Using SPREADSHEETS to make algebra more accessible

Part 1: Equations and functions

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Background

Mathematics is not about answers, it is about processes! One of the more interesting views on this appears in a paper by Robert H. Lewis (2000) titled *Mathematics — The Most Misunderstood Subject*. He uses the following analogy:

> When a new building is made, a skeleton of steel struts called the scaffolding is put up first. The workers walk on the scaffolding and use it to hold equipment as they begin the real task of constructing the building. The scaffolding has no use by itself. It would be absurd to just build the scaffolding and then walk away; thinking that something of value has been accomplished.
>
> Yet this is what seems to occur in all too many mathematics classes in high schools. Students learn formulas and how to plug into them. They learn mechanical techniques for solving certain equations or taking derivatives. But all of these things are just the scaffolding. They are necessary and useful, sure, but by themselves they are useless. Doing only the superficial and then thinking that something important has happened is like building only the scaffolding.
>
> The real "building" in the mathematics sense is the true mathematical understanding, the true ability to think, perceive, and analyze mathematically (p. 1).

"[T]he true ability to think, perceive, and analyse mathematically." What does this mean? Some might argue that it is the ability to solve problems. Others might say that it has more to do with advanced applications of procedures in particular contexts. I would like to put a slightly different spin on this by viewing it, at least partly, in terms of one's ability to use understanding in one concept to bridge understanding into another. The making of such connections will help to make the teaching and learning of mathematics more interesting and more attainable especially for those who might struggle with the discipline.

This article is the first in a series of two papers that attempt to address such an idea. The articles endeavour to give teachers some practical, spreadsheet-based ideas for helping students make appropriate connec-

tions between particular algebraic concepts, in order to understand, through a contemporary context, some of the processes by which mathematics develops.

Equations and functions

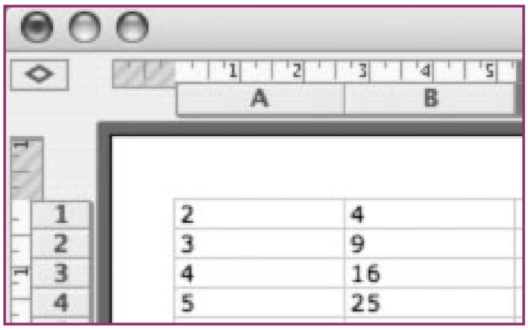

Figure 1

The first example is illustrated by the spreadsheet in Figure 1. The spreadsheet contains paired data values (ordered pairs): 2 is paired with 4; 3 is paired with 9, and so on. Students are asked to look for a relationship between the number that is placed in cell A1 and the number that is placed in cell B1, between the number that is placed in cell A2 and the number that is placed in cell B2, and so on. They are asked to represent the relationship as a series of equations, for example, $B1=(A1)^2$, $B2=(A2)^2$, $B3=(A3)^2$.

Once students understand this, they can put an arbitrary value into cell A5 and predict the value that they should put into cell B5, so that the same relationship is maintained between the two columns of numbers. Students should now be ready to generalise their thinking by stating the relationship (function) as $B = A^2$. The variables X and Y are not used at this early stage. One of the benefits in using a spreadsheet to introduce the idea of an algebraic variable is that it gives the clear message that alphanumeric indicators are little more than a label for a box, in this case the box is the cell in the spreadsheet. Final calculations cannot be performed unless a value is first assigned to that previously empty box. In other words, the variable has to be assigned a numerical value.

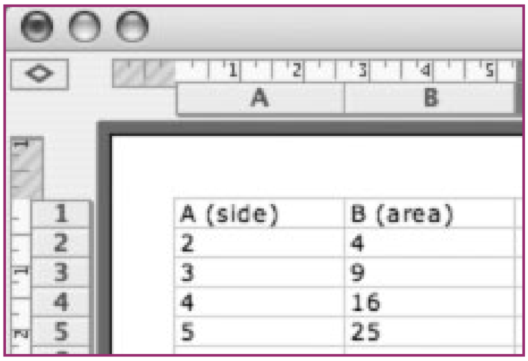

Figure 2

The next step to give the numbers on the spreadsheet a context by adding two labels (Figure 2). In this example the spreadsheet data could represent the measurements for the sides and the corresponding areas of four different squares. The relationship may still be expressed using the alphanumeric indicators A and B where A represents the side length and B represents the area.

This now provides a platform for instigating the back-tracking procedure for solving equations. Place 49 into cell B6 and have students back-track what should be in cell A6 (i.e., 7). Repeat this for (A7, B7), (A8, B8) and so on. Now have students represent what they did as a series of equations (i.e., $A1 = +\sqrt{B1}$, $A2 = +\sqrt{B2}$, $A3 = +\sqrt{B3}$). This thinking may now be generalised as $A = +\sqrt{B}$. (While the idea of one-to-one functional correspondence needs to be introduced at some point, it need not be discussed at this particular point in time.)

The relationship may just as appropriately be represented as (A2, B2), (A3, B3), (A4, B4), (A5, B5) with the values for these being drawn from the cells in the spreadsheet. In other words, the relationship may be represented as (2, 4), (3, 9), (4, 16), (5, 25). Another way to describe this

relationship is to say that it is made up of the paired values (A, B) where $B = A^2$ or $A = +\sqrt{B}$. It can also be seen that (A, A^2) or $(+\sqrt{B}, B)$ are equally legitimate representations of the relationship.

Using graphs is another method of representing a relationship between variables. The difficulty, however, is that many prerequisite skills have to be mastered before a student is able to draw a graph with the accuracy that is required to answer questions about the relationship. This takes time. The graphing tool in a spreadsheet gives students the opportunity to make an immediate start at analysing some basic relationships. Figure 3 illustrates the necessary steps.

The data is first selected using a standard click and drag operation. The *xy* (scatter) and line graph options are selected from the charting tool menu. This gives the first graph in Figure 3. Once the graph is obtained it is possible to remove (or add) paired values to the graph by clicking and dragging the grow-bar on the right-hand side of the 49. Dragging up will remove paired values while dragging down will create new spaces into which new pairs of values can be inserted. The graph is instantly rescaled and redrawn. This is one of the more powerful and useful features of the graphing tool in Excel.

In the example it has previously determined that the equation for the relationship is $B = A^2$. There will be times, however, when it is difficult to make such a determination because the relationship is not obvious. In such a situation Excel's predictive capabilities can be useful. Figure 4 illustrates the necessary steps.

The graph is first selected by moving the pointer to the graph and clicking. (You will know that the graph has been selected when you see the eight small 'growbars' as illustrated in the graph in Figure 4). *Add trendline* is then selected from the *Chart* menu. Choose *Power* for the Trend/Regression type. Under the *Options* submenu tick the *Display equation on chart* option. This is little more than a 5 second (rote) task. It is not necessary (especially at the middle school level) that students understand the advanced regression techniques used by Excel to predict this

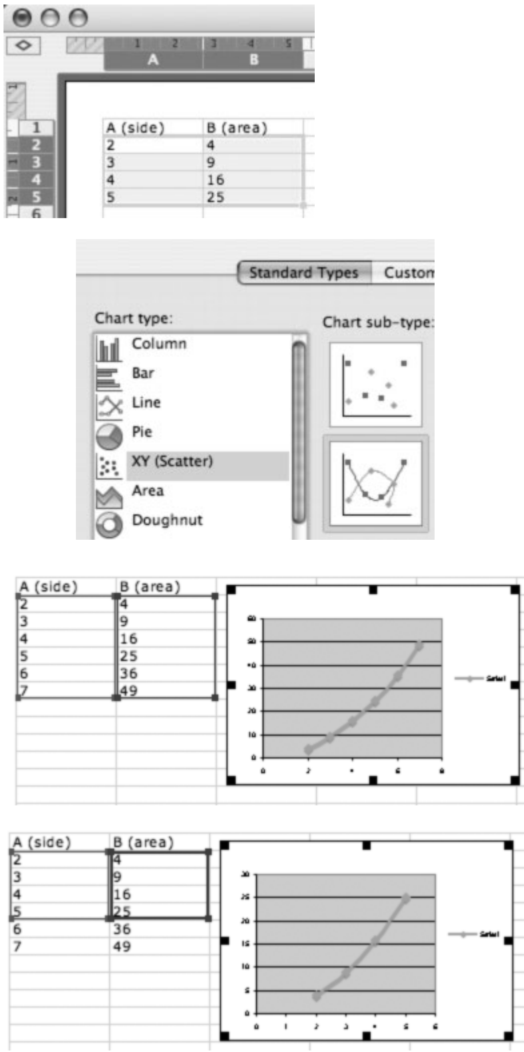

Figure 3

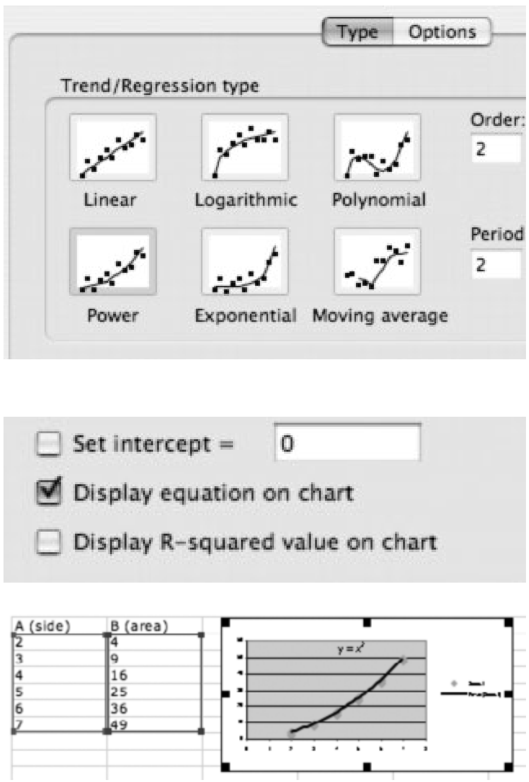

Figure 4

equation. Excel writes the equation using the variables *X* and *Y* rather than the variables *A* and *B*.

This particular relationship was so obvious that this tool was not necessary to predict what it was. A slightly more complex relationship exists in the following set of paired values (2, 1.5), (3, 2), (5, 3), (6, 3.5). For the purpose of this exercise students will need to be told that it is a linear relationship. Excel will predict the equation as being $y = 0.5x + 0.5$. With a little practice this now paves the way for working on more complex investigations such as temperature conversion equations or the formula for finding the circumference of a circle.

Further examples of investigations of functions

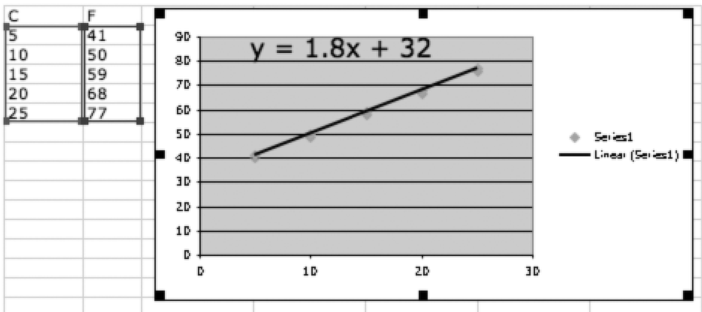

Figure 5a

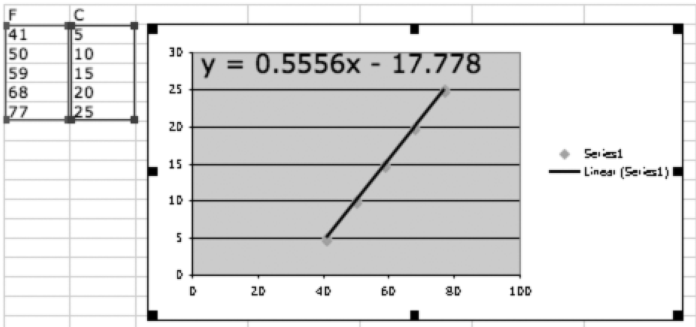

Figure 5b

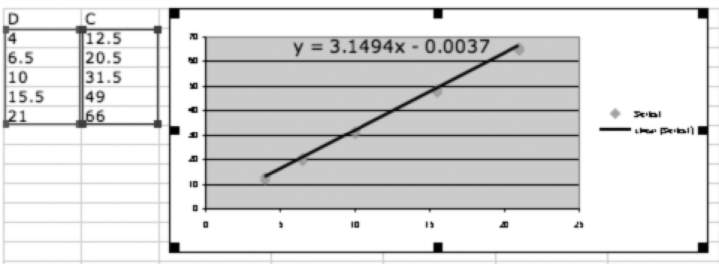

Figure 6a

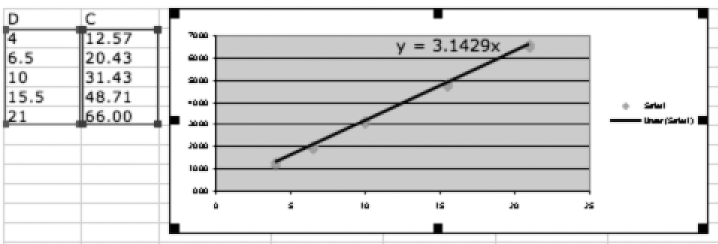

Figure 6b

The first example is the generation of a formula for converting Celsius temperatures to Fahrenheit temperatures. A dual Celsius/Fahrenheit thermometer may be used to obtain some common conversions such as 0 degrees Celsius equals 32 degrees Fahrenheit, 10 degrees Celsius equals 50 degrees Fahrenheit, and so on. The data is then entered into a spreadsheet and analysed using the spreadsheet's predictive capabilities (Figure 5a). The end result states that to convert to Fahrenheit from degrees Celsius, the Celsius reading is multiplied by 1.8 and 32 added. To obtain the equation for converting in the opposite direction (i.e., from Fahrenheit to Celsius) the data is reversed (Figure 5b). Later on, when students have learned techniques for solving equations they may simply change the subject of the formula to obtain the conversion formula for working in the opposite direction.

The second example generates a formula for finding the circumference of a circle. It was Euler who first proposed the existence

of a constant relationship between the circumference of a circle and the diameter of the circle. Excel can make it a little easier for students to follow Euler's reasoning. Figure 6a illustrates how students will record their paired value data resulting from measurements of circumferences and diameters of a variety of circular objects.

From the results, in the example, Excel predicts the circumference as 3.1494 times the diameter less 0.0037. It would not be too difficult to explain to students that this is a very good approximation of the theoretical relationship $\pi \times D$. The differences in the third decimal place of π and the subtraction of the very small constant can be attributed to rounding errors in the measurements. If, for example, the more accurate measurements of circumference in Figure 6b were to be used then the relationship given should be much closer to the theoretical relationship.

Conclusion

Contrast the ideas above with a more "traditional" way of teaching and sequencing such topics. Often students are taught how to mark ordered pairs on a plane and produce a graph before they are conceptually ready and have some understanding of how a graph represents "functional" relationships. The role of equations has to be carefully thought out and contextualised using examples that students will relate to. Graphing and algebraic relationships can be integrated into the work across a number of strands including measurement and geometry.

Adopting approaches such as those contexts described in this article will enable students to investigate higher level tasks that involve solving equations, finding functional relationships, and changing the subject of a formula. It is an approach that promotes inference making, connections and understanding over rote learning and procedural repetition. In a later edition of this journal, Part 2 of this series of papers will continue with an investigation of these ideas into the topic of equation-solving.

References

http://www.fordham.edu/mathematics/whatmath.html

From Helen Prochazka's

Scrapbook

"It can be of no practical use to know that Pi is irrational, but if we can know, it surely would be intolerable not to know."
E. C. Titchmarsh

"To speak algebraically, Mr. M. is execrable, but Mr. C. is (x + 1)ecrable."
Edgar Allen Poe discussing fellow writers Cornelius Mathews and William Ellery Channing.