Finding the AREA of a TRAPEZIUM

theme and variations

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An important message to convey to our students is that mathematics is founded on reasoning and is not just a collection of rules to apply. Many mathematical rules such as area formulas can be derived in ways that are age-appropriate to students, and it is an important part of mathematics education that students engage in these derivations to some extent.

A review of several Australian Year 8 textbooks showed that they generally included the reasoning behind the area formulas (Stacey & Vincent, 2008). For the area of the trapezium, there was a surprising variety of methods. Each of the textbooks deduced the rule from the rule for a known shape: parallelogram, rectangle or triangle. Some books used a general derivation, with pronumerals a, b and h for the dimensions of the trapezium. Others used specific cases, for example, giving the parallel side lengths of 6 cm and 10 cm. We assume that the authors intended students to see these as generic, that is, that the method would apply to any trapezium, although often this is not stated. A very abbreviated style of explanation (diagrams with few words) is common in textbooks, with certain steps of reasoning omitted, perhaps because the authors assume that extra reasoning would distract from the main argument. It is therefore critical that teachers help students fill in some of these gaps. However, as is illustrated below, there is a rich variety of methods for finding the area of a trapezium, and many students will enjoy discovering these variations on a theme.

Method 1

The most common method was to place two congruent trapezia to form a parallelogram (Figure 1), and then using the rule for the area of a parallelogram that had already been presented. The area of the trapezium is then half the area of the parallelogram with base $(a + b)$ and perpendicular height h.

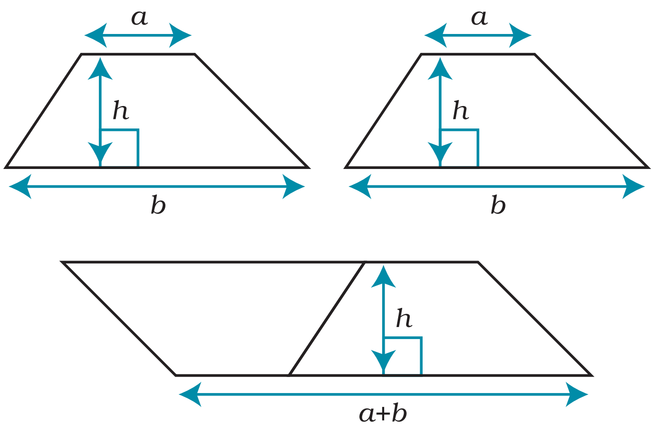

Figure 1

Method 2

The trapezium was dissected and rearranged to form a rectangle of the same height, but with length equal to the average of the two parallel sides of the trapezium (see Figure 2a, b). In one textbook that used this derivation, the justification that the length of the rectangle is indeed the average of the two parallel sides of the trapezium was omitted. In another, students were asked to find the length. This can be argued from similarity using the diagram in Figure 2c. The length PQ, and hence the length of the rectangle, can be seen to be

$$a + \frac{b-a}{2} = \frac{a+b}{2}$$

Students unable to follow the similarity argument may still be able to grasp intuitively that as the parallel line PQ moves from side a to side b, it will attain their average length when it is halfway between them.

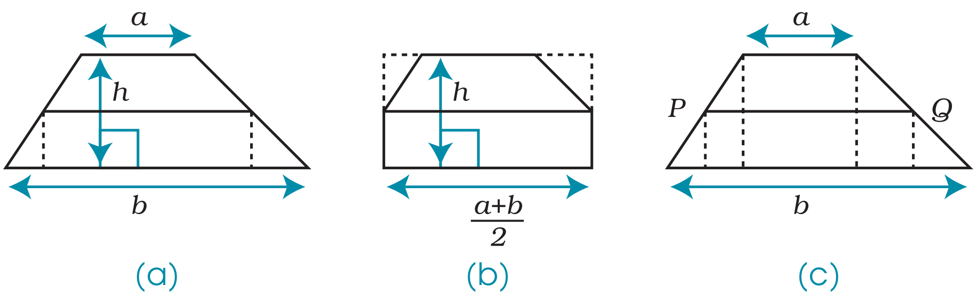

Figure 2

Method 3

A diagonal was used to dissect the trapezium into two triangles, each of which had the same height as the trapezium and base equal to one of the parallel sides of the trapezium (see Figure 3).

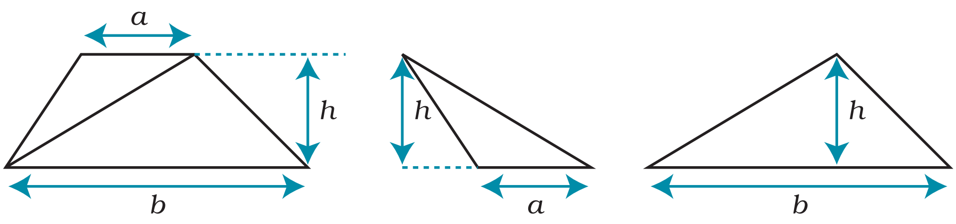

Figure 3

Method 4

The trapezium was dissected to make a parallelogram with the same height and base equal to the average of the two parallel sides (see Figure 4). The length added to the shorter side a is equal to the length taken from the longer side b.

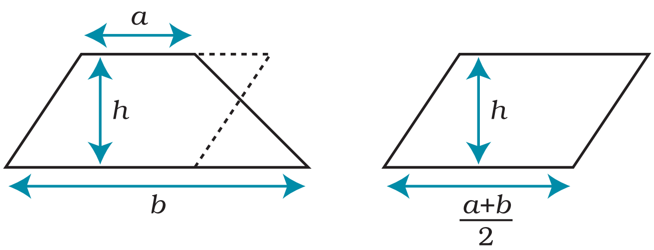

Figure 4

In symbols, this requires knowledge that

$$\frac{a+b}{2} = a + \frac{b-a}{2} = b - \frac{b-a}{2}$$

although it can also be seen geometrically.

Method 5

The trapezium was dissected into a rectangle and two triangles (see Figure 5). The two triangles were then moved together to form a single triangle with height h and base equal to $b - a$.

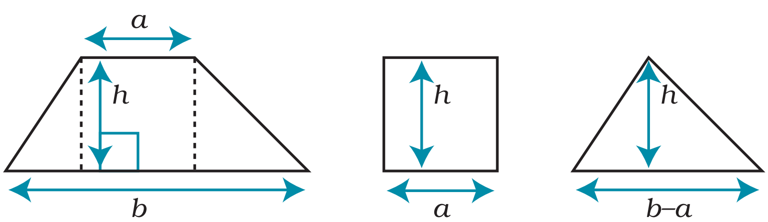

Figure 5

Guided discovery

One textbook, instead of deriving the rule and then setting exercises, placed two multi-step problems leading to the area of a trapezium area within an exercise problem set. Each problem guided students to dissect a trapezium of specific dimensions and rearrange into shapes of known area. Only in a final section were the rules explicitly stated and practice exercises provided. This approach foregrounded the importance of students being able to find areas of polygonal figures of varied shapes by dissecting into areas of known shapes, rather than relying on memorised rules. Although specific measurements are used in these problems, we judge that students are intended to see the generality in the particular, and to recognise that the dissection methods would apply to any trapezium.

Discovery approaches based on dissections provide worthwhile activities for students before they are presented with area rules for special quadrilaterals. However, when students are subsequently presented with area rules, it is important that links are made between the rules and the students' discovered methods.

Conclusion

If students are unable to remember the area rule for a particular special quadrilateral, they should be able to call on an appropriate dissection method. The important lesson from studying the area of a trapezium is not to learn the formula, but to learn that areas of a wide variety of shapes can be found by dissecting them into shapes of known areas and to see some of the ways in which this can be done. By exposing students to a variety of dissection methods and encouraging them to find their own variations, we promote flexible thinking, greater understanding and less reliance on memorised area formulas.

Reference

Stacey, K. & Vincent, J. (2008). Modes of reasoning in explanations in Year 8 textbooks. In M. Goos, R. Brown, K. Makar (Eds), *Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia* (pp. 475–481). Brisbane: MERGA.

From Helen Prochazka's
Scrapbook

Margaret Wertheim in the introduction to her book "Pythagoras' Trousers: God, Physics, and the Gender Wars" (Times Books, 1995) writes:

When I was 10 years old, I had what I can only describe as a mystical experience. It came during a math class. We were learning about circles, and to his eternal credit our teacher, Mr Marshall, let us discover for ourselves the secret of this unique shape: the number known as pi. Almost everything you want to say about circles can be said in terms of pi, and it seemed to me in my childhood innocence that a great treasure of the universe had just been revealed. Everywhere I looked I saw circles, and at the heart of every one of them was this mysterious number. It was in the shape of the sun and the moon and the earth; in mushrooms, sunflowers, oranges and pearls; in wheels, clock faces, crockery and telephone dials. All these things were united by pi, yet it transcended them all. I was enchanted. It was as if someone had lifted a veil and shown me a glimpse of a marvellous realm beyond the one I experienced with my senses. From that day on, I knew I wanted to know more about the mathematical secrets hidden in the world around me.