Technology Focus: Multi-Representational Approaches to Equation Solving

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Most mathematical functions can be represented in numerous ways. The main representations typically addressed in school, often refer to as "the big three," are graphical, algebraic, and numerical representations, but there are others as well (e.g., diagrams, words, simulations). These different types of representations "often illuminate different aspects of a complex concept or relationship" (NCTM, 2000, p.68), and each has its own features, advantages, and limitations. For students to have a conceptual understanding of functions, they not only need to understand each representation on its own, but also need to be able to link different representations meaningfully. Solving equations using a function approach, with different representations, can help students learn to better connect representations. Here are two sample activities we use with our pre-service teachers to help them think about teaching with multiple representations.

Activity 1: Solving a "Complex" Exponential Equation

We ask our students to find *algebraically* all solutions to the equation: $(x^2-5x+5)^{(x^2-9x+20)}=1$. All re-visit the basic rules of exponents. About half of them consider when the base polynomial is equal to one and find solutions $x=1$ and $x=4$, and then consider when the exponent polynomial is equal to zero and find solutions $x=4$ and $x=5$, thereby arriving at three solutions. The other students find these three solutions in the same way, but also recognize the need to consider the case of $x^2-5x+5=-1$, and thus find the additional solutions $x=2$ and $x=3$. So, the algebraic approach works, but it may not lead to any new insights.

After solving algebraically for all five solutions, we ask our students to solve for x *graphically*, on a graphing calculator. Most students initially do not appreciate the importance of the window settings. They rely on integer values for their scales without considering the screen's resolution. They generate graphs similar to the first graph in Figure 1, and again find only the first three solutions (i.e., x-values where the function has a value of 1) found earlier algebraically.

Students are confused as to why only three of the previously found five solutions are evident on the graph. They are generally unaware of the connection between window range and the number of pixels in a screen, and hence do not realize that the pixels and integer solution values don't "line up" in this window. The second graph in Figure 1, showing all five solutions, was generated using an x range of 9.4 with a calculator having 94 pixels across. Hence, the graphical approach also works well, but only when students choose a window appropriate for the specific task.

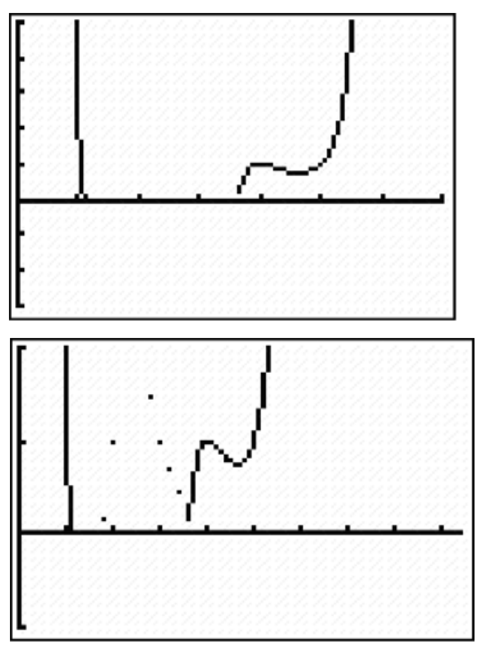

Figure 1: A window with x-max = 7 and a window with x-max = 9.4

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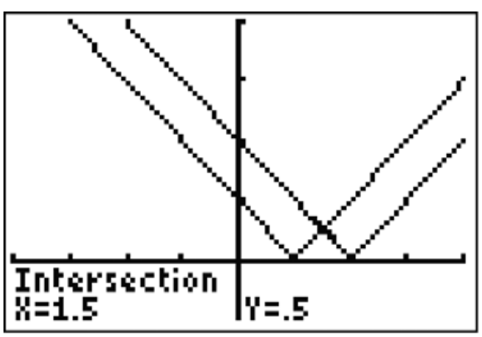
The real benefit of this activity comes from the class discussions connecting the two solution strategies, exploring the pixel issues, and analyzing the missing pieces of the second graph. This task can be used to address various topics, such as pixels, roots, exponents, domain, range and complex numbers.

Activity 2: Solving an Absolute Value Inequality

We ask our students to solve the inequality $x-1 < x-2$ in as many different ways as they can. They often resort to the traditional *algebraic* method they learned in high school, which involves breaking the task up into several cases. This solution method works, but could take from three to seven minutes of class time and is subject to various types of errors. Worse is the fact that too many students using the case method often lose sight of what is being asked. Hence, the resulting solution may not have any meaning to some students.

Many students use their graphing calculators to generate a graphical solution, as shown below in Figure 2.



A graphical solution to an inequality

A majority of them will find a correct solution using the graphical approach, but others will have a bit of difficulty. A few students will focus on the intersection and give an answer to a different question; others will be confused about how to interpret the graphs to provide the correct answer. And, a few will use a *numerical* approach, either guessing and checking or using the calculator to generate a table. The tabular method can still have interpretation problems – a student may use an ineffective increment or have trouble with interpreting the table in a way that will yield a correct solution. Note however, that the above difficulties can be capitalized on to create good teachable moments.

Never have any of our students used a pure *verbal representation*; that is, put the task into words. This approach would involve understanding the meaning of absolute value and understanding inequalities. When prompted to verbalize the inequality, students do come up with verbal representations of the tasks such as, "find the set of all x values whose distance from one is less than their distance from two" or "which values of x are closer to one than to two." At that point they quickly arrive at the solution: all x values less than 1.5. A person with the inclination to use a verbal approach can obtain a meaningful solution in a matter of seconds. This rarely used verbal solution strategy is the most efficient for this type of task. We have found that these activities can be used as springboards for in-depth discussions of various aspects of equations, functions, and problem solving. Also, the activities support the recommendation of the National Council of Teachers of Mathematics that instruction should enable students to "select, apply, and translate among mathematical representations to solve problems" (NCTM, 2000, p.67).

Reference

National Council of Teachers of Mathematics (2000). *Principles and standards for school mathematics*. Reston VA: NCTM.