EXPLORING OPEN-ENDED TASKS AS TEACHER LEARNING

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describe how tasks were trialled by teachers as part of a professional learning experience.

Introduction

The Task Types and Mathematics Learning project is investigating the opportunities and constraints that teachers experience when using particular types of mathematics tasks. Some assumptions underlying this aspect of the project are:

- that teachers seeking a balanced curriculum choose to use a mix of types of tasks;
- open-ended questions offer students and teachers opportunities for both creativity and different ways of thinking about mathematics (Christiansen & Walther, 1986);
- trialling exemplars of particular types of tasks, reflecting on the experience by writing a report, and reporting back to colleagues involves substantial teacher learning, and can form the basis of sustainable professional development (Runesson, 2008; Stigler & Hiebert, 1994).

This article explains what is meant by content specific open-ended tasks, illustrates the ways that teachers wrote up trials of such tasks in their classrooms, and uses excerpts from teachers' reporting on their trials at meetings of project teachers from a cluster of nearby schools, to illustrate the type of teacher learning that resulted from this process.

Content specific open-ended tasks

The basis of this aspect of the project is that teachers develop and trial particular types of tasks and report on the students' learning and any challenges they experience in using the task type. The particular task type that is the basis of this report is described by the project as: "students investigate specific mathematical content through open-ended tasks (task type 3)." The definition provided to teachers as part of the project is:

> Content specific open-ended tasks have multiple possible answers, they prompt insights into specific mathematics through students seeing and discussing the range of possible answers. An example is:
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> *Draw as many different triangles as you can that have an area of 6 cm^2.*
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> Such tasks allow unambiguous focus on specific aspects of mathematics while still allowing opportunities for creativity and active decision making by the students with the advantage that one task can be applicable to a wide range of levels of understanding.

For the example task given, an important first step would be to consider when a particular triangle is the same as another and when they are different. Next, the students would explore a range of right-angled triangles, such as: base 3, height 4; base 2, height 6; etc., and then move on to consider triangles that do not have a right angle. It is this latter aspect that allows students to explore the general rule for the area of a triangle.

The project also made the following suggestions about the associated pedagogies for this task type:

> It is assumed that the teacher will pose the task, will clarify terms, will explain that there are multiple possible responses, but will not tell the students what to do or how to do it. The teacher will orchestrate a class discussion after students have engaged with the task to hear interesting responses (that teachers have specifically identified while the students are working), and will seek to draw out commonalities, and generalisations.

So, in the case of the example task, the teacher might invite students to present examples of the different triangles with an area of 6 cm^2 that they had found, and would also highlight commonalities and differences between the various triangles that are drawn. The advantage of using this type of task is expressed as:

> Tasks of this type offer considerable choice in strategy and solution type, in that students might approach the tasks arithmetically, or they might seek more generalised solutions. Students experiencing difficulty can be posed similar less complicated tasks.

For the example task, the various approaches students use can be highlighted, and the steps toward generating the rule can be emphasised or at least encouraged. Students experiencing difficulty can be posed enabling prompts that present alternate tasks that have the demand reduced in some way such as, "Draw as many different rectangles as you can with an area of 12 square centimetres," and, "Draw some triangles on squared paper, and work out the areas."

Writing up lessons based on open-ended tasks

To illustrate how the process of teacher learning occurs, the following describes lessons that were taught by Mel and Chris. The first lesson is about constructing some timetables and the second is about exploring multiplication. Using a prepared format, each of the teachers wrote a report of the lesson soon after the teaching. The intention is that these reports will ultimately form a resource that can be shared with other teachers. The following is an extract from the written lesson report on timetables prepared by Mel, who was reporting also on behalf of Hayley, who contributed to the planning and teaching of the lessons.

Task description

Students were to create a timetable for a movie theatre. They were given the length of the movies and the opening times of the theatre. The students were also told that the movies needed 15 minutes between them. The task involved the students choosing 5 movies with the duration times of: 1 h 30 min; 1 h 30 min; 2 h; 2 h 20 min; and 1 h 45 min. The theatre opened at 10:00 a.m. and closed at midnight. The students then needed to choose which movies they were going to see on that day and work out how they could see the largest number of movies possible (See Figure 1 for one student's written representation of the task). They were allowed to visit different cinemas and needed to take into account travel time.

Assessment

To be working at Level 4 (typically grades 5 and 6), according to the Victorian Essential Learning Standards, it is expected that students be able to determine the start and finish times of the movie when given the duration. Students are also expected to create the timetable that can include the 5 movies.

Teacher advice

In the trial, this task took students three one-hour sessions to complete. They were then given another session to make advertising posters which led to the next task of choosing which cinema they were going to visit and what movies they could watch in a movie marathon.

Possible enabling prompts

Some suggested prompts to assist students experiencing difficulty in starting the activity including providing clock faces or offering examples of session timetables.

Extension suggestions

Students could vary the order of session times to enable them to see the greatest possible number of movies on the one day.

Feedback

The trial of this task was run with Year 5 students. Most students reported that the task was fun and they wished it was real. They could relate it to their experiences as they had all been to a cinema. Teachers found that students had a limited understanding of time and found it difficult to put the timetable together. We [the teachers] did not expect this finding and needed to spend more time than expected on the construction of timetables.

This is a sophisticated report of the lesson in a form that can be read and appreciated by other teachers. It illustrates both the power of the open-ended task and some suggested strategies that other teachers can use to optimise the potential effectiveness of this task. It is an example of the way that such reports provide evidence of teacher learning.

The second of the lessons, involving a task in three parts, engaged students in exploring aspects of multiplication. This extract is from the report by Chris:

Task description

Task 1: Using the digits 8, 7 and 6, how many different problems and solutions can you find to make this equation.

____ ____×______=

You can only use each digit once in each equation.

Task 2:

______×______= 2280

How many different equations can you find?

Task 3: St Kilda scored 108 points to win a game of football. If a goal is 6 points and behinds are 1 point, what might the score have been? Find as many possibilities for the number of goals and behinds as you can.

Students could choose any order in which to solve the problems. Some students spent all their time doing one problem, while others completed more.

Key mathematical concepts

- explain and use mental and written algorithms for the multiplication and division of natural numbers
- students recognise that multiplication and division are inverse operations
- students establish equivalence relationships between mathematical expressions, using properties such as the commutative property of multiplication.

Difficulties experienced

- Some students needed revision of multiplication.
- Task adaptations for students who finished quickly.

Again this is a sophisticated recording of the process of the lesson development in such a way that other teachers can use it. Clearly both Mel and Chris have appreciated the potential of tasks of this type, and have reported effectively on the opportunities that such tasks offer.

The reports are evidence of significant teacher learning about the process of implementing this particular task type.

Reflecting by reporting back to colleagues

The second aspect of teacher learning from this process was when teachers came together in a cluster of six schools to report on their classroom trialling of the tasks. The following is a selection from Mel's verbal report on the timetable task. One aspect of interest was the origin of the idea:

> It was a closed task in (a text series) as one worksheet and we decided that the kids, because they were Grade Fives, would really love something that's to do with the movies. … So we ended up taking something that they had put in there for half an hour, into four sessions.

The way the lesson went was reported as follows:

> It went over a period of time and we were looking at timetables and we were looking at time … they had to make a theatre, so they could choose the movies that they wanted, and they had a time. So the theatre opened at 10 o'clock and it had to close by midnight. So they had five movies and one of the movies had to go for 2 hours 20 minutes,

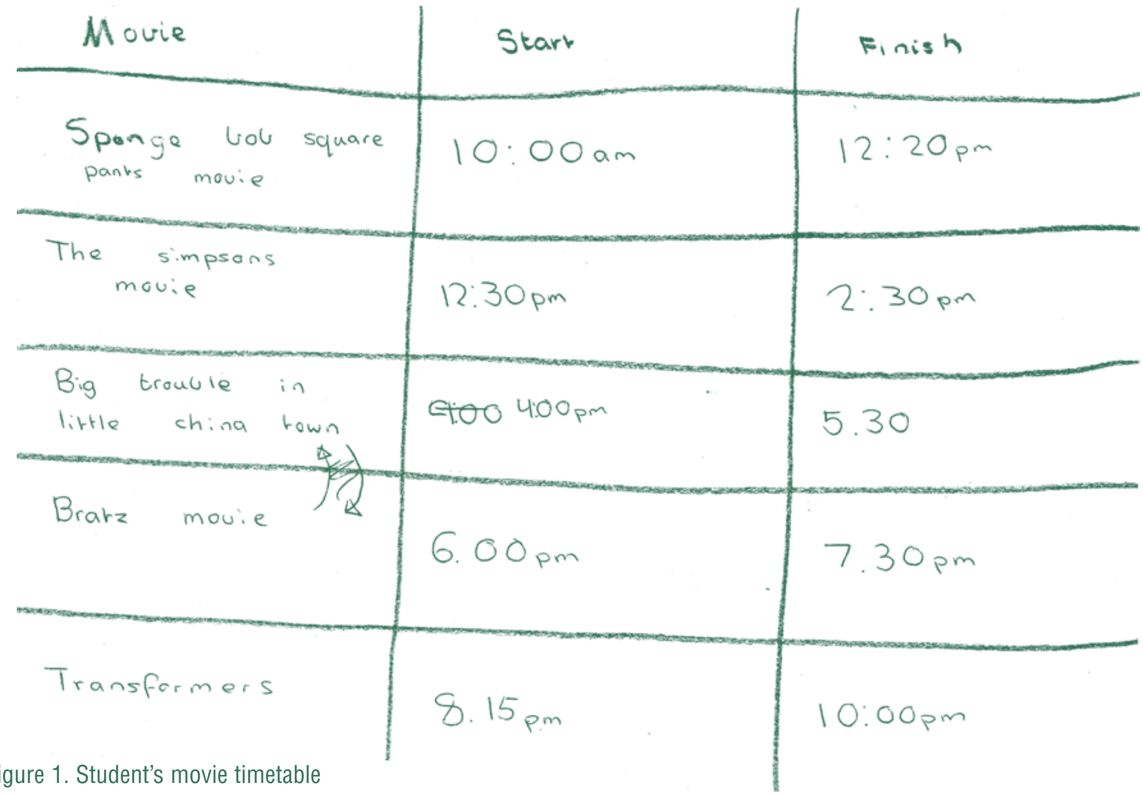

Figure 1. Student's movie timetable

> one was 2 hours, one was 1 hour 30 I think, 1 hour 45. So each of them had their own times to put in … They then selected which movie went for which amount of time, and then they had to write a sequence of when the movie was going to start, when it would end with 15 minutes in between. So the first task was doing their timetable.

This task is actually a combination of Task Type 3 with the context-based Task Type 2 (see Clarke & Roche in this issue). The report continued:

> The kids then had to do their own movie marathon, so they had a day some times, to go to the movies and choose which movie they were going to see at which cinema so that it all fitted into the time. So we were looking for things like whether they were choosing movies that were going to overlap. … So they had to really look carefully and calculate their times so that everything fitted in perfectly.
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> So then the kids … could see how many different combinations of movies they could see to fit into the time slot, so that they could do it on the same day. So that was our activity, and it did go for quite a while because then they, we gave them time to do a proper theatre and go through and write their own movie marathons up.

The lesson is an example of teacher adaptability, of using a context to assist in both posing a problem and maintaining student interest, of using an open-ended format to allow students to make their own decisions, including about the modes of reporting, and the way that the open-ended nature of the task was both engaging and allowed focus on an important aspect of mathematics, in this case time calculations.

Chris also reported on the tasks she had used, including during the multiplication lesson. The report in the meeting went on to highlight a powerful learning opportunity that arose, and some significant multiplicative insights that the students had:

> So it was interesting to see that what most students gained from the lesson was that some students very quickly realised that when you are doing multiplication, if you keep doubling one side of the expression and halving the other side of the expression, the solution won't change. That was probably the most important strategy that they discovered. Once they realised this they were asking, "Ok, does it work in reverse?" If I halve this side of the expression and double that side of the expression does the answer stay the same?" And of course it does, so they were impressed to discover that for themselves. Then they wanted to know, "If I switch halfway, what happens then?" "So if I am doubling this side and halving this side, and then I go back to the reverse after so many different numbers, what happens then?" Then they realised that they will just come back up to where they began, but that was the important thing, for them to discover that themselves through trial and error. They really wanted to know!
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> Once they started that, they were asking themselves, "Oh, is that going to work the other way?" and "How far could I go?" was the other thing they wanted to know. How far can they go doubling one side and halving the other? One of the students, when he got to 375 and he had to halve that, he said, "Now it's getting hard, because it's not going to be a whole number, when I have to halve 375." They had to keep going because they really were motivated to know how far they could go. I actually had to stop them in the end, saying, "No, no, no: we have run out of time now." Some of them went home and continued working on it, using the pattern of halving and doubling, and brought it back to school the next morning to share with the others and show them how far they had been able to continue. That, for me, was the important learning that had come from this lesson.

The quality of the thinking that underpinned these reports, given without notes at an after-school voluntary meeting, is evidence of capacity to design interesting tasks, to implement those tasks in ways that allow students freedom to explore the context and the mathematics, to reflect on the experience and to communicate about that reflection.

Conclusion

In this aspect of our project, the teachers were focusing on content-specific, open-ended questions (i.e., Type 3 tasks). These teachers in the project developed sample tasks, created lessons in which the tasks were embedded, wrote reports on the tasks and their use, and then reported at meetings of colleagues who were also reporting on their lessons.

The samples of the lessons, with write up and reports in meetings, illustrate that the tasks can be used to engage students in creating mathematics for themselves, that teachers can construct effective lessons based on such tasks, and that the reports of such lessons constitute significant teacher learning both for the teacher reporting and those listening.

References

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