A Matter of Interest

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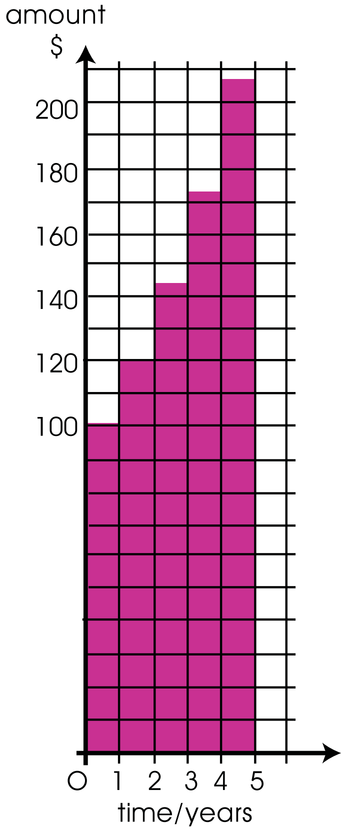
Be a prophet of profit

In these days of financial turmoil, there is greater interest in depositing one's money in the bank — at least we might hope for greater interest!

Banks and various trusts pay compound interest at regular intervals: this means that interest is paid not only on the original sum deposited, but also on previous interest payments.

The diagram at right shows the rate of increase of a deposit of $100 when a bank pays compound interest at the rate of 20% per year (which bank?!).



The first interest payment is

$$\frac{20}{100} \times \$100 = \$20$$

bringing the total amount to $120.

The second interest payment is

$$\frac{20}{100} \times \$120 = \$24$$

bringing the total amount to $144.

1. (a) Work out the next three interest payments and the corresponding total amounts.

   (b) Evaluate $\$100\left(1+\frac{20}{100}\right)^n$

   for n = 1, 2, 3... What do you find? Can you show how this expression is obtained?

You may know already that if r% compound interest is paid at regular intervals, then after n payments, $1 has grown to

$$\$\left(1+\frac{r}{100}\right)^n$$

Notice the exponential nature of this expression.

To celebrate its hundredth anniversary, the local bank decides to pay its customers compound interest at the rate of 100% per year for one year. Further, each customer can ask to have proportional amounts of interest paid into their account regularly throughout the year; for example, quarterly or monthly. What would be your choice?

Customer	A	B	C	D	E
Deposit	\$1	\$1	\$1	\$1	\$1
Number of interest payments	1	2	3	4	5
Proportion of interest	100%	50%	33.3%	25%	20%
Amount of interest	\$1	\$1/2			
New balance	\$(1+1) = \$2	$\$(1 + ½)^2 =$			

2. Customers Alf, Belinda, Charlie, Dora and Egbert have asked that interest be paid in once, twice, three, four and five times a year respectively. Complete this table to discover who has made the best choice. What formula gives the balance for customer Noodles who has interest paid in n times a year?

It is clearly a good idea to ask that interest be paid in as often as possible. When the number of interest payments is n, the new balance is

$$\$\left(1+\frac{1}{n}\right)^n$$

As n gets larger,

$$\left(1+\frac{1}{n}\right)^n$$

increases slowly and steadily towards e. This is indicated in the following table.

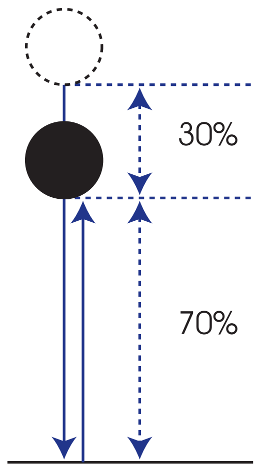
n	10	100	1000
$\left(1+\frac{1}{n}\right)^n$	2.5937...	2.7048...	2.7169...

Extension

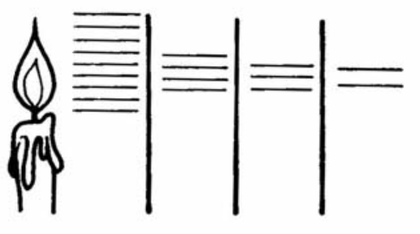
3. We have been discussing *compound* interest. Why does it have this name? In what way is it different from *simple* interest?

4. Since it is not celebrating its centenary, my bank only pays 2% interest once a year (pity!). If I initially deposit \$100, what will be my balance in three years? What will be my balance in six years? What formula gives the balance in t years?

5. A rubber ball rebounds to 70% of the height from which it is dropped. If it is initially dropped from a height of six metres, find the height to which it rises after:
(a) one rebound
(b) two rebounds
(c) three rebounds.
Show that after n rebounds it rises to a height of
$6\left(1-\frac{30}{100}\right)^n$ metres.



6. A certain non-reflecting glass absorbs 20% of the light falling on it, allowing the rest to pass through. Find a formula for the fraction of light passing through n parallel sheets of glass, as in the figure. How many sheets are required to reduce the fraction of light passing through to below a half?



7. (a) What number does

$$\left(1+\frac{1}{n}\right)^n$$

approach as n increases?

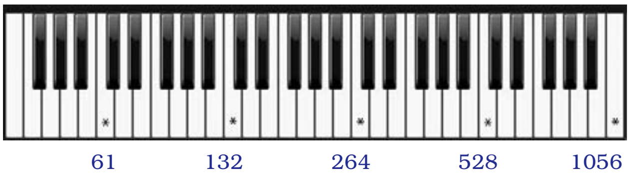
(b) Show that

$$\left(1+\frac{k}{n}\right)^n$$

approaches the number e^k as n increases, by setting $k/n = 1/m$ (i.e., $n = mk$).

Sing a song of six percent

The strings at the bottom end of a piano vibrate very slowly. As one moves up the keyboard, the rate of vibration increases. In fact, moving up one octave (eight whole notes) corresponds to a doubling of the rate of vibration. Thus, if the vibration of middle C is 264, then the C above it will have 528 vibrations and the C below it will have 132.



The same principle holds for adjacent notes. Each string vibrates approximately 6% faster than the string on its left. This means that the string vibrations of the black and white notes in the 264 – 528 octave are given by

$$264,\ 264\left(1+\frac{6}{100}\right),\ 264\left(1+\frac{6}{100}\right)^2,\ \ldots,\ 264\left(1+\frac{6}{100}\right)^{12}$$

By the way: If $264 is invested at 6% compound interest per year, how many years will it take to double itself?

Bibliography

Scott, P. R. (1974). *Discovering the mysterious numbers.* Cheshire.

From Helen Prochazka's

Scrapbook

I tell them that if they will occupy themselves with the study of mathematics they will find in it the best remedy against the lusts of the flesh.
German writer Thomas Mann in his book "The Magic Mountain" (1924)

Math is like love — a simple idea but it can get complicated.
Anonymous

My quest has taken me through the physical, the metaphysical, the delusional and back. And I have made the most important discovery of my career; of my life. That it is only in the mysterious equations of love that any logic or reasons can be found. I am only here tonight because of you. You are the reason I am. You are all my reasons.
John Nash in the film "A Beautiful Mind" (screenplay by Tom Stoppard)

A back two-and-a-half somersault with a half twist, from the pike position. It sounds like the Pythagoras Theorem.
Paul Kent, journalist, writing about a Sydney Olympics 2000 diving event

Life is complex. It has real and imaginary components.
Anonymous