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In-Service Middle and High School Mathematics Teachers: Geometric Reasoning Stages and Gender

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The purpose of this current study was to investigate the reasoning stages of in-service middle and high school mathematics teachers in geometry. There was a total of 148 in-service middle and high school mathematics teachers involved in the study. Participants' geometric reasoning stages were determined through a multiple-choice geometry test. The independent samples t-test with $\alpha = 0.05$ was used in the analysis of the quantitative data. The study demonstrated that the in-service middle and high school mathematics teachers showed all the van Hiele levels, visualization, analysis, ordering, deduction, and rigor, and that there was no difference in terms of mean reasoning stage between in-service middle and high school mathematics teachers. Moreover, there was no gender difference found regarding the geometric thinking levels.

Introduction

Various studies have documented that many students encounter difficulties and performed poorly in both middle and high schools geometry classrooms (e.g., Fuys, Geddes, & Tischler, 1988; Gutierrez, Jaime, & Fortuny, 1991). Usiskin (1982) has found that many students fail to grasp key concepts in geometry and leave their geometry classes without learning basic terminology. Moreover, research shows a decline in students' motivation toward mathematics (Gottfried, Fleming, & Gottfried, 2001). According to Billstein and Williamson (2003), "declines in positive attitudes toward mathematics are common among students in the middle school years" (p. 281). Among the variables that affect student learning, researchers have suggested that the teacher has the greatest impact on students' motivation and mathematics learning (e.g., Wright, Horn, & Sanders, 1997; Stipek, 1998). Burger and Shaughnessy (1986), along with Geddes and Fortunato (1993), claim that the quality of instruction is one of the greatest influences on the students' acquisition of geometry knowledge in mathematics classes. The students' progress from one reasoning (van Hiele) level to the next also depends on the quality of instruction more than other factors, such as students' age, environment, and parental and peer support (Crowley, 1987; Fuys et al., 1988).

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According to Stipek (1998), teachers' content knowledge has a significant impact on students' performance. Mayberry (1983) and Fuys et al. (1988) state that geometry content knowledge among pre-service and in-service middle school teachers is not adequate. Chappell (2003) says, "Individuals without sufficient backgrounds in mathematics or mathematics pedagogy are being placed in middle school mathematics classrooms to teach" (p. 294). In this study the researcher will investigate the argument that insufficient geometry knowledge of in-service mathematics teachers might be another reason behind students' poor performance in geometry.

The van Hiele Theory and its Philosophy

Level-I: Visualization or Recognition

At this level students recognize and identify certain geometric figures according to their familiar appearance. However, students do not perceive the geometric properties of figures. When students call a figure a square, they react to the whole figure and not to its right angles, equal side lengths, and equal diagonal lengths. For example, at this level students can recognize certain squares very easily because they look like the outline of a window or frame (Figure 1 left). However, they do not call the second shape in Figure 1 a square because it does not look like the outline of a window or frame.

Figure 1. Two perspectives of a square.

Level-II: Analysis

At this level students analyze figures in terms of their components and relationships among these components. For instance, a student's analysis may assert that opposite sides of a rectangle are congruent or all of its angles are right angles. Students can also identify and name geometric figures by knowing their properties. They would correctly identify only the second and fourth shapes in Figure 2 as parallelograms. Although at this level the students are able to acknowledge various relationships among the parts of the figures, they do not perceive any relationship between squares and rectangles or rectangles and parallelograms; students perceive properties of one class of shapes empirically, but can not relate the properties of two different classes of shapes. For example, students would not see rectangles or squares as parallelograms because they do not see one set of figures as a subset of another.

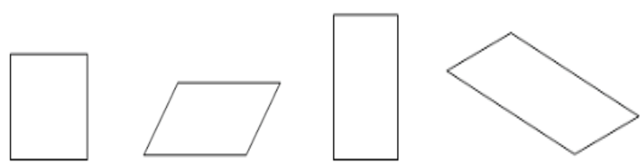

Figure 2. Examples of parallelograms.

Level-III: Ordering

At this level students logically order and interrelate previously discovered properties by giving informal arguments. Logical implications and class inclusions are understood and recognized. At this level the students are able to see the relationships among the quadrilaterals in Figure 3: they can easily say that a square is also a rectangle and a rectangle is also a parallelogram. Students are aware of relationships among different types of figures. These relationships may have been unclear to the students at level-II (Analysis). According to Hoffer (1988),

> they even may be able to observe various such relationships themselves and they only have an implicit understanding of how these relationships link to justify their observations. In other words, the students have not yet developed the ability to prove theorems. (p. 239)

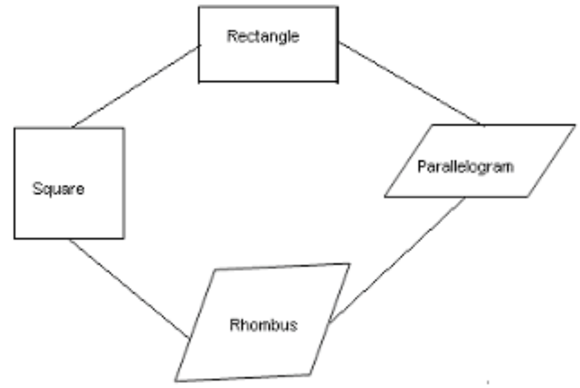

Figure 3. An example of ordering parallelograms.

Level-IV: Deduction

At this level students can analyze and explain relationships between figures. They can prove theorems deductively, supply reasons for statements in formal proofs, and understand the role of axioms and definitions. In other words,

> the students can follow the line of argument in proofs of statements presented to them, and they can develop sequences of statements to deduce one statement from another. What may have been an implicit understanding at the previous level, Ordering, of why certain statements were true now develops into reasoning patterns that enable the students to create sequences of statements to formally explain, or prove, why a statement is true [see Figure 4] (Hoffer, 1988, p. 239)

Students operating at level-IV can state that if a figure is a rhombus and a rectangle then it must be a square and prove this statement deductively. Students cannot analyze or compare various deductive systems. For example, students cannot establish theorems in different axiomatic systems.

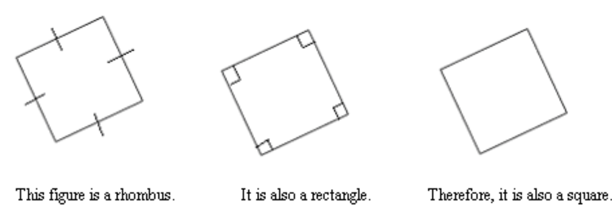

Figure 4. Showing that a rhombus is also a square.

Level-V: Rigor

At this level students are able to analyze and compare various deductive systems. A student should be able to know, understand, and give information about any kind of geometric figures (e.g., Fuys et al., 1988). Moreover, Hoffer (1988) says, “this is the most rigorous level of thought- the depth of which is similar to that of a mathematician” (p. 239).

Empirical Research on the van Hiele Theory

Since the proposal of the van Hiele theory, studies have focused on various components of this learning model at different grade levels. Wirzup (1976) conducted several studies and introduced the van Hiele theory in the United States. His work caught the attention of educators and researchers; four major studies were initiated by Hoffer (1988), Burger and Shaughnessy (1986), Usiskin (1982), and Fuys et al. (1986). Where Hoffer described and identified each van Hiele level, Burger and Shaughnessy focused on the characteristics of the van Hiele levels of reasoning. Usiskin affirmed the validity of the existence of the first four levels in high school geometry courses. Fuys et al. examined the effects of instruction on a student’s predominant Van Hiele level.

These research findings provide mathematics teachers insight on how students think and what difficulties they face while learning geometry. Several textbook writers have based their geometry sections or books on the van Hiele theory, such as Michael Serra’s (1997) geometry book and Connected Mathematics Project’s “Shapes and Designs” (Lappan, Fey, Fitzgerald, Friel & Phillips, 1996). The writers of both textbooks claimed that they implemented the van Hiele theory in their writings and designed their instructional approaches based on this theory.

Moreover, studies determined van Hiele reasoning stages in geometry of middle, high and college level students. For instance, Burger and Shaughnessy (1986) and Halat (2006, 2007) found mostly level-I (Visualization) reasoning in grades K–8. Fuys et al.’s (1988) interviews with sixth and ninth grade students classified as average and above average found none performed above level-II (Analysis). This finding supports the idea that many high school students in the United States reason at level-I (Visualization) or level-II (Analysis) of Van Hiele theory (Usiskin, 1982; Hoffer, 1988). These findings imply that neither middle nor high school students meet the expectations of NCTM (2000). At the end of 8th grade, students should be able to perform at level-II (Analysis) and at the end of 12th grade, students should be able to perform at level-III (Ordering) or level-IV (Deduction) (Usiskin, 1982; Mayberry, 1983; Crowley, 1987; Knight, 2006)). Usiskin, Mayberry, Burger and Shaughnessy, and Fuys et al. agreed that the last level (Rigor) is more appropriate for college students.

Some researchers have linked students’ mathematics performance to teachers’ content knowledge. For example, Chappell (2003) claims that high school students’ less than desirable background in geometry is due to middle school mathematics teachers’ superficial geometry knowledge. According to Gutierrez, Jaime, and Fortuny (1991), Duatepe (2000) and Knight (2006), pre-service elementary school mathematics teachers’ reasoning stages were below level-III (Ordering). Likewise, Mayberry (1983) stated that the 19 pre-service elementary school teachers involved in her study were not at a suitable van Hiele level to understand formal geometry and that their previous instruction had not help them to attain knowledge of geometry consistent with level-IV (Deduction).

Knight’s (2006) study with pre-service elementary and secondary mathematics teachers found that their reasoning stages were below level-III (Ordering) and level-IV (Deduction), respectively. These results are consistent with the findings of Gutierrez, Jaime, and Fortuny (1991), Mayberry (1983), Duatepe (2000), and Durmuş, Toluk, and Olkun (2002). None of these pre-service elementary and secondary mathematics teachers demonstrated a level-V (Rigor) reasoning stage in geometry. This is surprising because the van Hiele levels of pre-service elementary and secondary mathematics teachers are lower than the expected levels for students completing middle school and high school, respectively (Crowley, 1987; Hoffer, 1988; NCTM, 2000). Although most of these studies mentioned above were done with students, this study will investigate in-service middle and high school mathematics teachers.

Gender Differences in Mathematics

Research indicates gender should be included as a variable in analysis, even if it is not the main focus of a study (Forgasız, 2005; Armstrong, 1981; Ethington, 1992; Grossman & Grossman, 1994; Lloyd, Walsh & Yailagh, 2005). Over the past few decades, research suggests a difference between the achievement of male and female students in many content areas of mathematics, including spatial visualization, problem solving, computation, and measurement (e.g., Grossman and Grossman, 1994; Lloyd, Walsh and Yailagh, 2005). According to Armstrong, female students performed better at computation and spatial visualization than males. Fox and Cohn (1980) found males performed significantly better than females on the mathematics section of the Scholastic Aptitude Test. Similarly, Smith and Walker (1988) concurred with this finding in their study of tenth grade geometry students. According to Hyde, Fennema and Lamon (1990) and Malpass, O'Neil and Hocevar (1999), there is a significant increase in the gender gap among gifted or high scoring students on mathematics tests. Factors explaining gender differences in mathematics include prior achievement, attitudes towards mathematics, and support from others (Becker, 1981; Ethington, 1992; Grossman & Grossman, 1994; Fan & Chen, 1997).

However, in recent years there is a considerable decrease in the difference of the mean scores between male and female students' achievement (Halat, 2006). Although in the past female students had negative attitudes towards mathematics, today they are less likely to perceive mathematics as a male domain (Friedman, 1994; Fennema & Hart, 1994; Halat, 2006). For example, Fennema and Hart (1994) claimed that interventions designed to address inequalities in middle or high school mathematics classrooms played important role in the establishment of gender equity in learning mathematics. Likewise, Halat (2006) found no difference in the acquisition of the van Hiele levels between male and female students using van Hiele theory based-curricula. Instruction influenced by the van Hiele theory-based curricula may cause changes in females' attitudes towards mathematics courses (Halat, 2006).

The Purpose of the Study

The current study focuses on the reasoning stages of in-service middle and high school mathematics teachers in geometry. The following questions guided this study:

1. What are the reasoning levels of in-service middle and high school mathematics teachers in geometry?
2. What differences exist in terms of geometric reasoning levels between in-service middle and high school mathematics teachers?
3. Is there a difference in terms of geometric reasoning levels between male and female in-service mathematics teachers?

Method

Participants

In this study the researcher followed the convenience sampling procedure, defined as "using as the sample whoever happens to be available" (Gay, 1996, p.126). According to McMillan (2000), this is the most common procedure in today's educational research environment because of the difficulty of finding volunteers to participate and obtaining permission from schools and parents. The data was collected during the Spring and Summer of 2006. Of a total of 384 in-service secondary school mathematics teachers in a city located in the western part of Anatolia in Turkey, 148 teachers (39%) agreed to take the Van Hiele Geometry Test (VHGT). (See Table 1 for the grade level and gender of the teachers.)

Of the participating teachers, 110 were in-service middle school mathematics teachers—49 male and 61 female—and 38 were in-service high school mathematics teachers-31 male and 7 female. The participants teaching at the middle school level represented 54% of in-service middle school mathematics teachers in the city and the participants teaching at the high school level represented 21% of those in the city. The sample includes public and private school teachers of both geometry and algebra. The years of mathematics teaching experience varied from 1 to 26 years. The high school mathematics teachers took the test at their work places during the school day. However, the middle school teachers took the test at the end of an educational seminar. This seminar conducted by the researcher did not relate to van Hiele levels.

Data Collection

The researcher gave participants a geometry test called the *Van Hiele Geometry Test* (VHGT), consisting of 25 multiple-choice geometry questions. The VHGT was taken from a study by Usiskin (1982) with his written permission and is designed to measure the subject's van Hiele level when operating in a geometric context. This test was translated to Turkish by the investigator. Five mathematicians reviewed the Turkish version of VHGT in terms of its language and content. All participants' answer sheets from VHGT were read and scored by the investigator. Each participant received a score referring to a van Hiele level, guided by Usiskin's grading system.

Analysis of Data

The data were responses from the in-service middle and high school mathematics teachers' answer sheets. The criterion for success for attaining any given van Hiele level in this study was four out of five correct responses. The investigator constructed a frequency table to display the distribution of the mathematics teachers' van Hiele level. Independent samples *t*-test with $\alpha = 0.05$ was used to compare the

Table 1

Frequency Table for In-service Middle and High School Math Teachers' van Hiele Levels

Groups	*N*	Level-I (Visualization)		Level-II (Analysis)		Level-III (Ordering)		Level-IV (Deduction)		Level-V (Rigor)	
		n	%	*n*	%	*n*	%	*n*	%	*n*	%
A	110	19	17.3	19	17.3	54	49.1	12	10.9	6	5.5
B	38	0	0	14	36.8	18	47.4	3	7.9	3	7.9
Total	148										

geometric reasoning levels between teachers' genders along with level of students taught. The Levene's test with $\alpha = 0.05$ showed no violation of the equality of variance assumption in all the ANCOVA and the independent-samples *t*-test tables used in the study.

Results

In determining the reasoning levels of middle and high school mathematics teachers in geometry, Table 1 provides a summary of the distribution, indicating that van Hiele levels I through V were present. The most common stage for middle school mathematics teachers' reasoning stages was level-III (Ordering) (49.1%), but some showed a level-IV (Deduction) (10.9%) or level-V (Rigor) (5.5%) performance. According to table 1, none of the high school mathematics teachers showed level-I (Visualization) reasoning stage on the test; most were at level-II (Analysis) (36.8%) and level-III (Ordering) (47.4%). However, there were some performing at a level-IV (Deduction) (7.9%) or level-V (Rigor) (7.9%) of geometric reasoning.

Table 2 displays the mean score of in-service middle and high school teachers in order to help determine the differences that might exist between these two groups. High school mathematics teachers' van Hiele levels (2.87) was greater than that of the middle school mathematics teachers (2.70). The mean score difference in terms of reasoning stages was not statistically significant [$t = -0.88, p = 0.37 > 0.05$].

Table 3 presents the descriptive statistics for the mathematics teachers' van Hiele levels by gender. The table shows that the male mathematics teachers' mean score (2.88) is greater than that of the female mathematics teachers (2.59). However, according to the independent samples t-test, the mean score differences between male and female mathematics teachers on the Van Hiele Geometry Test (VHGT) is not statistically significant, [$t = 1.73, p = 0.086 > 0.05$].

Table 2

Descriptive Statistics and the Independent Samples t-Test for the In-service Teachers' van Hiele Levels

Group	*N*	van Hiele Geometry Test Mean	SD	SE	df	*t*	*p*
A	110	2.70	1.05	0.10	146	-0.88	0.37
B	38	2.87	0.87	0.14			
Total	148						

Note. A – In-service middle school mathematics teachers, B – In-service high school mathematics teachers.

Discussions and Conclusion

This study revealed that the in-service middle and high school mathematics teachers showed all reasoning stages described by the van Hieles. Although the proportion of mathematics teachers showing level-V (Rigor) was low in comparison to other levels, it is important to see some teachers operating at this level. This is important in a theoretical perspective because Usiskin (1982), Mayberry (1983), Burger and Shaughnessy (1986) and Fuys, Geddes and Tischler (1988) agreed that the last level, rigor (level-V), was not appropriate for high school students. It was more appropriate for college students or mathematics teachers. However, some studies noted that none of their pre-service elementary and secondary mathematics teachers indicated level-V (Rigor) reasoning stages in geometry (e.g., Mayberry, 1983; Gutierrez, Jaime, & Fortuny, 1991; Durmuş, Toluk, & Olkun, 2002; Knight, 2006). This study found that there were some mathematics teachers who operated at level-V (Rigor) on the test. Therefore, the finding supports the idea that level-V reasoning may be a realistic expectation of secondary teachers.

Table 3

Descriptive Statistics and Independent Samples t-Test for the In-service Mathematics Teachers' van Hiele Levels

Group	*N*	van Hiele Geometry Test Mean	SD	SE	df	*t*	*p*
Male	80	2.88	0.97	0.10	146	1.73	0.086
Female	68	2.59	1.04	0.12			
Total	148						

The study found that almost 83% of the middle school mathematics teachers' van Hiele levels were at or above level-II (Analysis). The reasoning stages of the middle school mathematics teachers involved in this study were higher than the level of their students; research has shown that most middle school students reason at level-I (Visualization) or at most level-II (Analysis) (Burger & Shaughnessy, 1986; Fuys et al., 1988; Halat, 2006). Sixty-three percent of the high school mathematics teachers were at or above level-III (Ordering), and only 15.8 percent of the secondary mathematics teachers were at or above level-IV (Deduction), the level at which high school students should be (NCTM, 2000). Mathematics teachers must have strong geometry knowledge and reasoning skills themselves in order to help high school students meet this expectation. The findings of this study imply that high school mathematics teachers' van Hiele levels may not be adequate for teaching geometry at the secondary level. This should be of particular interest to those charged with the task of preparing teachers of mathematics. The results of this study suggest mathematics teacher educators should assess the geometry knowledge of their pre-service teachers and modify programs to encourage growth in their geometric reasoning.

Furthermore, the study showed that there was no statistically significant difference with reference to geometric thinking levels between male and female mathematics teachers on the geometry test. As discussed earlier, research has documented that although there is a difference between the achievement of males and females in many content areas of mathematics (Grossman and Grossman, 1994; Lloyd, Walsh and Yailagh, 2005), there is no difference with respect to gender in reference to motivation and achievement in mathematics (Friedman, 1994; Fennema & Hart, 1994; Halat, 2006). The findings of the current study might support the latter group of research.

Limitations and Future Research

According to Mayberry (1983), students can attain different levels for different concepts. Likewise, Burger and Shaughnessy (1986) found that students may exhibit different levels of reasoning on different tasks. Because the researcher tested teachers using only questions on quadrilaterals, the results of this study should not be generalized to all geometry topics. Moreover, the results of the study should not be generalized to all in-service middle and high school mathematics teachers because of the differences in teacher preparation. Furthermore, the convenience sampling procedure followed in the study may limit the generalization of the findings. Additional research studies done with pre-service elementary and secondary mathematics teachers would be necessary in order to make a more general statement about teachers' geometric reasoning.

References

Armstrong, J. M. (1981). Achievement and participation of women in mathematics: Results of two national surveys. *Journal for Research in Mathematics Education*, *12*, 356–372.

Becker, J. R. (1981). Differential treatment of females and males in mathematics classes. *Journal for Research in Mathematics Education*, *12*, 40–53.

Billstein, R., & Williamson, J. (2003). Middle grades MATH Thematics: The STEM project. In S. L. Senk & D. R. Thompson (Eds.), *Standards-based school mathematics curricula. What are they? What do students learn?* (pp. 251–284). Mahwah, NJ: Lawrence Erlbaum Associates.

Burger, W. F., & Shaughnessy, J. M. (1986). Characterizing the van Hiele levels of development in geometry. *Journal for Research in Mathematics Education, 17*, 31–48.

Chappell, M. F. (2003). Keeping mathematics front and center: Reaction to middle-grades curriculum projects research. In S. L. Senk & D. R. Thompson (Eds.), *Standards-based school mathematics curricula. What are they? What do students learn?* (pp. 285–298). Mahwah, NJ: Lawrence Erlbaum Associates.

Crowley, M. (1987). The van Hiele model of development of geometric thought. In M. M. Lindquist, (Ed.), *Learning and teaching geometry, K-12* (pp.1–16). Reston, VA: National Council of Teachers of Mathematics.

Duatepe, A. (2000). An investigation of the relationship between van Hiele geometric level of thinking and demographic variables for pre-service elementary school teachers. Unpublished Masters' Thesis, Middle East Technical University.

Durmuş, S., Toluk, Z., & Olkun, S. (2002). *Sınıf öğretmenliği ve matematik öğretmenliği öğrencilerinin geometrik düşünme düzeyleri*. Orta Doğu Teknik Üniversitesi'nce düzenlenen 5. Ulusal Fen Bilimleri ve Matematik eğitimi Kongresi'nde sunulmuş bildiri, 16-18 Eylül: ODTÜ, Ankara.

Ethington, C. A. (1992). Gender differences in a psychological model of mathematics achievement. *Journal for Research in Mathematics Education*, *23*, 166–181.

Fan, X., & Chen, M. (1997). Gender differences in mathematics achievement: Findings from the National Education Longitudinal Study of 1988. *Journal of Experimental Education, 65*(3), 229–242.

Fennema, E., & Hart, L. E. (1994). Gender and the JRME. *Journal for Research in Mathematics Education, 25*, 648–659.

Forgasız, H. (2005). Gender and mathematics: Re-igniting the debate. *Mathematics Education Research Journal*, *17*(1), 1–2.

Fox, L., & Cohn, S. (1980). Sex differences in the development of precious mathematical talent. In L. Fox, L.A. Brody, & D. Tobin (Eds.), *Women and the mathematical mystique*. Baltimore, MD: Johns Hopkins University Press.

Friedman, L. (1994). Visualization in mathematics: Spatial reasoning skill and gender differences. In D. Kirshner (Ed.), *Proceedings of the Sixteenth Annual Meeting North American Chapter of the International Group for the Psychology of Mathematics Education*, (Vol.1, pp. 211–217). Baton Rouge: Louisiana State University.

Fuys, D., Geddes, D., & Tischler, R. (1988). *The Van Hiele model of thinking in geometry among adolescents*. (Journal for Research in Mathematics Education Monograph No. 3). Reston, VA: National Council of Teachers of Mathematics.

Gay, L. R. (1996). *Educational research: Competencies for analysis and application* (5th ed.). Upper Saddle River, NJ: Merrill Prentice Hall.

Geddes, D., & Fortunato, I. (1993). Geometry: Research and classroom activities. In D. T. Owens (Ed.), *Research Ideas for the Classroom: Middle grades mathematics* (pp. 199–225). New York: Macmillan Publishing.

Gottfried, A. E., Fleming, J. S., & Gottfried, A. W. (2001). Continuity of academic intrinsic motivation from childhood through late adolescence: A longitudinal study. *Journal of Educational Psychology*, *93*(1), 3–13.

Grossman, H., & Grossman, S. H. (1994). *Gender issues in education*. Needham Heights, MA: Allyn & Bacon.

Gutierrez, A., Jaime, A., & Fortuny, J. (1991). An alternative paradigm to evaluate the acquisition of the van Hiele levels. *Journal for Research in Mathematics Education*, *22*, 237–251.

Halat, E. (2006). Sex-related differences in the acquisition of the van Hiele levels and motivation in learning geometry. *Asia Pacific Education Review*, *7*(2), 173–183.

Halat, E. (2007). Reform-based curriculum & acquisition of the levels. *Eurasia Journal of Mathematics, Science and Technology Education*, *3*(1), 41–49.

Hoffer, A. (1988). Geometry and visual thinking. In T. R. Post (Ed.), *Teaching mathematics in grades K-8: Research based methods* (pp. 233–261). Newton, MA: Allyn & Bacon.

Hyde, J. S., Fennema, E., & Lamon, S. J. (1990). Gender differences in mathematics performance: A meta-analysis. *Psychological Bulletin*, *107*(2), 139–155.

Knight, K. C. (2006). An investigation into the change in the van Hiele level of understanding geometry of pre-service elementary and secondary mathematics teachers. Unpublished Masters Thesis,. University of Maine.

Lappan, G, Fey, J. T., Fitzgerald, W. M., Friel, S. N., & Phillips, E. D. (1996). *Shapes and design: Two-dimensional geometry*. Palo Alto, CA: Dale Seymour Publications.

Lloyd, J. E. V., Walsh, J., & Yailagh, M. S. (2005). Sex differences in performance attributions, self-efficacy, and achievement in mathematics: If I'm so smart, why don't I know it? *Canadian Journal of Education*, *28*(3), 384–408.

Malpass, J. R., O'Neil, H. F., & Hocevar, D. (1999). Self-regulation, goal orientation, self-efficacy, worry, and high-stakes math achievement for mathematically gifted high school students. *Roeper Review*, *21*(4), 281-228.

Mayberry, J. (1983). The van Hiele levels of geometric thought in undergraduate pre-service teachers. *Journal for Research in Mathematics Education*, *14*, 58–69.

McMillan, J. H. (2000). *Educational research: Fundamentals for the consumers* (3rd ed.). New York: Addison Wesley.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

Serra, M. (1997). *Discovering geometry: An inductive approach* (2nd ed.). San Francisco, CA: Key Curriculum Press.

Smith, S. E., & Walker, W. J. (1988). Sex differences on New York State Regents examinations: Support for the differential course taking hypothesis. *Journal for Research in Mathematics Education*, *19*, 81–85.

Stipek, D. (1998). *Motivation to learn from theory to practice*. (3rd ed.). Needham Heights, MA: Allyn & Bacon.

Usiskin, Z. (1982). *Van Hiele Levels and Achievement in Secondary School Geometry*. (Final report of the Cognitive Development and Achievement in Secondary School Geometry Project.) Chicago: University of Chicago. (ERIC Document Reproduction Service No. ED220288).

Van Hiele, P. M. (1986). *Structure and insight: A theory of mathematics education*. New York: Academic Press.

Wirszup, I. (1976). Breakthroughs in the psychology of learning and teaching geometry. In J. I. Martin & D. A. Bradbard (Eds.). *Space and geometry: Papers from a Research Workshops*. Columbus, OH: ERIC Center for Science, Mathematics and Environment Education.

Wright, S. P., Horn, S. P., & Sanders, W. L. (1997). Teacher and classroom context effects on student achievement: Implications for teacher evaluation. *Journal of Personnel Evaluation in Education*, *11*(1), 57–67.