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In Focus…

Deconstructing Teacher-Centeredness and Student-Centeredness Dichotomy: A Case Study of a Shanghai Mathematics Lesson

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Teacher-dominated classrooms with some student-centered elements are a perplexing phenomenon of Chinese mathematics classrooms. In-depth exploration of this phenomenon is helpful for understanding the features of mathematics teaching in China. This paper demonstrates how the teacher can encourage students to actively generate knowledge under the teacher's control from a perspective of variation and further deconstruct the legitimacy of teacher-centeredness and student-centeredness dichotomy.

Teacher-dominated classrooms in countries under the influence of the Confucian-heritage culture (CHC) are often seen as an environment not conducive to learning in western countries (Biggs, 1996). However, students from CHC countries have consistently performed well in recent international studies of mathematics achievement (Beaton et al, 1996; Mullis et al, 1997; 2000; 2003). The mismatch between the unfavorable learning environment and the outstanding achievement has prompted discussion on the so-called "paradox of the Chinese learners" which led to many studies about the teaching in CHC classrooms and the psychological and pedagogical perspectives about Chinese teaching and learning (Leung, 2001; Watkins & Biggs, 2001; Fan, et al, 2004). To crack the paradox, some studies have tried to explore the mechanism of mathematics teaching in CHC settings (Huang, 2002; Huang & Leung, 2004; Mok, 2003; Mok & Ko, 2001). One interesting observation was made that there are some student-centered features in mathematics classrooms in CHC although the teaching is teacher-dominated. So, there are some elements of good teaching in the teacher-dominated classrooms in CHC (Watkins & Biggs, 2001), and a dichotomy of teacher-centeredness and students-centeredness may not be suitable to characterize whole classroom teaching (Huang, 2002; Mok & Ko, 2000). There is not a single teaching method guaranteeing students' high achievement. Different countries share some common components of classroom teaching, but have different emphases and different combinations of those components (Hiebert et al, 2003). Although researchers have argued that there were many elements of student-centeredness in Chinese classrooms, there are seldom studies on how teachers encourage students to actively participate in mathematics learning in teacher-dominated classrooms (Huang, 2002; Mok & Ko, 2000; Mok & Morris, 2001). Looking at Chinese classrooms from a different perspective may shed new insights and understanding on what is really happening and whether it is conducive to student learning. A theoretical framework of variation was developed recently. It described how an enacted space of learning was constructed through creating certain dimensions of variation for students to experience (Marton & Booth, 1997; Mok, 2003; Marton et al, 2004; Gu et al, 2004). We will use this framework to analyze a Shanghai lesson and to demonstrate how students involve themselves in a process of learning, although the teacher controls the teaching.

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Theoretical Considerations

According to Marton et al (2004), learning always involves an object of learning. The authors refer to the object of learning as a capability that has a general and a specific aspect. The general aspect has to do with the nature of the capability such as remembering, interpreting and grasping. The specific aspect has to do with the subject on which these acts of learning are carried out, such as formulas and simultaneous equations. This object of learning is often conscious in the mind of the teacher and may be elaborated in different degrees of detail. What teachers are striving for is the "intended object of learning," which is an object within the teacher's awareness. However, what is more important is how the teacher structures the lessons so that it is possible for the object of learning to come to the fore of the students' awareness, which is called the enacted object of learning (Mok, 2003). According to Marton et al (2004), learning is a process in which we want learners to develop a certain capability or a certain way of seeing or experiencing. In order to see something in a certain way the learner must discern certain features of the object. Experiencing variation is essential for discernment, and is thus significant for learning. Marton et al (2004) further argues that it is important to pay attention to what varies and what is invariant in a learning situation. Moreover, based on a series of longitudinal mathematics teaching experiments in China, and heavily influenced by cognitive science and constructivism, a theory of mathematics teaching/learning, called teaching with variation, has been developed (Gu et al, 2004; Huang, 2002). According to this theory, meaningful learning enables learners to establish a substantial and non-arbitrary connection between their new knowledge and their previous knowledge (Ausubel, 1964). Classroom activities are developed to help students establish this kind of connection by experiencing certain dimensions of variation. This theory suggests that two types of variation are helpful for meaningful learning (Gu et al, 2004). One is called "conceptual variation," which provides students with multiple experiences from different perspectives. The other is called "procedural variation," which is concerned with the process of forming a concept logically or chronologically, arriving at solutions to problems (scaffolding, transformation), and forming knowledge structures (relationship among different concepts). According to this theory, the space of variation, which consists of different dimensions of variation in the classroom, forms the necessary conditions for students' learning in relation to certain objects of learning. For the teacher, the critical issue is how to create an adequate space of variation focusing on critical aspects of the learning object through appropriate activities. For the learner, it is important to be engaged in this 'space' of variation (called the enacted objective of learning). Gu et al. (2004) argued that by adopting teaching with variation, even with large classes, students could still actively involve themselves in the learning process and achieve meaningful learning. In this paper, this framework of variation is used to analyze a Shanghai lesson.

A Case Study

Setting and Data Source

Shanghai is one of the largest cities affiliated with the central government of the People's Republic of China. A total area of approximately 6,000 square kilometers contains the population of 13 million people, which is about 1% of the population in China. It is a center of commerce and has a well developed education system. In Shanghai, the children start their schooling at the age of six. They receive 9 years of compulsory education. In Shanghai, there is a municipal curriculum standard which is different from the national one, a unified textbook according to the syllabus. Usually, candidates who finish Grade 12 could apply for a four-year full-time teacher's training course, qualifying them to teach in secondary schools.

In this paper, a 40-minute videotaped lesson, which recorded the practice of a grade seven teacher working in a junior high school class in the countryside of Shanghai, constitutes the data source for this analysis. The videotape was an excellent lesson. The lesson illustrates well the teacher-dominated style of teaching, which is very common in China.

Description of the Lesson

The topic of the lesson is "corresponding angles, alternate angles, and interior angles on the same side of the transversal." By and large, the lesson includes the following stages: review, exploration of the new concept, examples and practices, and summary and assignment.

Reviewing and inducing. At the very beginning, the teacher drew two straight lines crossing each other on the blackboard, and asked students to recall their learned knowledge such as concepts of opposite angles and complementary angles. When the teacher obtained the correct answers to those questions from the students, the teacher added one more straight line to the previous figure (see Figure 1(a)), and asked students how many angles there are in the figure, and how many of them are opposite angles and complementary angles. After that, the teacher guided students to explore the characteristics of a pair of angles formed by the two lines with the transversal by asking students the question: "What characteristics are there between $\angle 1$ and $\angle 5$? [This actually is the new topic to be explored in this lesson]."

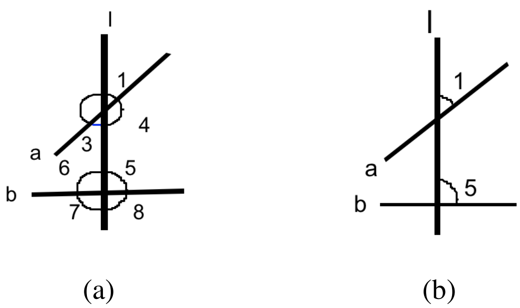


Figure 1. Basic diagram; two straight lines intersected by a third line.

Exploring new concepts. In order to examine the relationship between ∠1 and ∠5, a particular diagram was drawn separately as shown in Figure 1(b). Through group discussion, the students found many features between these two angles, such as "∠1 and ∠5 are both on the right hand side of line 1, and both are above the relevant line (line a and line b)." Based on students' responses, the teacher summarized the students' explanations and stated the definition of the "corresponding angles." Then, students were asked to identify all the "corresponding angles" in Figure 1(b). Similarly, the two concepts "alternate angles" and "interior angles" were explored respectively.

Example and exercise. After introducing the three angle relationships, students were asked to identify them in different configurations. The problems are as follows:

Task 1: Find the "corresponding angles, alternate angles, and interior angles on the same side of the transversal" in Figure 2.

Task 2: Find the "corresponding angles, alternate angles, and interior angles on the same side of transversal" in Figure 3.

Task 3: In Figure 4, please answer the following questions: (1) Is ∠1 and ∠2 a pair of corresponding angles? (2) Is ∠3 and ∠4 a pair of corresponding angles?

Task 4: Given ∠1 is formed by lines l and a as shown in Figure 5. (1) Please draw another line b so that ∠2 formed by lines l and b and ∠1 is a pair of corresponding angles. (2) Is it possible to construct a line b so that ∠2 formed by lines l and b is equal to ∠1?

Summary and assignment. The teacher emphasized that these three types of relationship are related to two angles located at different crossing points. These angles are located in a "basic diagram" which consists of two straight lines intersected by a third line. The key to judging these relationships within a complicated figure is to separate out a proper "basic diagram" which includes the angles in question. Moreover, the teacher demonstrated how to remember these relationships by making use of different gestures.

Finally, some exercises from the textbook were assigned to students.

Enacted Object of Learning

From the perspective of variation, and in order to examine what learning is made possible, we need to identify the dimensions of variation. In the following section, we look at the lesson in greater detail from this particular theoretical perspective focusing on the enacted object of learning and the possible space of learning.

Reviewing and Inducing

At the first stage of the lesson, a variation was created by the teacher through demonstration and questioning: varying two straight lines crossing each other to two straight lines intersected by a third one. By opening with this variation, the relevant previous knowledge was reviewed and the new topic was introduced in a sequential and cognitively connected manner. Thus, this variation is a procedural variation.

The next stage of the lesson was a stage of introducing and practicing new concepts. Two dimensions of variation were alternately created which are crucial for students to generate an understanding of the new concepts**.**

Representations of New Concepts

During the process of forming new concepts, the representations of the new concepts have been shifted among the following forms: rough description, intuitive description, definition, and schema. After a group discussion, the students were invited to present their observations, and based on students' explanations, the new concepts were built through teacher guidance, finally, the concepts were imbedded in a "basic diagram," i.e. "two straight lines intersected by a third line" (see Figure 1).

Different Orientations of the "Basic Diagram"

After the concepts of three types of angle relationship were constituted in a "basic diagram," the teacher provided students with Task 1. By doing so, a new dimension of variation was opened for students to experience how to identify those relationships in different orientations of the figures. The teacher purposely varied the figures in position and number of angles in the figures (see Figure 2).

By providing students with these variations, the students were exposed to the concepts from different orientations of the diagram, which may make students aware that these concepts are invariant even when the orientation of a diagram is varied.

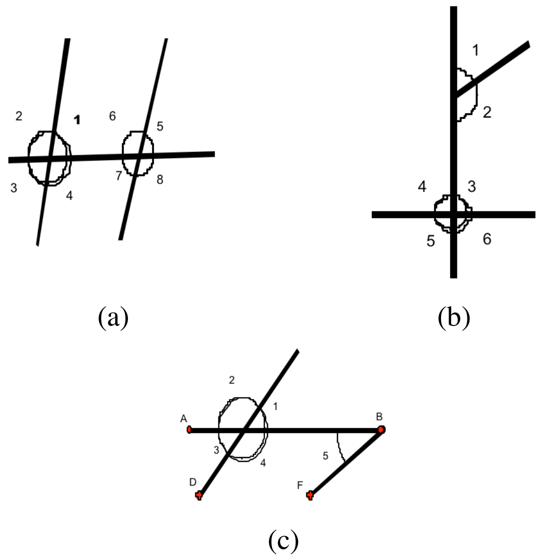


Figure 2. Variations of the basic diagram.

Different Contexts of the "Basic Diagram"

After students received a rich experience of these concepts in terms of the different orientations of the basic diagrams, the teacher then provided a group of tasks in which the "basic diagrams" were embedded in complex contexts (Task 2 and Task 3). Through identifying the angle relationships in different contexts of the "basic diagram," an invariant strategy of problem solving, i.e. identifying and separating a proper "basic diagram" from complex configurations, came to the fore of students' awareness. In general, separating a proper sub-figure from a complex figure is a useful strategy when solving a geometric problem (see Figure 3).

Different Directions for Applying the New Concepts

As soon as the students answered Task 2(see Figure 3), the teacher posed a new and challenging question: "Conversely, if $\angle 1$ and $\angle 5$ are a pair of corresponding angles, which basic diagram contains them?" After students were given some time to think about the question, one of them was nominated to answer the question. The student gave a correct answer by saying that the basic diagram is "straight line a and b intersected by straight line c." Similarly, by searching for a pair of interior angles on the same side of the transversal of $\angle 3$ and $\angle 12$, students identified a basic diagram, "straight lines c, d intersected by straight line a." Once the students completed the above questions, the teacher summarized that the key points for solving these problems is to pick out a "basic diagram" (for instance, two straight lines a, and b intersected by a third straight line c) by deliberately covering up one line (d) from the figure. Through identifying the three angle relationships within basic diagram or separating relevant 'basic diagrams' so that the given angle relationship is true, the students not only consolidated the relevant concepts, but more importantly, learned the separation method of problem solving as well, i.e. separating a basic sub-figure from a complex configuration.

Contrast and Counter-example

After the preceding extensive exercise, the students might believe that they had fully mastered the focus concepts. At this point, the teacher posed Task 3 (see Figure 4) to test whether they had really mastered the concepts and methods of problem solving. Through separating a basic diagram as shown in Figure 6(a), students were asked to justify that " $\angle 1$ and $\angle 2$ are

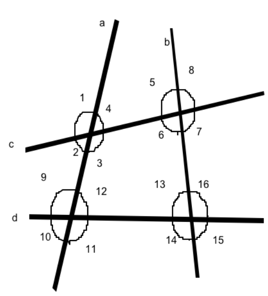


Figure 3. "Basic diagram" embedded in complex contexts.

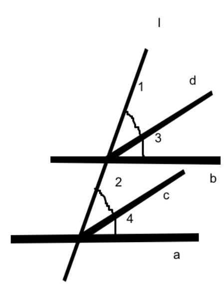


Figure 4. What are the corresponding angles?

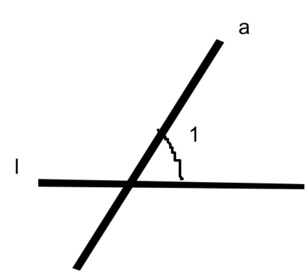
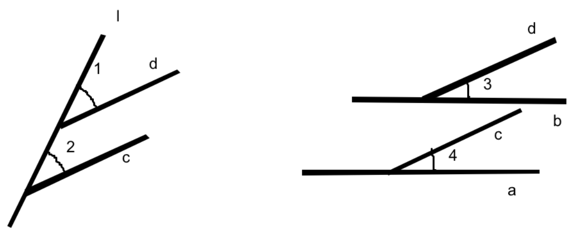


Figure 5. Can you draw another line b so that $\angle 2$, formed by lines 1 and b, is equal to $\angle 1$?

corresponding angles.” However, since students could only identify a diagram as shown in Figure 6(b), they failed to see that “∠3 and ∠4 are a pair of corresponding angles.” Thus a new dimension of variation of experiencing corresponding angles was opened: example or counter-example of the visual judgment.



(a) (b)

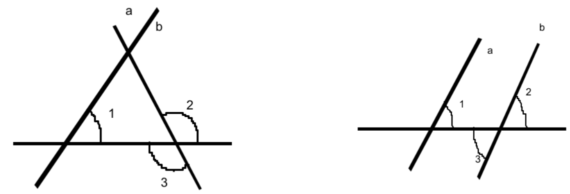
*Figure 6. E*xample and counter-example.

Creating a Potential for Learning a New Topic

After solving the above problems through observation and demonstration, the teacher presented a manipulative task (Task 4). First, through playing with colored sticks, the first question of task 4 was solved (see Figure 7(a)). Then, based on drawing and reasoning, the second question was also solved (see Figure 7(b)). During the process of problem solving, *the students’ thinking was shifted* along the following forms: concrete operation (by playing with the colored sticks) (enactive); drawing (iconic); and then logical reasoning (abstract). This exercise had two functions. On the one hand, the “previous proposition: opposite angles are equal” was reviewed; on the other hand, “a further proposition: if the corresponding angles are equal, then the two lines are parallel” was operationally experienced. That means a potential space of learning was opened implicitly.

Consolidation and Memorization of the Concepts

As soon as the key points for identifying three angle relationships in a variety of different situations were summarized, the teacher skillfully opened a new variation by making use of some gestures of the fingers (see Figure 8) in helping students to memorize the two concepts: mathematical concept and physical manipulation. This experience is helpful for visualizing and memorizing mathematical concept.



(a) (b)

Figure 7. Construction corresponding angles.

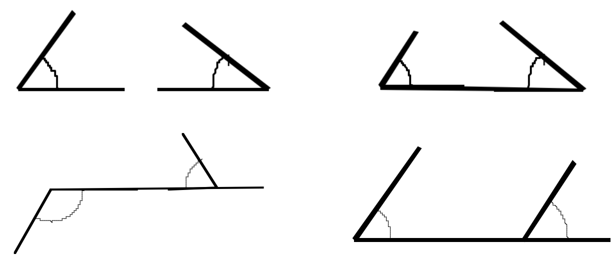


Figure 8. Demonstration angle relationships by gestures.

Summary and Discussion

Summary

According to the theoretical perspective, it is crucial to create certain dimensions of variation bringing the enacted object of learning to students’ awareness. These objects of learning can be classified into two types. One is the content in question (such as concepts, propositions, formula), and the other is the process (such as formation of concepts, or process or strategy of problem solving). Based on the categories of variation: conceptual variation and procedural variation, it was demonstrated that conceptual variations served the purpose of building and understanding concepts, while procedural variations were used for reviewing previous knowledge and introducing the new topic, consolidating new knowledge, developing a strategy to solve problems with the new knowledge, and preparing for further learning implicitly. These two dimensions of variation were created alternatively for different purposes of experiencing the enacted objects of learning (see Table 1).

Table 1.

Dimensions of variation and their functions

	Dimensions of variation	Functions of variation
Reviewing and inducing	Procedural variation 1	Revising previous knowledge; Introducing new topic
Exploring new concepts	Conceptual variation 1, 2	Building, clarifying and consolidating the new concept
Examples and exercises	Procedural variations 2, 3, 4	Consolidating the new concept; Learning separation method of problems solving
Summary and assignment	Procedural variation 5 Conceptual variation 3	Creating a potential topic of further learning Visualizing and memorizing the new concepts

The aforementioned lesson unfolded smoothly, strictly following a deliberate design by the teacher. It is likely that it would be labeled as a teacher-dominated lesson from a Western perspective. However, if students' involvement and contribution to the creation of these variations (i.e. enacted object of learning) are taken into consideration, it is hard to say that students are passive learners. This paper intends to demonstrate that the teacher can still encourage students to actively generate knowledge through creating proper and integrated dimensions of variation although the whole class teaching is under the teacher's control. Thus, it seems to suggest that creating certain dimensions of variation is crucial for effective knowledge generation in large classrooms. Wong, Marton, Wong, & Lam (2002) argued that in teaching with variation, the space of dimension of variation constituted jointly by the teacher and the students is of crucial importance for understanding what the students learn and what they cannot possibly learn.

In order to re-conceive the dichotomy of teacher-centeredness and student-centeredness, Clarke and Seah (2005) adopted a more integrated and comprehensive approach, by analyzing both public interactions in the form of whole class discussion and interpersonal interactions that took place between teacher and student and between student and student during between-desk-instruction. They found that the style of teaching in both Shanghai schools was such that the teachers generally provided the scaffold needed for students to reach the solution to the mathematical problems without "telling" them everything. Hence, one could find quite a few math-related terms, which the teacher had not taught, that were introduced by the students during public discussion. The practices of the classroom in Shanghai sample school provided some powerful supporting evidence for the contention by Huang (2002) and Mok and Ko (2000) that the characterization of Confucian-heritage mathematics classrooms as teacher-centered conceals important pedagogical characteristics related to the agency accorded to students; albeit an agency orchestrated and mediated by the teacher.

> A unique teaching strategy consisting of both teacher's control and students' engagement in the learning process emerges in Chinese classrooms. (Huang, 2002, p. 227)

Once the distribution of responsibility for knowledge generation is adopted as the integrative analytical framework, the oppositional dichotomization of teacher-centered and student-centered classrooms can be reconceived as reflecting complementary responsibilities present to varying degrees in all classrooms (Clarke, 2005).

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