Scaffolding Students' Thinking in Mathematical INVESTIGATIONS

NATALIE MCCOSKER and **CARMEL DIEZMANN** draw some pertinent lessons about effective scaffolding from four episodes in which the teacher's attempts at scaffolding did not have the desired effects.

Introduction

Mathematical investigations are loosely-defined, engaging problem-solving tasks that allow students to ask their own questions, explore their own interests and set their own goals (Jaworski, 1994). The value of investigations for students lies in their complexity. Scaffolding plays an important role in supporting students' high-level engagement by encouraging divergent and creative thinking (Henningsen & Stein, 1997). Scaffolding is "a process that enables a child or novice to solve a problem, carry out a task or achieve a goal which would be beyond his [or her] unassisted efforts" (Wood, Bruner & Ross, 1976, p. 90). Scaffolding provides the opportunity for students to develop their independence, sense-making and self-confidence whilst working mathematically (Williams, 2008). However it is incorrect to assume that all conversations between teachers and students amount to scaffolding because not all result in high-level thinking and reasoning in students. Understanding what is ineffective and why, is one way to improve our pedagogical practice.

This article describes some of the issues that teachers might encounter when scaffolding students' thinking during mathematical investigations. It describes four episodes in which a teacher's interactions with students failed to support their mathematical thinking and explores the reasons why the scaffolding was ineffective. As a background to these episodes, we first provide an overview of the mathematical investigation. Our paper concludes with some recommendations for scaffolding during investigations.

The mathematical investigation

The mathematical investigation, undertaken by a Year 3 class of 24 students, emerged from a picture book entitled *Counting on Frank* (Clement, 1990). On the second reading of the book to the class, the teacher paused on a page that showed a boy surrounded by a large number of peas (Figure 1). She challenged the students to think of a way to find out how many peas had been pushed off the plate and what materials they would need for this investigation.

Small groups of students began their investigations of how they might determine the number of peas by creating a plan using drawings and text. Students were then invited to test their strategies. Upon completion, each group shared their investigatory approach and results with the rest of the class. Students then returned to their plans and recorded what their group did, using various diagrams, calculations, written explanations and flow charts.

The teacher as scaffolder

The teacher assumed the role of scaffolder many times during the peas investigation. Some of these scaffolding interactions impacted positively on the students' understanding of the task and their success with the investigation. However, in order to learn more about the characteristics of scaffolding, we examine four episodes, where the scaffolding appeared to be ineffective.

Episode 1: The need to press for meaning

In the following interaction, the teacher attempted to scaffold Marnie's thinking about the peas problem through questioning. Particularly pertinent phrases are in bold.

Teacher: I'm just having a look at the picture here that you have drawn, and I can see that you have put numbers and arrows. **Can you tell me a little bit about what you have done?**

Marnie: Um…

Teacher: I can see that number one here [teacher points to the number

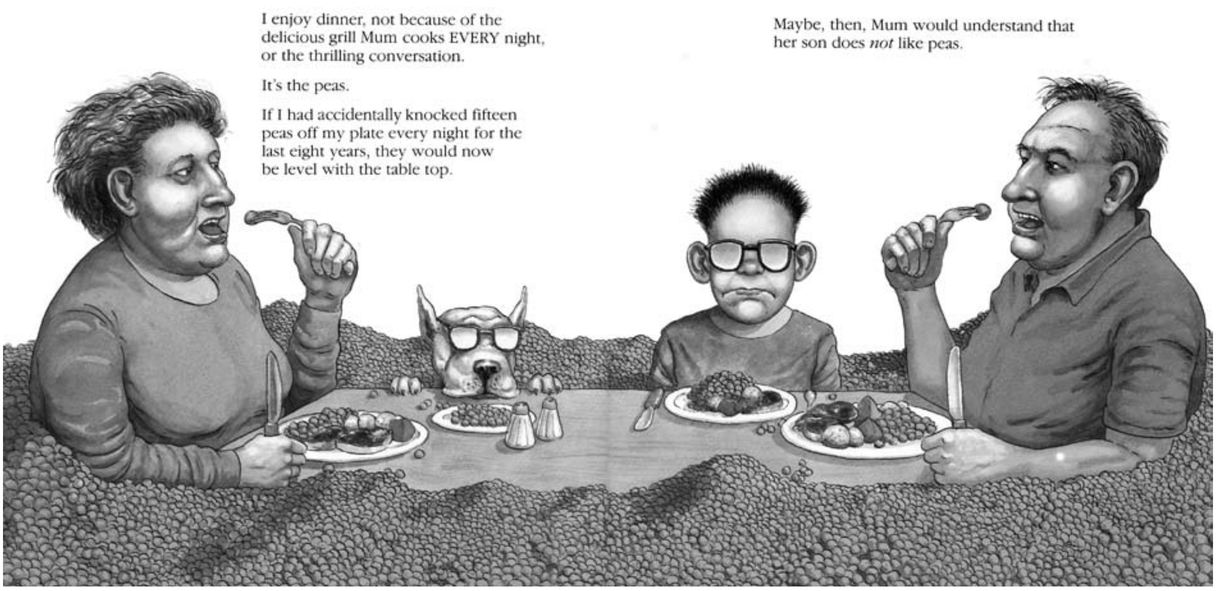

Figure 1. Excerpt from "Counting on Frank" by Rod Clement; reproduced with permission of HarperCollins Publishers.

one] is the desk, and this is a hundred block [teacher points to the drawing]. **Can you tell me what you are thinking?**

Marnie: **Measure a hundred block** on the desk and count in hundreds to see how much peas.

Teacher: Uh… So measure… count… and then you're going to come up with your answer… **What an interesting idea.**

Marnie's response to the first question, "Can you tell me a little bit about what you have done?" was "Um," which gave no insight into what she was thinking. The teacher followed up her response with another question: "Can you tell me what you are thinking?" Marnie's response to this question gave some insight into her approach to the task but she did not fully explain her thinking. Rather than press Marnie for an explanation of what she did or why she was going to "measure the hundred block", the teacher terminated the interaction with the comment, "What an interesting idea." This comment by the teacher limited the level of mathematical thinking and the communication required. Consequently, the potential for her questions to elicit Marnie's thinking were not realised and the scaffolding proved ineffective. Henningsen and Stein (1997) argue that consistently pressing students to provide meaningful explanations supports high-level mathematical thinking and reasoning.

Episode 2: Supporting an understanding of the problem

In the following interaction, the teacher attempted to scaffold Josiah's problem solving skills by prescribing a possible solution strategy to the peas problem.

Josiah: **What are we supposed to do?**

Teacher: Well, **how about you start with drawing the picture at the dinner table?**

Initially, Josiah was kept busy by producing his representation (Figure 2) of the illustration in the book (Figure 1). However, his time was wasted because the production of a realistic drawing as prescribed by the teacher lacked the cognitive advantage of the strategy "draw a diagram." Rather than supporting Josiah's thinking, the teacher's suggestion misled him. He

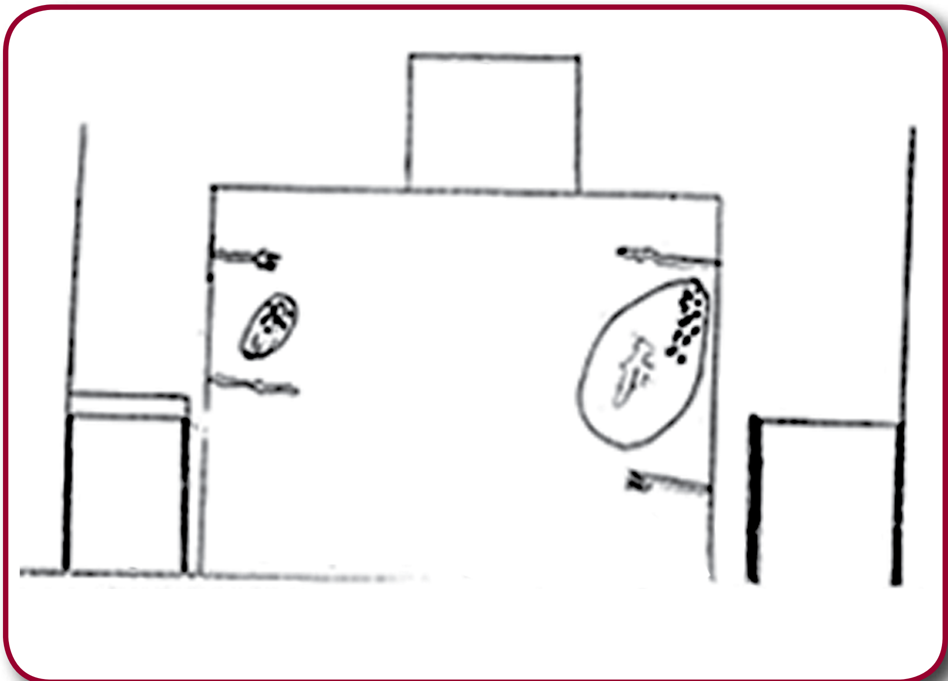

Figure 2. Josiah's representation.

continued to ask for assistance and ultimately failed to reach a solution. Josiah's thinking would have been more effectively scaffolded by developing his understanding of the peas problem. Jacobs and Ambrose (2008/2009) suggest three approaches to developing a student's understanding of a problem:

- ask the student to explain what they already know about the problem;
- rephrase or elaborate the problem; and
- re-contextualise the problem so that it is familiar to the student.

Episode 3: Engaging students in mathematical thinking and reasoning

The following interaction highlights the difference between scaffolding and encouragement. The teacher supported Conrad, who had drawn the picture shown in Figure 3, with positive comments such as, "Oh, I love that picture," and, "That's right. Keep going". However, her contributions were not explicitly directed towards promoting higher-order thinking and reasoning, and hence did not constitute scaffolding.

In the following exchange, the teacher's response to Conrad's admission that he was unsure of how to solve the pea problem was unhelpful because it assumed that Conrad had an accessible solution in his mind.

Teacher: Oh, I love that picture.
Conrad: Is that what we're meant to do?
Teacher: That's right. Keep going.
Conrad: Do we draw the picture now?
Teacher: Well, you can if you want to… What are you going to do now with it, Conrad?
Conrad: I don't know.
Teacher: You're a bit stuck are you? **Keep thinking. Something might come to you.**

It is important to note that for the remainder of the activity, Conrad was unproductive. During the post-activity reflection he wrote, "I didn't no wot [sic] to do," indicating that the teacher's comments had failed to support his thinking. Conrad's need for further guidance highlights a key component of scaffolding which is more than just encouraging the students' actions. It involves the teacher acting as a facilitator so the student is able to achieve more than he or she could without the scaffolding. Conrad could have been asked to think about how

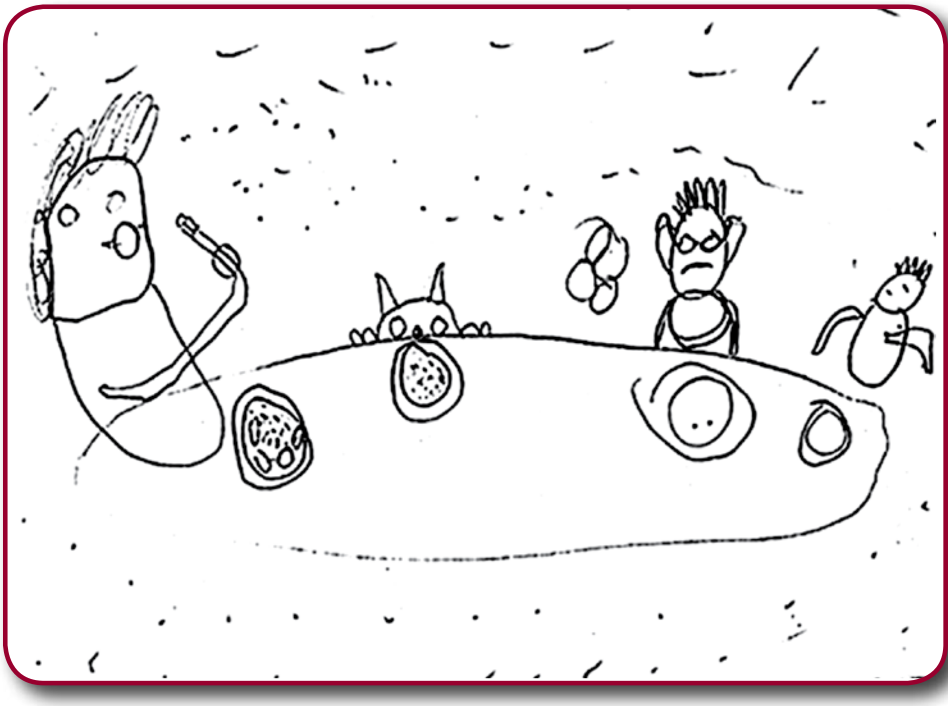

Figure 3. Conrad's picture.

he might investigate the number of peas in a box, therefore scaling the quantity down to a box full of peas instead of a room full of peas.

Episode 4: Fostering mathematical activity

During the post-activity reflection, students were asked to illustrate how they had determined the number of peas in the picture. The following interaction between the teacher and Luke's group highlights the difficulty faced by students who used a mathematical operation as their problem solving strategy. Luke's response to the teacher's instruction of "Just give me a little drawing," was "I don't really know what to draw."

Teacher: Darryl, Jacinta and Luke you probably won't have to draw that… Um… Do that but if you could just draw for me how… Well… **Just give me a little drawing.**

Luke: **I don't really know what to draw.**

Teacher: **Just draw something about it.** Draw a calculator.

In response to Luke's remark, the teacher suggested, "Just draw something about it." The teacher's comment proved counterproductive in developing this group's reflection skills because it conveyed the notion that the reflection activity was tokenistic and effectively gave them 'permission' to bypass reflective thinking. Instead, the group engaged in 'busy work.' For example, Luke drew a picture from the story which was unrelated to how he had originally calculated his answer (Figure 4) and Jacinta took up the teacher's suggestion and drew a calculator (Figure 5). This gave little insight into her problem solving processes beyond the tool she used.

These students' decline from working mathematically to no mathematical activity at all can be attributed to the "lack of suitably specific task expectations" (Henningsen & Stein, 1997, p. 537). To ensure students achieve the benefits of engaging in a mathematical investigation, task expectations need to be clear and explicit. Such a focus supports the students to operate in the "right cognitive and affective space" which in turn encourages high-level thinking and task progression (Henningsen & Stein, 1997, p. 537). Teachers need to precede student activity with an appropriate task

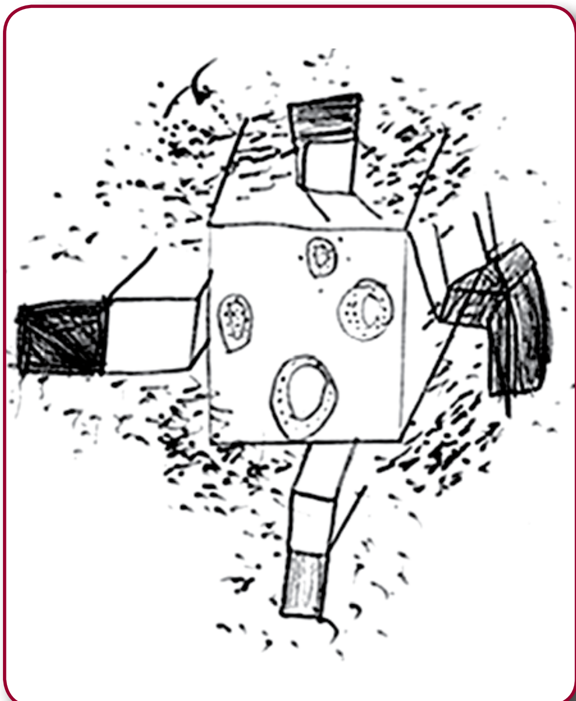

Figure 4

Figure 5

set-up. They will then find that their students are able to use multiple-solution strategies and multiple representations, and produce explanations and mathematical justifications in the majority of situations (Stein, Grover & Henningsen, 1996).

Conclusion and recommendations

Scaffolding in mathematical investigations benefits both students and their teachers. Scaffolding can foster students' creative and divergent thinking skills, and enhance their independence, sense-making and self-confidence in mathematics. By noting students' responses to scaffolding, the teacher is able to identify future topics for mathematical instruction. At the core of judicious scaffolding is awareness of and responsiveness to the students' thinking. Teachers need to:

1. Press students to provide meaningful explanations of problem solving strategies by asking them to explain their actions; for example: "Explain how you solved this problem step by step."
2. Support students' understanding of the problem working from the students' ideas rather than the teacher's ideas through questioning: "What are you trying to find out? Where did you start? What is confusing for you?"
3. Distinguish between positive encouragement and cognitive scaffolding and be willing to provide the latter.
4. Provide unambiguous task instructions and clear expectations whilst ensuring the investigation remains open-ended in approach.

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