# EARLY SPATIAL THINKING AND THE DEVELOPMENT OF NUMBER SENSE

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explains how teachers can build on young children's spatial abilities to develop important foundations of number sense.

Number sense has been recognised as central to young children's development of mathematics for a number of decades (Australian Education Council, 1990; National Council of Teachers of Mathematics [NCTM], 2000; Sowder, 1988). Although still rather nebulous to describe, a student with a 'good' sense of number normally has a thorough understanding of relationships among numbers and operations—being able flexibly to partition and combine numbers in convenient ways to allow appropriate estimations and mental calculations to be made. Components of number sense also include a facility with basic facts, an understanding of place value, the ability to use meaningful benchmarks or referents for numbers (such as 0.48 is nearly ½) and an understanding of the relative and absolute magnitude of numbers.

More recently, the significance of children's spatial abilities, particularly their ability to perceive spatial structures, has gained recognition for its influence on young children's development of mathematics. For example, Mulligan, Mitchelmore and Prescott (2005) report that young mathematically gifted children's representations used to help solve mathematical tasks are more structured and involve dynamic imagery; whereas low achieving students' representations tend to show little or no underlying structure and only use static images. They explain 'spatial structuring' as a process of constructing an organisation or form in space, and include the ability to identify spatial features and recognition of the relationships between these features. So a young child with well-developed spatial structuring abilities would be able successfully to construct and continue a triangular pattern of dots like that shown in Figure 1 (Mulligan et al., 2005, p. 5).

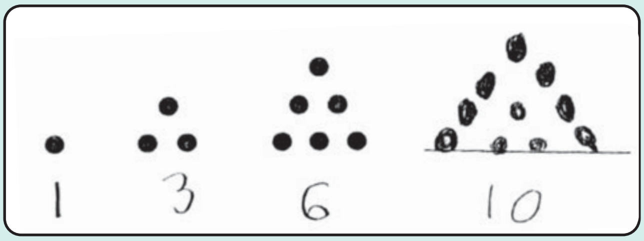


Figure 1. Spatial structuring task involving a pattern of triangular numbers.

The aim of this paper is to show how young children's spatial structuring abilities can contribute to an emerging sense of number. The result is a collection of activities that may encourage the parallel development of children's spatial and number sense. The relationship between the two has been recognised for some time (Bobis, 1996), with spatial sense-type activities already incorporated into the number content of early years curricula (Board Of Studies [BOS], NSW, 2002). However, emerging evidence suggesting that the link is far greater than previously considered (e.g., van Nes & de Lange, 2007), means that there is a need to revisit and more fully explore the relationship between the emerging spatial and number sense of young children.

## Part–whole knowledge and number sense

A fundamental numerical concept is the understanding that all numbers can be conceptualised as composites of other numbers. For instance, knowing that 5 can be partitioned into "3" and "2" or "4" and "1" means that number facts do not need to be treated in isolation and then memorised, but that each number can be considered in relation to other numbers and thus, make more sense. The more relations a child 'sees,' the more flexible they can be in their mental strategies for solving complex computations.

Number relationships such as part–whole relationships exist for children only when they have constructed meaning for them in their own minds. We cannot 'make' children understand relationships between numbers in any direct way, but we can provide activities designed to enhance the construction of these relationships. Such activities utilise the ability of young children to recognise spatial structures or arrangements almost instantly, without the need for counting individual items. Although counting is certainly an important skill, we often ignore the natural capacity of young children to identify and compare quantities through visualisation strategies when small numbers of objects are present; for example, when a die is thrown and we instantly distinguish between a "5" or a "6" arrangement of dots. The development of such a skill can provide a basis for enhancing children's knowledge of part–whole number knowledge, which has implications for learning basic addition and subtraction facts and has been linked to a child's meaningful understanding of the place value system later in their development of number knowledge (Hunting, 2003). For instance, such knowledge can allow primary-aged students easily to move between the different place value units of hundreds, tens and ones, thus allowing more flexible mental computation strategies to be used. Thus, when faced with the computation of, for example, $43 \times 5$ a child may be able mentally to multiply the digits in the tens and ones place value columns by 5 separately and then combine the answers to find the product. Alternatively, they might simply multiply 43 by 10 and then halve the answer. Either strategy relies on an understanding of number relationships, and particularly knowledge of combining and partitioning.

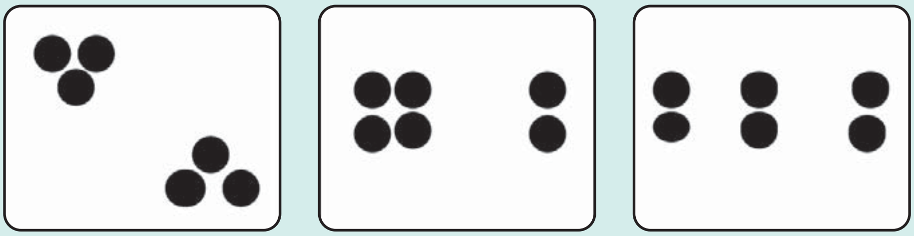

Figure 2. Three arrangements of dots for "six" suggesting different ways of structuring or 'partitioning' that number.

## Spatial thinking and subitisation

The process of instantaneously recognising the number of items in any spatial structure without counting is known as "subitising," which comes from the Italian word "subito" meaning "immediately." It is well known that even pre-school children are capable of instantaneously recognising the number of objects in spatial arrangements of up to four objects. It has also been found that children can usually subitise before being able to count and that it may even be a necessary antecedant to counting (Klein & Starkey, 1988). Having been surrounded by groups of objects from a young age, children develop stable mental images of these spatial arrangements and they are soon able to reconstruct them for themselves without the need for physical items. Constant exposure to the number names used to describe the arrangements help children form pattern-name associations. For example, children who are familiar with the arrangement of dots on a die do not need to count each dot every time a number is rolled. However, it is important to stress that recall of a number name does not necessarily mean that a child fully understands the concept of "3" but may merely be recognising the name for a spatial arrangement of dots. This type of subitising is referred to as "perceptual subitising" and is a primitive ability that is also evident in animals and babies (Clements, 1999). Nevertheless, it is still an important basis for further learning. Utilising the capacity of young children to visualise arrangements of dots similar to those that occur on a die or domino can provide a basis for developing children's understanding of relationships between numbers. An advantage of presenting different arrangements of dots for the one number is that they suggest different ways of partitioning the whole. For example, a child might recognise an arrangement of "two threes" or an arrangement of "two, two and two" for "six" depending on how the dots are spatially arranged (see Figure 2).

Organising the dots into recognisable subgroups facilitates recognition and naming of groupings with number words. Joining and separating visual patterns provides a basis for developing part–whole number relationships. Children who can partition a spatial arrangement of dots into its composite parts *and* recognise the whole in this way are using a more advanced form of subitisation: *conceptual subitisation*. This type of subitisation is evident when a child can explain that an arrangement of six dots, for example, comprises combinations of "4" and "2" or "3" and "3". For most children, perceptual subitising occurs quite naturally without specific instruction, but conceptual subitising (usually) must be learned.

## Spatial activities that develop number sense

The following activities may stimulate children's development of both spatial and number sense. The activities utilise commercially available dominoes and dice (with dots rather than numerals), and teacher-made dot cards and ten frames. Many of the activities have been played in classrooms for generations, but the full extent to which such activities can benefit children's understanding of number relationships has often remained unrecognised by teachers. However, such advantages are now emphasised in many of the activities used by teachers involved in numeracy programs like *Count Me In Too* (for a more extensive list of activities, see NSW Department of Education and Training, 1998) and are referred to in current state and territory syllabuses (e.g., BOS NSW, 2002).

The set of dot cards should include numbers from one to ten represented by at least three different arrangements (e.g., see Figure 2 for arrangements of the number "six"). The exact size of the cards is not significant. The important requirement is that they utilise the ability of children to subitise and that they encourage mental shifts from thinking about a number as one type of dot arrangement (or part–whole combination) to another (conceptual subitisation). Ten-frames, on the other hand, are rectangular flash cards divided into two rows of five equal-sized boxes where counters or dots are placed to illustrate numbers less than or equal to ten (see Figure 3).

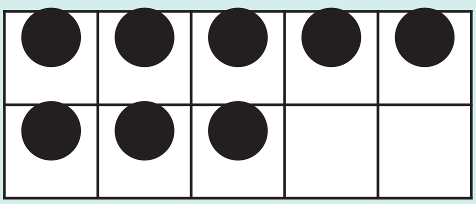

Figure 3. A ten-frame for eight.

## Circle champion (Pre-K–6, small groups or whole class)

This activity works best with large foam dice, but ordinary-sized dice will also work. All children except for one sit in a circle while the one child stands behind a seated child who has a die. The game starts with these two children. For pre-school or kindergarten classes, the die is rolled into the centre of the circle and the child standing must race to say the number name for the arrangement of dots rolled before the child seated immediately in front of him or her. The other children in the circle "judge" who is the fastest. If the child standing wins, he or she remains standing and moves behind the next child (or challenger) seated in the circle. The die is passed around the circle. If the challenger wins, they must swap places with the child standing and move to stand behind the next child in the circle. The game continues in this way until the teacher decides to stop. This game can be modified to suit children from pre-school to Year 6. For example, while still using one die, children have to say the number that is one more (one less, two more, three more, double, five times greater, etc.) than the number rolled. As children need more challenges, introduce more dice and ask them to add or multiply the numbers according to their abilities.

## Dice and dot card match (Pre-K–3, small groups)

Use two or three sets of dot cards containing up to 6 dots. Place the dot cards face up in the centre of the children. Players take turns to roll the die and match the number of dots on the die with the corresponding dot card. The winner is the child with the most dot cards when all the cards have been removed. For children in Years 2 and 3, use two or more dice and include dot cards up to 18. Children can record the different combinations of numbers.

## Flash-and-tell with dot cards (Pre-K–2, whole class)

Begin with dot patterns for numbers less than five. Add new ones to the familiar cards gradually. Flash a card and ask children to tell you what they saw. Encourage discussion that relates to the different ways a number may be partitioned. For instance, a pattern for seven could be seen as three dots, two dots and two dots; or as three and four. Children could also be encouraged to say the number "before," "after," or "one or two more" than the number represented by the dot card. Alternatively, flash a card and ask the children to copy the arrangement with counters. Ask them to record how their number is arranged by drawing or tracing the counters. Different colours could be used to indicate different groups of dots.

### Dominoes
### (Pre-K–3, small groups)

Small groups of children could play the traditional game of dominoes. By Year 2, children could be assigned a number between 15 and 30 and be asked to find four dominoes that have the total number of dots for their number. The children could then check each other's answers (perhaps with the help of a calculator) and write down the number sentence that their four dominoes make. This process should be repeated several times with a new target number for each student. Discuss the many different combinations to make each target number. As a further challenge, children could work in pairs to find the largest possible sum or the smallest possible sum from any set of four dominoes.

### Ten-frames flash-and-tell
### (K–2, whole class)

Children can learn to recognise arrangements of dots and their number names according to the number of dots and empty frames occurring on a ten-frame. For example, if the ten-frame for eight was flashed, the child could be asked:

- How many dots were on the card?
- How many empty spaces?
- How many dots will there be if we add one more (or less) to the number shown?
- How many more dots to make ten? How do you know?

### Make a number
### (K–2, whole class)

Provide children with empty ten-frames. Allow them to select the number of ten-frames with which they feel comfortable working. Ask them to arrange counters to make a specific number (or number of their choice). Encourage arrangements that are easy to read without counting (see Figure 4).

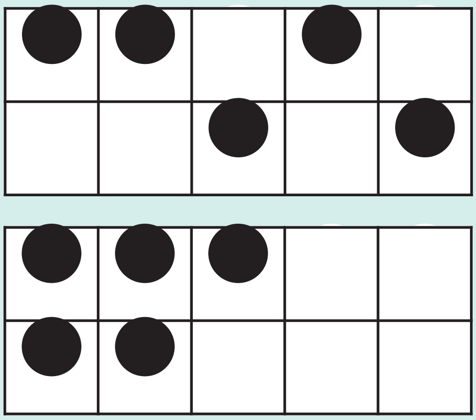

Figure 4. Two arrangements for five; the second arrangement is easier to subitise.

Allow some freedom for those children in your class who intuitively can work with larger quantities. I recall one 5 year-old girl who, when asked to "imagine" an arrangement of dots, visualised 20 (2 groups of 10) but because she had only been given a single ten frame, she had to 'squash' two counters into each empty box of the frame.

Ask the children to add one more counter (or remove some) and encourage them to notice how many spaces are now filled and how many remain empty. What arrangements of dots do the children 'see' in their new number?

### Addition and subtraction to 20
### (K–2, whole class)

Develop the notion of a group of 10 and some more (pre-place value activity) to enhance the visualisation of numbers between 11 and 20; e.g., 10 and 5 make 15. For addition problems involving 8 and 9, encourage the children to make 10 and then add the remaining numbers. For example, for 8 and 6, make 10 by adding 8 and 2, and then add the remaining 4 dots to get 14. This should be done first with concrete materials (ten frames and counters on an overhead projector) and then mentally (see Figure 5).

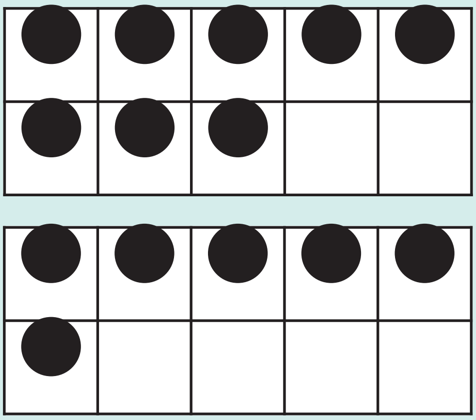

Figure 5. Take 2 dots from the 6 dot arrangement to make 10 with 4 remaining.

### Numbers to 20 (K–2, small groups)

Using dot cards or ten-frames, provide the children with two copies of the numbers 0 to 10 and one set of numeral cards with the numbers 0 to 20. Place the dot cards or ten frames face down and the numeral cards face up. Children take it in turns to flip over two dot cards and add the two cards together. Children should be encouraged to count-on from the larger number or use visualisation strategies such as those described in the previous activity. If the total corresponds to a numeral card still remaining on the table, the child takes the numeral card. The game ends when all the numeral cards have been taken.

## Conclusion

The activities presented here focus on the visualisation of spatial structures or arrangements of dots to solve simple problems, rather than just counting one-by-one. They also require that children focus on part–whole relationships of various number combinations—especially the decomposition of ten.

Research has shown that when given appropriate materials and activities, children are capable of mentally combining and partitioning numbers, thus enabling them to recognise number wholes and their related parts (Bobis, 1996). This is an important skill to develop prior to the introduction of addition and subtraction in the symbolic form and aids the development of children's sense of number. Counting is an important skill, but teachers should also consider the benefits gained by developing children's spatial thinking abilities through activities involving dice, dominoes, dot cards and ten-frames.

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