Technology Focus:
Enhancing Conceptual Knowledge of Linear Programming with a Flash Tool

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Mathematical knowledge can be categorized in a different ways. One commonly used way is to distinguish between procedural mathematical knowledge and conceptual mathematical knowledge. Procedural knowledge of mathematics refers to formal language, symbols, algorithms, and rules. Conceptual knowledge is essential for meaningful understanding of mathematical ideas (Hiebert & Carpenter, 1992) because it is knowledge rich in relationships. Students use conceptual knowledge to develop underlying meaning for procedural knowledge, and to make decisions and problem-solve (Bransford, Cocking, and Brown, 2000; Hiebert & Lefevre, 1986). Competency in mathematics requires both procedural and conceptual knowledge.

Traditional approaches to mathematics teaching and assessment too often overemphasize procedural knowledge at the expense of conceptual knowledge. Not surprisingly, research shows that students' performance on procedural tasks is often adequate but that their conceptual knowledge of mathematics is poor (Grouws, 1992; Silver & Kenney, 2000). The National Council of Teachers of Mathematics (NCTM, 2000), other organizations (e.g., CBMS, 2001), researchers and educators (e.g., Ball, 1991) advocate for placing more importance on the development of conceptual mathematical knowledge.

Illustration: The Corner Point Theorem in Linear Programming

In intermediate algebra and pre-calculus courses, linear programming is often taught as a three-step procedure in which, given an objective function and a list of constraints, students must (1) graph the given constraints in two-space to construct a feasible region, (2) find the corner points of this feasible region, and (3) determine which point or points in the feasible region maximize or minimize the given objective function. In step (3), rather than testing each and every point inside and along the boundaries of the feasible region, the Corner Point Theorem reduces the task to simply testing the corner points. Sometimes called the Fundamental Theorem of Linear Programming, the Corner Point Theorem states:

"Let S be the feasible region for a linear problem, and let $z = ax + by$ be the objective function. If S is bounded, then z has both a maximum and a minimum value on S and each of these occurs at a corner point of the S. If S is unbounded, then a maximum or minimum value of z on S may not exist. However, if either does exist, it must occur at a corner point of S" (Barnett, Ziegler, & Byleen, 2001, p. 472)

We have observed through years of informal conversations and discussions at workshops and conference sessions around the country, and through a small study, that a majority of those who teach linear programming do not understand why the Corner Point Theorem is true. This is the case despite the fact that teachers know and teach procedures associated with this theorem year after year. Essentially, they do not have a conceptual understanding of the Corner Point Theorem. One teacher stated "I know that's what happens but as far as the why's...that's a little sketchier." A second teacher told us "The one thing I felt like I really didn't know was how to do a good explanation of why the max and minimum points occur at the vertices," and a third added "Well, [the corner points] must have some meaning in terms of maximizing or minimizing something."

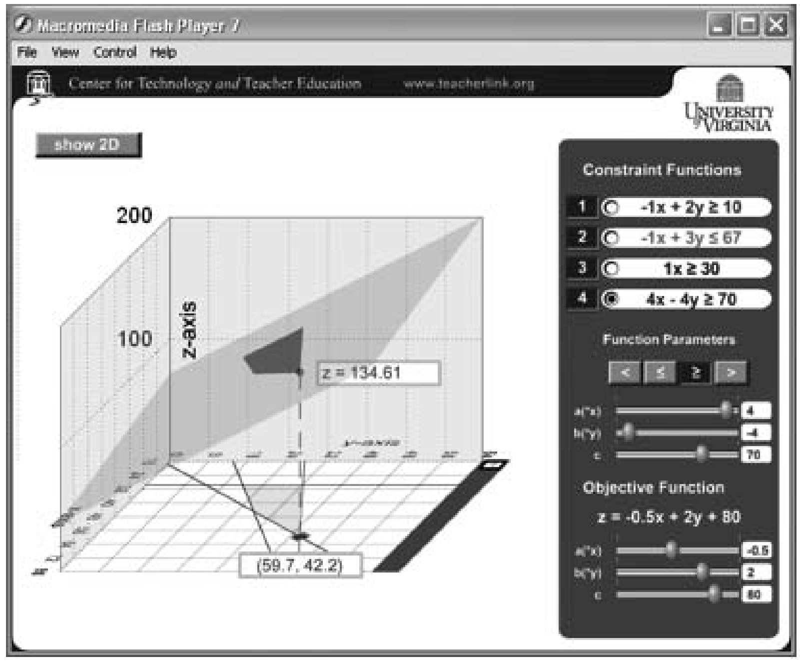
Some teachers try to verify the Corner Point Theorem by having students substitute into the objective function various points from inside, along the boundaries, and at the corners of the feasible region to determine where the objective function appears to have the highest and lowest values. Others do not even try to convince students that the theorem is true. Indeed, one teacher told her students "People have done enough of them that they know that the highest and lowest end up at the corners. Take my word for it."

Linear Programming Tool

Digital technologies offer ways to advance conceptual mathematical knowledge. The NCTM (2000) recognized technology's potential in its Technology Principle: "Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning" (p. 24). In particular, appropriate technology can facilitate the use of multiple representations (numerical, algebraic, and graphical) of mathematical concepts to promote development of conceptual understandings (Garofalo, Drier, Harper, Timmerman, & Shockey, 2000; Jiang & McClintock, 2000).

We developed an interactive Flash tool to help teachers and students visualize and deduce why the Corner Point Theorem works. Our Linear Programming Tool allows one to input up to four constraint functions and an objective function to be optimized (see Figure 1). One can graph these constraint functions and display the feasible region in two dimensions, like one would see in a textbook. As the cursor is moved over the feasible region, the x and y coordinates are displayed, along with the corresponding value of the objective function. Such two-dimensional representations, while often helpful, do not necessarily lead students to understand the linear programming algorithm. Once the feasible region and the graph of the objective function are displayed in *three dimensions*, however, students can easily see that the graph of the objective function is a plane that goes up or down linearly (or stays flat) in each dimension. Because there are no bumps or humps in the graph, as a point in the feasible region is moved in any dimension toward a boundary, its image point is either going up or down on the feasible region's projection on the objective function surface. Students can then make sense of the fact that the objective function reaches its extrema over corner points.

Figure 1. Linear Programming Tool



Teacher's Responses to the Linear Programming Tool

After working with this Flash tool the most high school teachers said they believed using it would help their students develop conceptual understanding of the Corner Point Theorem. They made comments such as "That's exactly what we need to see in class because it would be awesome to make them see why that happens," "This would make it a lot easier to understand the concept - I would use it in a heartbeat," and "This would make it a lot easier to understand the concept without having to actually graph it." These teachers thought the tool would be helpful in presenting the Corner Point Theorem, even though it involved three-dimensional ideas. While a few teachers expressed concerned about the use of a three-dimensional graph, others thought the end result would be worth any extra time needed to discuss three-dimensional graphing. For example, one teacher stated "It would take a day to go over, and look at the big impact you have on linear programming."

Using the Linear Programming Tool

We encourage you to download and use our Linear Programming Tool (Windows or Mac) in your own classes. And, we welcome your comments and your suggestions for improving this tool. You can freely download the tool from our Flash download page at: http://www.teacherlink.org/content/math/interactive/flash/home.html. We have other Flash tools, addressing topics that range from basic fractions through calculus, which can also be downloaded from this page. Feel free to try these tools as well.

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