

Conceptual Development of Einstein's Mass-Energy Relationship

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Einstein's special theory of relativity was published in 1905. It stands as one of the greatest intellectual achievements in the history of human thought. Einstein described the equivalence of mass and energy as "the most important upshot of the special theory of relativity" (Einstein, 1919). In this paper, we will discuss the evolution of the

concepts of Einstein's mass-energy relationship during these 100 years. Challenges, issues and implications for curriculum development, instruction and assessment are also discussed.

Keywords: Einstein, Mass-Energy relationship, $E = mc^2$

愛因斯坦的質量 - 能量關係的概念演變

愛因斯坦的狹義相對論發表於1905年，這是人類思想史上的一大突破。愛因斯坦(1919)提出質量和能量是等價的，這是狹義相對論其中最重要的一個結論。在這篇論文裏，我們將探討愛因斯坦關於質量——能量關

係的概念，在這一百年的發展過程。我們也將探討這個理論在課程發展、規劃、測試中所面臨的問題。

關鍵詞：愛因斯坦、質量-能量、 $E=mc^2$

Introduction

Are not gross Bodies and Light convertible into one another, and may not Bodies receive much of their Activity from the Particles of Light which enter their Composition?

Newton, *Opticks* (4 ed. , 1730)

The vast impact of $E = mc^2$ can be justified by how it is coined as the "most famous equation in Science". Harald Fritzsche (1997) described this impact in his book "An Equation that changed the world". In addition, there is a large amount of published work, both in books and journals, which look at this equation from many different perspectives. One disturbing fact, according

to Warren (1976), is that many of the authors who misrepresent mass-energy were trained in the period when the available textbooks were mostly correct and clear. Today, the situation is even worse when many more textbooks and websites are explaining the concepts in various different ways. Some explain that mass will change with velocity, some prefer mass to remain the same instead. Students may find this equation more confused than before. Perhaps it could be called the “most confused equation”.

During a recent email correspondence with two famous Nobel Laureates, one is a theoretical particle physicist and the other is an experimental particle physicist, they were asked to express their views on Feynman's statement that “A photon of frequency f has the energy $E = hf$. Since the energy E has the gravitational mass E/c^2 the photon has a mass (not rest mass) hf/c^2 , and is “attracted” by the earth.” (The Feynman Lectures. Vol 2, 42-11). The theoretical particle physicist disagreed with Feynman, he preferred “gravity acts on energy and momentum; there is no need to mention mass at all.” He also explained that “Energy is a more basic concept than mass; mass is just one form of energy. If the energy of a system is E AND its momentum is zero, or negligible ($\ll E/c$) then we can assign it mass E/c^2 .” The reply from the experimental particle physicist was affirmative, “In general physicists include mass when they speak of the energy of a system. Also, Feynman's statement is quite correct and the bending of a beam of light as it grazed the sun on its way to a telescope on the earth was the first verification of the Theory of Relativity.”

If these two prominent Nobel Laureates do not concur on Feynman's statement, it is not a surprise that many textbooks explain the subject very differently. Interestingly, when Okun worked in a commission created by the former Ministry of Education of USSR to determine the best textbook, he was struck by the fact that all the 20 textbooks submitted, adopted a

velocity-dependent mass. In our recent survey of more than 30 textbooks, and more than 100 “popular” books from UK and USA (published after Okun's paper in 1989), apparently many UK textbooks still use the term “relativistic mass”, or the term “mass” which may vary with velocity, that is, they are holding the traditional view of Einstein. However, slightly more textbooks in US, though not all, prefer the term “mass” which is not velocity-dependent, a Lorentz invariant quantity. Essentially, slightly more USA textbooks follow the “modern view”, that is, mass means “rest mass”. (The comparison between USA and UK textbooks will be discussed in another paper.) Nowadays, many popular books and websites still explain that energy and mass are completely equivalent. It is also important to note that astrophysicists, or cosmologists, have the habit of using mass density ρ and energy density ε interchangeably. In a sense, astrophysicists hold the “traditional view” instead of the “modern view” proposed by the theoretical particle physicists.

With the controversy in mind, we would like to present parts of the conceptual development of Einstein's mass-energy relationship that could be useful to educators, and some of the practical applications developed during this one hundred years. This may provide educators some ideas on how to redesign the curriculum. Challenges, issues and implications for curriculum development, instruction and assessment are also discussed.

Historical Development of $E = mc^2$

The development of the concepts of Einstein's mass-energy relationship during these 100 years is summarised as follows:

1. Before 1905 (The forthcoming of $E = mc^2$)

1881	J. J. Thomson proposed that a charged conductor in motion increases its mass by $(4/15) \mu e^2/a$
1904	H. A. Lorentz proposed that $m_L = m_0(1 - v^2/c^2)^{-3/2}$ based on deformable spherical charge.
1904	Hasenöhl derived an apparent mass increase of a moving cavity containing electromagnetic energy E , obtaining $\delta m_e = 8/3 E/c^2$.
1905	On Abraham's suggestion, Hasenöhl corrected this to $\delta m_e = 4/3 E/c^2$
Based on the study of the historical development of concepts, one may deduce that the equation $E = mc^2$ would have been discovered by other physicists sooner or later. However, it is likely that Einstein merely accelerated the development.	

2. From 1905 to 1924 (The beginning of $E = mc^2$)

1905	Einstein proposed that $m_L = m_0(1 - v^2/c^2)^{-3/2}$ and $E = mc^2$. A body's mass diminishes by L/c^2 if it gives off energy L .
1907	Einstein formulated rest energy explicitly. Energy and mass are equivalent. $E = m_0c^2 (1 - v^2/c^2)^{-1/2}$
1907	M. Planck proposed that mass change in the absorption and emission of heat energy, from the momentum of energy. (He is the first physicist to relate $E = mc^2$ to binding energy, estimate molecular binding energy for a mole of water.)
1907	Einstein asserted mass-energy relationship to be true for gravitational energy.
1908	M. Planck proposed that mass-energy relationship holds for all types of energy transfer
1922	Fermi in a paper entitled, "Correction of a serious discrepancy between the theory of electromagnetic mass and the theory of relativity", tried to resolve the difference between electromagnetic mass, $m = (4/3) E/c^2$ and $m = E/c^2$.
There were at least two main different interpretations on the meaning of $E = mc^2$. In one interpretation, the relation expresses the inter-convertibility of mass into energy, with one entity being annihilated and the other being created. The other interpretation expresses a proportionality factor between the two attributes, energy and mass, and they were considered to be manifestations of one.	

3. From 1925 to 1944 (The proof of $E = mc^2$)

1929	Eddington proposed that the distinction between mass and energy is artificial.
1932	Cockcroft and Walton studied the bombardment of a lithium atom (Li) by a proton (p), which produces two alpha particles (a). (First experimental verification of $E = mc^2$ and to an accuracy of better than 1%)
1933	Bainbridge discussed the degree of accuracy of the early quantitative measurements of nuclear reactions which confirmed the mass-energy relationship.
1934	The first photograph showing the creation of a pair of particles, revealed by the fog spots they made in passing through the wet air of a "cloud chamber." The two particles, curving apart under the influence of a magnet, were created in the annihilation of a particle of light. (First photograph of $E = mc^2$)
Einstein further developed the philosophical interpretation of $E = mc^2$. There are two realities: matter and field. Field represents energy, matter represents mass. The greatest part of energy is concentrated in matter; but the field surrounding the particle represents energy, though in incomparable smaller quantity. Mass-energy equivalence implies that we can no longer distinguish between "matter" and "the field".	

4. From 1945 to 1964 (The shock of $E = mc^2$ and the meaning of $E = mc^2$)

1945	Aug 6: The United States dropped a uranium atomic bomb on Hiroshima, Japan, killing over 100,000. Aug 9: The United States dropped a plutonium atomic bomb on Nagasaki, Japan, killing over 40,000.
1948	Russell pointed out that "atoms" are merely small regions in which there is a great deal of energy. He suggested that "mass is only a form of energy, and there is no reason why matter should not be dissolved into other forms of energy. It is energy, not matter, that is fundamental in physics."
1956	The first major nuclear power plant opened in England. (Application of $E = mc^2$ for daily living)
1962	Wheeler coined the phrase "mass without mass" to indicate the possibility of removing the term "mass" from the fundamentals of physics.
The meaning of $E = mc^2$ was lively discussed after the landing of atomic bombs on Hiroshima and Nagasaki. The interpretation of "mass can be converted into energy" was suggested to be a misconception. There was also debate on whether "the law of conservation of mass still holds" and this could be "purely a question of definition."	

5. From 1965 to 1984 (The application of $E = mc^2$ in daily lives)

1965	David Bohm suggested that the transformation of “matter” into “energy” is just a change from one form of movement (inward, reflecting to-and-fro) into another form (outward displacement through space). Internal transformations taking place on the molecular, atomic, and nuclear levels can change some of this to-and-fro, reflecting “inward” movement, into other forms of energy whose effects are “outwardly” visible on the large scale.
1983	Torretti regarded the terms “mass” and “energy” as designating properties of physical systems. The apparent difference between mass and energy is thus an illusion that arises from “the convenient but deceitful act of the mind by which we abstract time and space from nature”.
1984	According to Zahar, Einstein showed that “energy” and “mass” could be treated as two names for the same basic entity. The apparent difference between mass and energy arises from the contingent fact that our senses perceive mass and energy differently.
Warren revived the debate on the misconception in the expression “converts mass into energy”. There are more textbooks written to explain the meaning of $E = mc^2$ in various different ways resulting in even more alternative conceptions.	

6. From 1985 to 1999 (The application of $E = mc^2$ to save lives and the continuous debate on the concept of mass)

1987	Positron Emission Tomography (PET), an imaging system that uses radioactive substances introduced into the body, was introduced. (Application of $E = mc^2$ in Medical Science)
1989	Lev Borisovich Okun emphatically declared that in the modern language of relativity there is only one mass, and the concept of relativistic mass is misleading.
1991	T.R. Sandin defended the concept of relativistic mass based on aesthetic reason.
1997	S. Carlip explained the observational evidence on kinetic energy contributes to gravitational mass, based on general theory of relativity.
The term “relativistic mass” was questioned by the particle physicists especially. The debate was also extended to “rest mass”. Taylor and Wheeler discouraged the use of this concept because it leads to the belief that increase in energy, alias “mass”, of a particle with velocity results from some change in the internal structure of the particle and not in the geometric properties of space-time itself. The crux of this controversy was proposed to be a matter of aesthetic simplicity, terminological convention, practical applicability, the result of different mathematical approaches etc. Please refer to Jammer (2000) for more details.	

7. Beyond 2000 (The future and uncertainty of $E = mc^2$)

2001	<p>Giovanni Amelino-Camelia proposed the third postulate, “Existence of observer– independent skill of mass or length (k or k^{-1}) in the Theory of Doubly Special Relativity, and suggested a correction factor for $E = mc^2$:</p> $E = \frac{MC^2}{1 + \frac{MC^2}{E_p}}$
2001	<p>Steven Weinberg wrote that “No one knows how to calculate the spectrum of the iron nucleus, or the way the uranium nucleus behaves when fissioning, from quantum chromodynamics. We don’t even have an algorithm; even with the biggest computer imaginable and all the computer time you wanted, we would not today know how to do such calculations.”</p>
2003	<p>Joao Magueijo claimed that “according to varying speed of light, c is not constant, hence energy or mass cannot be conserved.” In other words, a varying speed of light allowed for matter to be created and destroyed.</p>
2004	<p>Frank Wilczek in the Nobel lecture, Dec 8, 2004, explained that “Mass comes from energy”. (There is energy stored in the motion of the quarks, and energy in the color gluon fields that connect them. This bundling of energy makes the proton’s mass.) He described $m = E/c^2$ as Einstein’s 2nd Law.</p>
<p>We still do not fully understand $E = mc^2$. More physicists begin to question the validity of $E = mc^2$. In the modern view, the particle physicists explained that energy is not equivalent to mass. (One argument is based on the concept that photon has energy, but no mass.) However, there are still many physicists adopt the traditional view. John W. Luetzelschwab (2003) recommended and explained on how apparatus can measure the relativistic mass increase of beta particles.</p>	

In the historical development of $E = mc^2$, it is interesting to note that this concept has been evolving continuously. With the advent of internet, the pace of development on the concepts can be faster, and the coverage of knowledge could be even wider and deeper. Can educators keep up to date with the new knowledge? Would not the students of the new generation gain access of the internet easily and surpasses the knowledge of educator now and then? Will the educators be able to answer all the questions on conceptual development of $E = mc^2$ during the 100 years as outlined above? Can educator cope with the new challenges of teaching? What other challenges associated with $E = mc^2$?

Challenges associated with $E = mc^2$

Theoretical speaking, how the energy transfer can cause a change in mass is not yet completely known even though Einstein worked out the scale of transfer a century ago. On the other hand, the variation of mass predicted by the theory of relativity is immeasurably small and practically cannot be detected by direct weighing on even the most sensitive scale. Perhaps the most disturbing fact is that the situation is worse when many textbooks are explaining the concepts in various different ways. Students may find this equation more confused than before.

There are three common alternative conceptions on the energy-mass relationship that can give educators and students some problems. There are many others which will not be discussed in the paper.

1. On “conservation of mass”:

“Traditional view”: Hobson (2003) proposed that it is sometimes said, incorrectly, that Einstein’s relation means that “mass is not always conserved.” It is true that matter (rest-mass) is not always conserved. But mass (inertia) is always conserved, because mass equals energy divided by c^2 , and energy is always conserved.

“Modern view”: When the particle move with different velocities, there is no change in the internal structure of the particle. It is meaningful to define a mass which is invariant in all inertial frame of reference. Hence, mass is not equivalent to energy, and the definition of “relativistic mass” being equivalent to energy, is a redundant concept. “Conservation of mass” is only approximately true.

2. On “conversion of mass”:

Conventional view: Einstein (1905) explained mass can be converted to energy. Lange (2002) explained that “The ‘conversion’ of mass into energy is not a real process like the metamorphosis of a caterpillar into a butterfly... The ‘conversion’ of mass into energy occurs because we have shifted our perspectives, not because the nucleus has decayed.”

Alternative views: Hobson (2003) proposed that it is sometimes said, incorrectly, that Einstein’s relation means that “mass can be converted to energy.” It is true that rest-mass (matter) can be converted to nonmaterial forms of energy such as radiation. But mass is always conserved, so mass can never be converted to anything else! Baierlein (1991) proposed ‘One should bear in mind that neither inertial mass nor energy is a substance; rather, both are attributes of a physical system. “Matter,

” however, is most certainly a thing. An attribute cannot be converted into a thing, and so energy cannot be converted into matter.’ However, some other may simply explain that there is no conversion of energy, since mass is already a form of energy.

3. On “mass of light”:

Einstein did not seem to be consistent when he discussed the need for relativistic mass, how mass changes with speed. This has led to some debate on the interpretations of Einstein’s statement. Some physicists prefer rest mass of photon to be zero, whereas some others prefer relativistic mass of photon to be E/c^2 . Below are some common quotations on Einstein.

But there is, fortunately, a grave fault in the reasoning of the inside observer, which saves our previous conclusion. He said: “A beam of light is weightless and, therefore, it will not be affected by the gravitational field.” This cannot be right! A beam of light carries energy and energy has mass.

Einstein, A. and L. Infeld (1938) *The Evolution of Physics*

It is not good to introduce the concept of the mass $M = m\omega/(1 - v^2/c^2)^{1/2}$ of a body for which no clear definition can be given. It is better to introduce no other mass than ‘the rest mass’ m . Instead of introducing M , it is better to mention the expression for the momentum and energy of a body in motion.”

Einstein A, in a 1948 letter to Lincoln Barnett

Due to these common alternative conceptions, here are some questions which different physics teachers may give or accept different answers.

Question 1: A nuclear power station differs from one burning coal or oil as it converts mass into energy according to the law $E = mc^2$.

A. True B. False

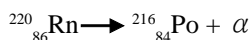
J W Warren (1976) in “The mystery of mass-energy” shared that when this “true-false” question was set to 147 university science and engineering freshmen, only 32 students recognized this as false. In a small group of postgraduate students training as physics teachers, one half accepted the statement as true. It should be interesting to know the responses of current students and teachers.

Question 2: During the radioactive decay, which of the conservation laws is/are conserved?

A. Mass B. Energy C. Momentum

In 2001, an Examination Board in United Kingdom asked the following similar question:

The decay of a radioactive nuclide may be represented by the equation



State and explain whether mass is conserved in this decay.

Currently, one popular worked solution found in the book, “Ten Year Series”, commonly used by Singapore students, also gave the answer that mass was not conserved during this decay. (Whether the marking scheme would accept both answers, that is, conservation of mass is conserved or not conserved depending on the system chosen, we are not sure. The Examination Report did not give any comment on this part of the question.)

It is expected that more students or even educators will add up the masses and conclude that the sum of masses after decay is less than before. Hence, their answer is simply mass is not conserved. One possible reason is they view energy and mass as separate entities. Besides, they would also explain that the “conservation of mass” is only approximately true. The approximation holds in many examples in secondary curriculum such

as chemical reactions and mechanical interaction. In fact, many physicists would regard only momentum and energy are conserved. However, some other physicists or physics teachers may also conclude that mass is conserved. This is because they consider energy and mass as one entity. In other words, they hold the same view as Hobson (2003) or they look at everything as a whole system.

Question 3: What will happen when you raise a book by a height of 1 metre.

- A. There is no change in mass in the book.
 B. There is an increase in mass in the book.
 C. There is a decrease in mass in the book.
 D. Not sure.

This question is also interesting as some physics educators felt that it seems somewhat strange that the mass of the book would change with the change in potential energy of the book. We expect the most common answer is “There is no change in mass in the book.” Many physics teachers and students may choose this because they reason that there is simply no change in the quantity of matter though there is change in potential energy.

Applying the concept of equivalence of mass and energy, some may deduce that the change in energy will correspond to a change in mass. Since the mass gain potential energy, so it should have an increase in mass. One similar question given by the physics notes, commonly available in United Kingdom, was “Explain the change in the mass of the following systems: the heating of some cold water.” The answer for the effect on heating of some cold water is “the increase of heat energy, and hence the gain in mass by $\delta m = \delta E/c^2$.” However, some students may argue that there will be a decrease in the mass of water due to evaporation. So, would you consider this a good assessment question for the students?

Issues and Implications

We suggest that there are a number of issues which experts and educators need to be cognizant in introducing this mass-energy relationship into any curriculum.

1. Knowledge limits (Experts and Educators): There is no consensus between experts on some of these alternative conceptions. It may not be resolved in the near future. Many educators may not be aware of the nature of controversy. Limitation of scientific progress and technology advancement may also affect the concept of mass and “mass of light”. Example in other fields may include the theory of evolution. Educators who believe in creationism may have different interpretations. Some key questions here, are as follows: “Do all experts and educators know the limitations of scientific knowledge in their fields of specialization?” “Where is the boundary of scientific knowledge?” “Can educators always be aware of the current progress of scientific knowledge?”

Here is another example. One frequently-asked-question in Physics is the mass of photon. Most educators might follow the conventional textbooks’ explanation that mass of light is zero. What if the student quotes the results of Jun Luo (2005)? (The new experimental limit on photon mass, less than 1×10^{-51} gm or 6×10^{-17} eV. This was established by an experiment in which light is aimed at a sensitive torsion balance.) Similarly, some other students may challenge the educator that light has screening mass, Meissner mass or Debye mass, which is easily available in the internet. Should not the educators admit their lack of knowledge? Would the students feel disappointed with their educators’ limited knowledge? How would you response when the doctor that you consult, check his textbooks or encyclopaedia in front of you to confirm the

diagnosis? Is he incompetent, forgetful or very cautious? Or something better?

2. Conceptual limits (Educators and Students): The concept of mass is not as simple as it seems. Though we may commonly consider mass as “quantity of matter”, some physicists prefer mass like Wilczek (2004), as “unitary irreducible representations of the Poincaré group in Hilbert Space”. There are many other conceptual meanings of mass too. Some physicists may prefer the introduction of “velocity-dependent mass” in explaining $E = mc^2$, however, how to conceptualize the increase in mass due to the increase in velocity? (If you imagine you are moving the same velocity as the particle, then you might consider that there is no change in the internal structure of the particle, however some may conceptualize that mass is dependent on the environment etc instead.) Do we know the limitations of our students thinking ability or their ability to conceptualize? Can students grasp the abstract concept like energy and mass? Can these concepts really be grasped or most of us are memorizing the definitions?
3. Pedagogical limits (Educators): Controversy is at the heart of science inquiry, challenging scientists, motivating debate, and stimulating research. How should educators deal with some of the controversial concepts? Some educators may not feel comfortable in teaching some of the controversial concepts. Some educators may explain that mass of light is zero and some may choose to differ. Are educators in our countries ready for the challenge?

Besides, the concept of energy is already very abstract. If Feynman believed that even during his times, that we had no knowledge of what energy was, it is debatable how to teach something which “we may not have real knowledge of”. On the other hand, we do know something about energy, and Feynman’s comment on textbooks’ limited

explanation on energy is still relevant today.

I turned the page. The answer was, for the wind-up toy, "Energy makes it go." And for the boy on the bicycle, "Energy makes it go." For everything, "Energy makes it go." Now that doesn't mean anything. Suppose it's "Wakalixes." That's the general principle: "Wakalixes makes it go." There's no knowledge coming in. The child doesn't learn anything; it's just a word!

What they should have done is to look at the wind-up toy, see that there are springs inside, learn about springs, learn about wheels, and never mind "energy." Later on, when the children know something about how the toy actually works, they can discuss the more general principles of energy. It's also not even true that "energy makes it go," because if it stops, you could say, "energy makes it stop" just as well. What they're talking about is concentrated energy being transformed into more dilute forms, which is a very subtle aspect of energy. Energy is neither increased nor decreased in these examples; it's just changed from one form to another. And when the things stop, the energy is changed into heat, into general chaos. But that's the way all the books were: They said things that were useless, mixed-up, ambiguous, confusing, and partially incorrect.

Feynman

The Editor of The Textbook Letter, Bennetta (1999) quoted Feynman's observation and further commented that "This is why we continue to see, in state after state, the same absurdities that Feynman saw thirty-five years ago."

4. Assessment limits (Educators): Should educators confront students with alternative answers to qualitative problems? It is perhaps more important to use such controversies to improve students reasoning skills. Do educators usually accept only

one answer? For the question, State and explain whether mass is conserved in this decay, do you award students who give "different" answers to this question?

Whether the educators have strong feelings toward the traditional view, modern view or even other views, it is hoped that educators are not going to simply penalize the students for subscribing to the other school of thought. The controversy on the alternative conceptions relating $E = mc^2$ has been unfortunate; it has been rather emotional or indirectly personal. In 1989, Okun explained in his famous paper, "The concept of mass", that the term "rest mass" was superfluous, and "relativistic mass", archaic concept. Okun also added many other negative comments in addition to describing the "traditional view" as nonsense and nonrational. This has stirred the feelings of many physicists to respond immediately and some later. (Baierlein 1991, Sandin 1991 etc) On the other hand, Baierlein (1991) explained that "mass is converted into energy" as heretic, and Okun was possibly more passionate than reasoned. A Soviet physicist, Khrapko (2000) described Okun's concept as having psychological barrier and problem.

Currently, we have no solution to all the problems outlined above. However, we believed that the educators' willingness to share their knowledge, the network between physics educators, or the formation of more email discussion lists will certainly help. The question may not be just the changing role of educators, but the changing relationship between the Ministry of Education, the Institute of Education and the school teachers. Will students gather more knowledge by surfing the internet independently without the help of educators? If Daniel Tsui's success can be attributed to the glut of overqualified teachers, would not it possible for students to benefit even more from the valuable and almost unlimited information in the internet?

Conclusion

In the historical development of $E = mc^2$, it is interesting to note that this concept has been evolving continuously. With the advent of internet, the pace of development on the concepts can be faster, and the coverage of knowledge could be wider and deeper. Educators will have greater challenge to keep up to date with the new knowledge. Besides, educators should be aware of the knowledge limits, conceptual limits, pedagogical limits, and assessment limits when they are introducing this mass-energy relationship to the students.

If the concept of Einstein's energy-mass relationship has evolved into many areas of application and many alternative conceptions are formed during these 100 years, this may occur in many other concepts as well. We hope that this paper has demonstrated some of the controversies surrounding the conceptual development of $E = mc^2$ and the need to pay attention to its inclusion in any curriculum.

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- ▶ 各試卷包含不同類型的題目，另備有評估表，方便記錄和評估學生於不同範疇及單元的表現
- ▶ 附有答案、評分參考和各題的評估重點，供批改時使用
- ▶ 可配合《基本能力訓練》使用



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- ▶ 鞏固學生在數、圖形與空間、度量、數據處理四個範疇的基本能力
- ▶ 各練習清楚註明相關內容出現的年級，方便教師及學生使用
- ▶ 備有評分參考及注意事項，供教師或家長使用
- ▶ 可配合《基本能力評估模擬試卷》使用

