitly without knowing anything about matrices in general or orthogonal matrices in particular. (Sandifer 2007). This was done with purpose that the students should see how mathematics as a deductive science has developed and such that they got an idea of how scientific research often is based on solving a specific problem and then later on generalizing this to a proper theory.

For the students knowing almost nothing about mathematics and absolutely nothing about mathematics as a science when they were matriculated at the university "Mathematical Bildung" was new, surprising, and challenging. They also became more interested in doing mathematics on their own and they became more active and personally engaged. So now it was exciting to experience how their oral exam in mathematics would turn out.

EVALUATION CRETERIA

From September 2007 all Danish university exams should be assessed using a scale consisting of 7 grades: 12, 10, 7, 4, 02 (which are passing grades), and 00, and -3 (which are failure grades). The top grades are 12 and 10, the middle grades are 7 and 4, and the grade 02 is given for just passing the exam. The failure grades are 00 and -3, where -3 is only given if almost nothing is correct. Compared to the international ECTS-scale we have that 12 = A, 10 = B, 7 = C, 4 = D, 02 = E, 00 = Fx, and -3 = F.

Between the percentage of scores and the ECTS grades the following scale of equivalence is basically used for assessing written exams:

Percentage	0 – 10	11 – 49	50 – 59	60 – 67	68 – 83	84 – 91	92 - 100
Grade	-3	00	02	4	7	10	12

The Oral Exam at The Department Of Economics

The examination in the first year course "Mathematics A" at The Department of Economics is oral with both internal and external examiners. Each student is examined during approximately 25 minutes.

All students have preparation time of approximately 25 minutes after having drawn out one of the following 16 examination questions:

1. Differentiability for a function of one real variable and differentiation rules.

2. The Chain Rule, higher order derivatives, and implicit differentiation.

3. Inverse functions and exponential and logarithmic functions.

4. The Mean-Value theorem.

5. Taylor polynomials.

6. Anti-derivatives, the definite integral and the Fundamental Theorem of Calculus.

7. The method of substitution and integration by parts.

8. Integrals of rational functions.

9. Sequences and infinite series.

10. Functions of several variables and partial derivatives.

11. Higher order derivatives of functions of several variables.

12. The General Chain Rule and homogeneous functions.

13. Extreme values for functions of several variables.

14. Improper integrals.

15. Probability.

16. First order differential equations.

During preparation the student can read in all relevant textbooks and use all the material from the teaching. But it is strictly prohibited to communicate with any other person or via the Internet or by using telephone.

The examination itself is a conversation between the student and the examiners and it is expected that the student introduces the topics from the drawn examination question. During the examination the student is allowed to use a short outline worked out during preparation.

After the examination and the discussion between the examiners the internal examiner will tell the student the grade that is obtained.

In January 2009, 145 students were examined in the first year course "Mathematics A" at The Department of Economics. The result was interesting and also very positive which is seen from the table below:

Grade	-3	00	02	4	7	10	12
Numbers	4	10	10	19	29	24	49

The typical grade was 12, and only 9.7 per cent of the students failed. The average grade for all students was 7.7 and for the students passing the exam the average grade was 8.6. Such an excellent result had never been seen before. (Olesen (19) 2009).

The Written Exam at The Faculty Of Pharmaceutical Sciences

The written exam in mathematics has duration of 4 hours. The students are allowed to use their textbooks but not any electronic calculator or computer. The set of exam problems consisted of 4 problems—2 in linear algebra (vectors, matrices, systems of linear equations) and 2 in mathematical analysis (functions of one and several variables, partial derivatives, exact differential forms, integration, complex numbers).

On January 14 2009, 207 pharmacy students participated in the written first year exam of mathematics. The result (shown in the table below) was the worst ever seen at The Faculty of Pharmaceutical Sciences (Olesen (18) 2009):

Grade	-3	00	02	4	7	10	12
Numbers	6	76	45	29	41	7	3

The typical grade was 00 which is a failure grade. The rate of failure was 39.6 per cent. The average grade for all students was only 2.8 and the average grade for the students passing the exam was 4.8. We notice that only 10 students got the top grades 10 and 12. Furthermore 45 students or 21.7 per cent just passed the exam with the grade 02. This was a very poor result, and therefore it was specifically interesting to investigate all the students' solutions to find out what exactly had gone wrong. This close investigation showed that the lack of elementary mathematical knowledge and training in logical argumentation were the main causes for failure. It was also interesting to see that many students just passed because they were able to solve the two problems in linear algebra at an acceptable level. Linear algebra is not taught at all at Danish high schools and although linear algebra might be abstract the students were able to learn this new mathematical discipline much better than analysis building on elementary high school teaching. This showed that most of the students of pharmacy actually had mathematical capability but the bad teaching at high school had given them an almost invincible problem that caused them failing. (Olesen (18) 2009).

It was now quite obvious that if the rate of failure should be decreased substantially the students must have much more teaching of elementary mathematics and training in mathematical thinking during their first year course at The Faculty of Pharmaceutical Sciences. This will require more lectures and classes and the teaching must be developed to engage the students and make them more active and productive.

Conclusion

We have noticed that the Danish high school teaching has become so weak and poor especially since 2005 that the students don't learn either mathematical thinking or enough of elementary mathematics. This implies that they will meet many unnecessary problems when they start a university study but we have also seen that these problems were redressed if the first year teaching is enlarged (with app. 50 per cent) and enhanced by introducing "Mathematical Bildung". If this is not done the rate of failure will increase to an unacceptable level such as we saw at The Faculty of Pharmaceutical Sciences. But if "Mathematical Bildung" becomes a main ingredient in the teaching of mathematics the students learn much more and they will be more motivated and engaged. Furthermore they will see mathematics as both an important scientific and a cultural discipline that plays a central role in many applications in human activity. And at last: The students will also obtain a stronger academic attitude and profile which is very important and desirable in modern mass universities.

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