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Quality of Instruction:

Examining Discourse in Middle School Mathematics Instruction

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Background

What role does discourse play in the middle school mathematics classroom and can various types of discourse differentially contribute to mathematics engagement? We know that mathematics instruction should be more than teachers' writing on their chalkboards and explaining algorithms as they solve problems, hoping that students can follow along. Mathematics instruction has evolved into a more democratic, collaborative, and conceptually based form of learning. However, the dialogic interaction that accompanies these pedagogical changes remains relatively unexplored. Discourse can take many forms. In the simplest form, the teacher tells and students respond. More complex forms

summary

According to the NCTM reform suggestions, when teachers are orchestrators of student interactions, students adopt a more active role in explaining and learning mathematics. This research, which mapped the nature and role of meaningful mathematical discourse, provides insights into discursive practices that lead to rich mathematical interactions. We observed, coded, and analyzed middle school algebra, number, and data lessons using a grounded theory approach. We organized the observed paths that emerged into a map depicting actual paths for mathematics discourse. The results indicated that communication pathways between the teacher and students occur in many ways, and certain student-initiated questions may trigger predictable teaching patterns. Conversation that originates with the teacher often results in dialogue that is one-dimensional, mostly provides factual information, and rarely results in rich, meaningful mathematical dialogue. However, when students engage in the teacher's conversation or they are persistent in their own questioning, teachers tended to provide more detailed explanations, and teachers often embellished with new examples and representations using nuanced solution methods. Although results seem to indicate that teaching children to be persistent with their questioning will enhance understanding, this behavior may be interpreted as threatening to some teachers. Therefore, caution is warranted when attempting to turn these findings into action. It is important that, before instructing students about being persistent with questions, teachers understand the students' intentions. Although participants did not have negative reactions to persistent student questioning, some children might experience negative responses without proper professional development for teachers.

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include situations in which both stakeholders ask questions of each other. This dialogic nature also can be simplistic, such as when the teacher asks closed questions and students do nothing more than supply a missing word; however, it also can lead to rich, meaningful discourse. The potential for rich, meaningful discourse can occur when teachers ask questions that resemble the logical thinking process for solving a problem (explanation), when the teacher engages students by asking if is there is another way to solve the problem (justification), or when the teacher asks questions that extend beyond rote memorization. When students ask questions that extend beyond the explicit, "Can you show me how to do that problem again?" and the implicit, "I don't understand," with more specific and/or more generalizable questions, the teacher can provide alternative explanations or justifications that may initiate rich, meaningful discourse.

The purpose of this research was to examine the nature of classroom discourse related to the teaching and learning of algebra, number, and data analysis. Specifically, we were interested in understanding how student-to-student or teacher-to-student dialogue resulted in changes to a questioning and question-explanation framework. For the purposes of this study, we defined rich, meaningful discourse as interactive and sustained discourses of a dialogic nature between teachers and students aligned to the content of the lesson that addresses specific student learning issues. This definition is closely aligned to Hicks (1995–1996), who also suggested that any process should allow for understanding how teachers and students collectively and individually construct disciplinary knowledge.

What a teacher says is not the only important factor for students' attainment of success in advanced academics. Classroom discourse practices are important in students' mathematical development. The nexus of teacher factors and student responsibility lies in classroom discourse practices that are likely to identify each student's maximal opportunity to learn. Vygotsky (1978) coined the term *zone of proximal development* to describe this situation. Research on the interactions between teacher and student emphasizes the teacher's role in facilitating conversational dialogue. It is this dialogue that precipitates significant mathematical ideas and makes this discourse foundational to meaningful insights for both students and teachers (Thompson & Thompson, 1996). However, discourse in and of itself should not be the goal (Ball, 1991); specific conversations must be justified in terms of learning outcomes. The desired outcome is communicating to learn mathematics rather than learning to communicate mathematically.

Classroom Questioning

Researchers have attempted to examine how student-teacher talk was related to student learning. For instance, Buggey (1971) and Aagard (1973) found that students exposed to a larger number of higher order questions performed better on an outcome measure than students exposed to fewer higher order questions. However, the Stanford Program on Teaching Effectiveness (Gage, 1976) found that students exposed to a greater quantity of higher order questions underperformed their counterparts. Ryan (1974) and Winne (1979) found no measurable effect between students experiencing a greater quantity of higher order questions as opposed to fewer higher order questions. The various methodologies employed in these studies lacked the necessary power to yield a unified theory. However, a meta-analysis of 20 articles on teacher questioning revealed a standardized effect of 0.72 in favor of students whose teachers asked higher order questions over students of teachers who did not use higher order questions (Redfield & Rousseau, 1981).

In application, the purpose of teacher questioning has been to evaluate what students knew (Dillon, 1988), but as sociolinguistic findings have begun to influence the field of education, research interest on effective questioning has grown (Bellack, Herbert, Hyman, & Smith, 1966; Carlsen, 1991; Cazden, 1988). Effective questioning consists of question combinations intended to *probe* or *evaluate* what students know about the topic (Dillon, 1988; Graesser & Person, 1994; Winne, 1979), *guide* toward specific understandings (van Zee & Minstrell, 1997), *elicit* discussion, or *check* progress of the lesson (Gall, 1984; Mehan, 1985; Stevens, 1912).

Classroom Discourse and Questioning

Persistent questions by both teacher and student can help facilitate the development of mathematical understanding in students (Kazemi, 1998; Knuth & Peressini, 2001; Martino & Maher, 1999; National Council of Teachers of Mathematics [NCTM], 1996). Research on classroom discourse often cites the NCTM (1991) teaching standards recommendation that mathematics teachers initiate and "orchestrate discourse by posing questions that elicit, engage, and challenge students' thinking," by "listening carefully to students' ideas," and by "asking students to clarify and justify their ideas orally and in writing" (p. 35). More recently, NCTM (2000) recommended that teachers encourage and enable students to "communicate their mathematical thinking coherently and clearly to peers, teachers, and others" and that students need to learn "what is acceptable as evidence in mathematics" (p. 60). Teacher questioning remains an important means to achieving this standard for students (Mason, 2000). Classroom discourse, properly managed, allows students to concentrate on sense making and reasoning; it allows teachers to reflect on students' understanding and to stimulate mathematical thinking.

Teachers can stimulate students' growth of mathematical knowledge through the ways they ask and respond to questions. One method of stimulation is asking open-ended questions that are designed to initiate problem solving and aid conceptual understanding (Martino & Maher, 1999). By accepting correct answers and issuing a series of related questions or by providing evaluative or neutral comments to incorrect answers followed by a rewording of the original question, Chin (2006) found that teachers "further elicited student responses, stimulated productive thinking, and extended lines of conceptual thought in students" (p. 1326). Teachers' questions can serve to scaffold students' thinking and lead students toward conceptual understanding through teacher-student discourse (Chin, 2006). For example, Japanese teachers, more than U.S. teachers, often orchestrate the kind of discourse that is advocated in reform documents and ask "more describe/explain questions, and fewer yes/no questions" (Stigler, Gonzales, Kawanaka, Knoll, & Serrano, 1998, p. 123).

Student Questioning

Classroom teachers in the United States find it difficult to provide a classroom environment rich with question-posing while ensuring the required curriculum is presented. Students have questions. So why, when classrooms are observed, do researchers not find students asking these questions (Nathan & Knuth, 2006)? One explanation may be that teachers do most of the talking in classrooms (Cazden, 1988; Stigler et al., 1998). Hervey (2006) proposed that because teachers ask most of the questions and do not encourage students to ask their questions, students are likely to refrain from questioning.

Researchers have shown that the questions students ask illustrate their focus on the content of the material (Palinscar & Brown, 1984; Scardamalia & Bereiter, 1991). When students pose questions, they are thinking about their thinking. These metacognitive strategies can help students learn to take control of their own learning by self-defining learning goals and self-monitoring progress in achieving those goals (Donovan & Bransford, 2005). Martinello (1998) suggested that many children do not know how to effectively pose questions without explicit instruction in asking questions. A review of intervention studies found that teaching students to generate questions improved their comprehension of material they had read, which led to gains in comprehension as measured on written tests (Rosenshine, Meister, & Chapman, 1996). Additionally, King (1994) and Lampert (1990) found that when students were encouraged to ask questions, they were able to generate links among thoughts within the content of the lesson and connect those links to their prior knowledge. Additionally, students who posed questions were more flexible with the content and demonstrated greater comprehension.

Responses to Student-Initiated Questions

In every lesson, teachers make discursive decisions spontaneously as they lead discussions and convey content knowledge (Hayes, 1999). For example, teachers wrestle with what to say next to connect prior knowledge to the material at hand. These spontaneous moments cannot be planned and often occur at inopportune moments. When students ask questions or make comments (provide a glimpse of their incremental conceptions), a process is triggered that challenges the teacher to quickly consider the following and respond: (a) assess student thinking, (b) formulate a plan, and (c) engage or dismiss the comment. The assessment process happens nearly instantaneously and simultaneously with plan formation. Depending on various real-life teaching demands, the teacher either engages the student by creating an opportunity for discourse, by asking a question(s), by reteaching, or in some cases by avoiding the question (King, 1991). These spontaneous and frequent decisions have an impact on the quality and direction of classroom discourse.

Examining Discourse

Previous studies have examined discourse in terms of utterances within small discrete time periods. These small time periods and isolated utterances allowed for the counting of words and phrases but rarely include sentences or entire discourse segments (cf. Cazden, 1988; Kazemi, 1998; Knuth & Peressini, 2001; Martino & Maher, 1999; Mehan, 1985; Winne, 1979). In the early 1990s, studies examined questioning as a structure for eliciting discourse as opposed to counting questions based on taxonomies, such as Bloom's taxonomy. Researchers examined the function of open-ended questions for provoking discussion, facilitating learning, and gaining insight into children's thinking (Ralph, 1999a, 1999b; Stenmark, 1991). Teachers can use open-ended questions to cue students, to improve on weak responses, to develop a discussion atmosphere, or to foster curiosity and inquiry.

Therefore, it is important to understand the influence of dialogic interactions in actual classroom settings as teachers and students explore new content. Thus, this study examined the nature of classroom discourse related to teaching and learning. Specifically, we examined how the dialogue, student-student or teacher-student, resulted in changes to a questioning and question-explanation framework.

Methodology

Data

The data for this study were 3 years of classroom video recorded after an intervention (72 videos from 2002–2003 representing 18 teachers, 71 videos from 2003–2004 representing 20 teachers, and 40 videos from 2004–2005 representing 10 teachers). All middle school mathematics teachers (grades 6–8) from 5 school districts participated (for more information, see Nelson, Kulm, & Manon, 2000). Each participant was scheduled to be videotaped four times per year. However, due to illness and professional commitments, substitutes taught some of the target lessons, resulting in fewer recorded lessons. Thus, there were three or four videos per participant. The lessons' topics were selected by the research team to match the scope and sequence for the districts. The lessons were taught throughout the year, but the lesson objectives were identical. The lessons revolved around (a) symbolic equations used to summarize how a quantity of something changes over time or in response to other changes; (b) equivalent forms of integers, fractions, decimals, and percents to interpret and compare numbers; and (c) measures of central tendency. The researchers conducted a one-week summer in-service to train participants in the discourse strategies under investigation. The training focused on viewing classroom videos and notating and commenting on the questions asked and answered by participants in the videos. As part of the training, teachers were encouraged to differentiate among question types (e.g., probing, guiding) to encourage them to become more aware of the different types of questions and the advantages these types of questions can have on the development of active learning by students (Sahin, 2007). Probing questions are defined as questions that encourage students to express their knowledge or understanding and to clarify, justify, interpret, or represent their knowledge or understanding (Martino & Maher, 1999). Guiding questions are classified as questions that are related to experiences or learning with real-world examples or representations by guiding students to interpret and reason about experiences or learning with real-world examples or representations. They generally provide hints or suggestions to help students to interpret and reason (Kawanaka & Stigler, 1999).

The allotted time for mathematics instruction was either 60 or 90 minutes; however, engaged time rarely exceeded 50 minutes regardless of allocated time. Engaged time for this study was time used to address the identified objectives; therefore, checking homework, reviewing prerequisite knowledge, doing silent seatwork, working on that evening's homework, and making announcements were among the activities not counted as engaged time. The mean engaged time was 42 minutes (SD = 15.23), the range was 10 to 72 minutes, and the mode was 46 minutes. For this study, almost 8,700 minutes of mathematics instruction were analyzed. The students in these classes represent 5 different school districts spanning rural, urban, and suburban communities. All districts were diverse with at least 55% to as much as 70% minority enrollment. The methodology for this study closely parallels that of Stigler et al. (1998), in which they examined classroom videotapes from the Third International Mathematics and Science Study, now known as Trends in International Mathematics and Science Study. The major difference was that those researchers used a random sampling of the videos available; however, like our work, they carefully selected the content, the criteria, and time period for coding enactments. The present research explored whether a rich, meaningful discourse might stem from “more describe/explain questions, and fewer yes/no questions” (Stigler et al., p. 123). The following questions are similar to the describe/explain questions discussed in Stigler et al.: “How could we figure out how many rhombuses we would need to make the figure we just made with triangles?” “How did you know?” and “Describe how to convert from a fraction to a percent. How can we prove they are equivalent?”

Coding

Coder training. Ten individuals were trained to identify interactive classroom discussions. Each video was chunked by a four-member subset of the 10 trained individuals. Chunking is segmenting a video into manageable units of data for analysis based on the criteria for discursive interactions. For purposes of this study, we only analyzed information chunked for interactive discussion and questioning. To ensure reliability, training included chunking and time-coding teachers across time (i.e., a teacher who participated all 3 years), across grade and content (algebra, number, and/or data objectives) and across curricula (i.e., *MathThematics, 1999*; *Mathematics: Applications and Connection, 1999*; and *Connected Mathematics Project, 1998*). Agreement among coders had to reach the 95% threshold before coders were authorized to chunk and time code on their own. Agreement was determined by meeting three criteria: (a) questions and discussions were all identified, (b) time codes matched within 20 seconds for each instance (i.e., for every question or discussion that was coded, the identified time period matched within 20 seconds), and (c) particular questions or discussions were captured by the coder. The 20-second window was selected to allow for contextualization of the specific observation. This window allowed researchers to disagree about the exact moment the instance started but sufficient time to ensure the coders were certain about who initiated the interaction, were able to ascertain the relevance of the interaction to the content, and were able to ensure that the instance was not carry over from the immediately preceding interaction.

Videos were assigned so that at least four different coders examined each video. The coders did not know who was also assigned the same videos because they worked individually. When they submitted their chunked and time-coded results, discrepancies were noted. When any coder deviated more than 10% from the other coders, the particular coder was retrained. However, final decisions about chunking discrepancies were judged by the primary researcher. To control coder change over time, groups of coders were not fixed, but variable, ensuring that individuals remained consistent to the initial training and that no group variation crept into the coding process. This resulted in 210 single-spaced pages of time codes and transcriptions. The entire research team reviewed each transcription by time code for final analysis. The time codes were organized into categories in an attempt to formulate conclusions about what leads to rich mathematics discourse. Each research team member organized each transcribed chunk into meta-categories. It was the content of these chunks that contributed to the organization of the chart and individual chunks, rather than linkages across chunks that demonstrated the pattern in responses that emerged. These meta-categories were then reviewed and named for the processes common to each meta-category. Because time was continuous within lessons, chunks were able to be sorted and resorted for meaning and finally identifiable patterns. Those patterns were then depicted in graphical form.

A grounded theory approach was utilized throughout the analysis of the data (Glaser & Strauss, 1967; Strauss & Corbin, 1990) to develop a mapping and theory explaining teacher and student questioning and the interactions originating from these questions. Grounded theory, as an inductive approach to analyzing the data, was used so the researchers could avoid any preconceived explanations for and about student and teacher questioning and the connections between emerging categories. All of these student-teacher interactions were analyzed by five researchers who were interested in the paths followed by the classroom communications. Separate paths were assigned depending on whether the interaction was initiated by the teacher or the student. Categories and pathways were refined through constant comparison of data to the emergent categories and pathways. Goetz and LeCompte (1981) noted, "As events are constantly compared with previous events, new topological dimension, as well as new relationships, may be discovered" (p. 58). These paths included (a) where the communication started, (b) how it progressed, (c) how discourse initiated, and (d) how it ended. Unlike previous researchers, this team did not attempt to judge the quality or quantity of the interactions, nor did it presuppose paths that would occur. Rather, the observed paths organized the emergent question types and were graphically displayed in a map based on the initial framework.

Results and Discussion

The coding process afforded several insights that, while not conclusive or definitive, led to some assumptions about the efficacy of the coding training and implementation. For instance, once the first year's coding was established, the map was relatively fixed. While some basic refinement occurred and occasionally a new path emerged, in general, teacher questioning changed little over time and the frequency of the paths remained stable in subsequent years.

The categories were organized into a map that emerged from the analysis of communication paths between the teacher and students showing that rich mathematics discourse can occur in many ways (see Figure 1, Table 1, and Table 2). The Dynamic Student-Teacher Communications Pathways (DSTCP) map illustrated that rich mathematics discourse began with either the student or the teacher and could begin with a variety of questioning techniques and was not limited to higher order question prompts.

A questioning interaction was classified as teacher-generated or student-generated according to who asked the initial question that began the conversation. Both teachers and students utilized "cloze" or "cloze-type" questions—fill-in-the-blank questions or

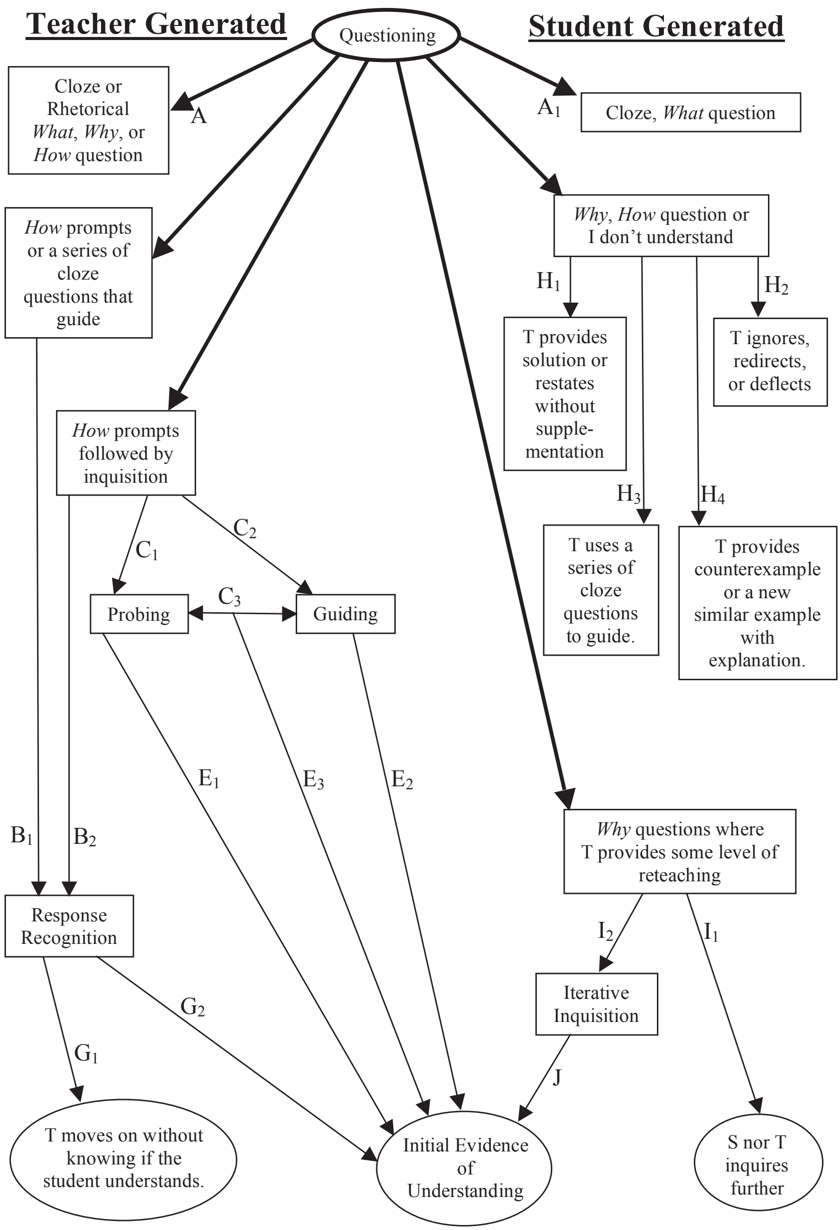

Figure 1. **Dynamic Student-Teacher Communication Pathways map.**

Table 1

Paths From Teacher-Initiated Interactions

Path	Description
A	*What*, *Why*, or *How* questions that are cloze or rhetorical. Never lead to rich discussion or deeper interactions. These *Why* questions are often dismissive.
B_1	*How* prompts or a series of cloze questions that guide followed by *Response Recognition* (e.g., T—How did you get that answer? S—I multiplied by X. T—Very good. . . . This is correct.).
B_2	*How* prompts followed by inquisition and *Response Recognition* (e.g., T—How did you get that answer? S—I multiplied by X. T—Why did you multiply by X? . . . Could you have done ___? . . . Are you sure . . . or How do you know you are correct?).
C_1	*How* prompts followed by inquisition and *Probing* (e.g., Explain that again. . . . Is that like [the example]? . . . Does your answer seem right to you? . . . How did you get ___ again? . . . Explain the relationship again. Do you think your answer is correct?).
C_2	*How* prompts followed by inquisition and *Guiding* (e.g., Can you explain the previous example? . . . How did I get ___for this problem? Could ___ be an answer [counter example]? . . . Why did you multiply? . . . What is X times Y again?).
C_3	*How* prompts followed by inquisition and a combination of *Probing* and *Guiding*.
E_1 E_2 E_3	*How* prompts followed by inquisition and *Probing* or *Guiding* (codes of C_1, C_2, and C_3, respectively) and *Initial Evidence of Understanding is demonstrated*.
G_1	*How* prompts or a series of cloze questions that guide followed by *Response Recognition* or *How* prompts followed by inquisition and *Response Recognition*, then *the teacher moves on without knowing if the student understands.*
G_2	*How* prompts or a series of cloze questions that guide followed by *Response Recognition* or *How* prompts followed by inquisition and *Response Recognition*, and *Initial Evidence of Understanding is demonstrated.*

Table 2

Paths From Student-Initiated Interactions

A_1	*What* questions. Most are cloze. Never led to rich discussion or deeper interactions (e.g., What was/is the answer?).
H1	*Why* or *How* questions. T—provides solution. *or* I don't understand ___. T—restates without supplementation.
H2	*Why* or *How* questions or I don't understand . . . T—ignores, redirects, or deflects.
H3	*Why* or *How* questions or I don't understand . . . T—uses a series of cloze questions to guide.
H4	*Why* or *How* questions or I don't understand . . . T—provides counterexample or a new similar example with explanation.
I1	*Why* questions. T—Clarification of question . . . S—Restates or explains what he or she is unsure of . . . T—Interpretation of ambiguity and provides some level of reteaching. S—May seem sure or unsure but does not inquire further and neither does teacher.
I2	Iterative inquisition *Why* or process questions . . . T—Clarification/Interpretation . . . S—Restates or explains what he or she is unsure of . . . T—Interpretation of ambiguity and provides some level of reteaching. T—Is this clear . . . do you understand? Focus is teacher-elicited evidence of understanding.
J	I2 interaction and *Initial Evidence of Understanding is demonstrated.*

questions for which a limited set of specific and correct answers will suffice (Pimm, 1987). This category of questions is borrowed from the literature on reading comprehension and language development (cf. Bellon-Harn, Hoffman, & Harn, 2004). An example of this type of question is "The definition of a fraction is [pause]" where an acceptable answer of "part of a whole" would be provided by the student(s). Questions that also were categorized as cloze or cloze-type questions were those that were lower level, factual questions that also required a limited number of correct or acceptable responses such as, "What was the answer to number two?" These were labeled as A, for teacher questions, and A_1, for student questions. In our study, these cloze or cloze-type questions never led to rich discussions. When asked by the teacher, this type of question usually started with "what," and to a lesser degree, "why" or "how." Generally "what" questions could be answered with one or two words in a convergent setting so we coded these as cloze questions on the map. Less often, teachers used "why" and "how" questions that were either rhetorical or were answered by the teacher. For example, "How do we add two fractions with unlike denominators?" and then the teacher starts the lesson, or "Why did you add those two numbers?" and then the teacher answers the question. In other studies (Hiebert & Wearne, 1993; Klinzing, Klinzing-Eurich, & Tisher, 1985), these questions would have been classified as higher order process questions. When in actual usage, those questions were neither higher order nor related to a process. This is in contrast to "why" or "how" questions for which teachers provided wait time and encouraged answers that reflected higher order process skills. Therefore, in our decision process, rather than classifying a question based only on the interrogative word, student responses also were used to decide if a question was a cloze question or not.

Teacher-Initiated Questions

With "A" questions, the teacher seemingly had a single response that he or she was trying to elicit. These cases were typically characterized by the teacher calling on one student after another until the predetermined answer was given. It is important to note that at times divergent answers were passed over to finally arrive at an expected response. In a few instances, the teacher asked the question in another way to the same student in hopes of getting the predetermined answer. Questions such as, "What operation do we use?" "How many sheep would I want here?" "What is this graph called?" "How would I represent that as a decimal?" and "How would I change it to a percent?" were typical for this code. These last two examples illustrated times when questions were used to elicit choral process skill responses such as, "move the decimal place two places when converting between decimal and percent."

When teachers asked procedural questions and then asked follow-up questions to a student's response, the path was labeled with a B_1 or B_2 according to the amount of additional inquisition. Teachers who initiated a series of repeated cloze questions were seemingly responding to a student's inability to explain his or her reasoning or to articulate difficulty in either understanding or answering the question. An example of a B_1 question was the teacher asking a series of cloze questions (mainly "how" prompts). This questioning exemplified classroom discourse in which the teacher asked questions not to ascertain whether students gained conceptual understanding but merely to make sure students were able to verbalize procedural steps for solving the problem.

T: So, I am going to do 40 divided by what?
S: 40 divided by . . .
T: How many movies do we have?
S: 10.
T: So, 40 divided by 10. Right?
T: How did you know that?
S: Because 10 times 4 is 40. (videotaped dialogue, February 13, 2003)

Another type of questioning sequence that earned a B_1 code was when the teacher asked a series of questions leading to an affirmation that students were on the right track.

T: If I want to find the mean of this, what am I going to do?
T: What did you learn, probably last year, about how to find average? (pause) Laura?
S: Look at the book?
T: I want to find the average of this data. Okay, David.

S: Add it up.
T: Very good. The first step is to add it up. (videotaped dialogue, December 1, 2003)

This sequence contains a rhetorical question with the teacher answering her own question with a question. In addition, the teacher went from student to student until she found a student who had the "prescribed answer." She did not appear to be interested in understanding the answer given by Laura. She wanted someone to give the answer, "add it up." When that answer was verbalized, the discourse on this topic was over. From the conversation we are somewhat certain that David knows to "add it up," but the teacher has not ascertained what the other students know. Therefore, this B_1 question on the map led to a G_1 (i.e., the teacher moves on without knowing if the students understand).

If the teacher asked probing questions during classroom discourse, the interaction received a label of C_1. A series of probing questions was defined by the research team to mean a cohesive series of questions that further investigated a student's understanding of a concept or process by utilizing student-generated ideas about the concept or process. The label C_2 was used for guiding questions. A series of guiding questions were a series of related and sequential questions that guided a student's understanding of a concept or process. On several occasions, a teacher would alternate between C_1 and C_2, and when this occurred, the interaction was given the designation C_3.

The following series of interactions exemplify each of the C_1, C_2, and C_3 labels. The first interaction shows a teacher asking a series of probing questions. This interaction was coded as a C_1. Notice the ways the teacher further investigated student understanding of the problem using the student's previous responses.

T: An amusement park charges \$3 to enter and \$2 for each ride. What is the price of going to the park and riding on *k* rides? Let's see. That's gonna be what, Susan?
S: $C - [3 + 2k]$

T: Why?
S: Because it takes $3 like when you get in, they charge you $3, and then 2 times how many times you are gonna ride.
T: Excellent, because the 3 is not gonna change, right? That's just a flat fee. But what, how much is it gonna cost if I ride on one ride?
SS: [*different answers*]
S: Five.
T: Five dollars. Why $5?
S: 'Cause you're adding it to the [price] to get in.
T: Right. Now how much is it gonna cost if I ride on two rides?
SS: Seven.
T: Because I still have to pay my?
S: . . . flat fee.
T: Plus how many 2s am I gonna pay?
S: One.
T: Well, two 2s, right? So two tickets. Does that make sense to you? (videotaped dialogue, March 19, 2003)

The next interaction shows a teacher utilizing a series of guiding questions to lead a student to a deeper understanding of the mathematical ideas being taught. This interaction was coded as C_2.

T: [Distance, rate, and time] An equation means we have an equals sign, right? Okay. And, one side they want what?
S: Distance.
T: Her distance. And on the other time, on the other side, they want what?
S: Travel time.
T: Travel time. . . . Hmm. How did we do this anyway? How did we do this? Well they're saying that we should be able to write an equation relating the distance to the travel time. Okay, fine. So here's the

distance and here's the travel time. Right? So what's one of the distances we have?

S: One hundred and ten.

T: One hundred and ten and how long did it take? What was the time?

S: Two hours.

T: Two hours and we have an equals sign. What are we missing? . . . How did we get this?

S: Times it [multiply].

S: . . . by 55.

T: So we times it by 55, right? . . .

S. So whatever the distance was, we took the time [and multiplied by] 55. Could we just say that no matter what, that's how we got the answer? We would take it and try to find the distance, we're gonna take the time and multiply it by 55. (videotaped dialogue, February 19, 2003)

The teacher transitioned from understanding how the components of the formula function to having students work on representing the equation using symbols and numbers.

The next set of interactions shows a teacher's use of a series of related questions to sometimes guide and probe for understanding. In these interactions, there was some evidence that the student was beginning to show understanding, and therefore, required a coding beyond just C_3. The labels E_1, E_2, and E_3 were used to indicate that these probing or guiding interactions led to evidence that the student was beginning to show understanding depending on the type of question: probing or guiding. In the interactions below, the teacher probes the class and individual students with questions about their understanding of tabular numbers about cats. When necessary, the teacher guides individuals with specific questions intended to help the students understand the table.

T: Okay. For Firesmoke. What do you notice? Raise your hand if you notice anything about Firesmoke that could help you describe Firesmoke from this information? Sienna.

S: She's a female.

T: Okay, she's a female. Anything else?

S: She is 25⁄100 old.

T: Okay wait a minute, 25⁄100 what, what do you think that is?

S: It is her age.

T: Uh, think about it. 25⁄100. What does that tell you about Firesmoke's age? What does it tell you, Sarah?

S: She is not a year old yet.

T: How do you know that, Sarah?

S: I think that it [25⁄100] is only part of a year so there is no unit.

T: That we have a whole number, right . . . she . . .

S: Yes, we would have a whole unit or a mixed number.

T: So, Sienna, what do you think about that 25⁄100?

S: I have never heard of someone being 25⁄100 of a year old. I think it is wrong.

T: Okay, but you are really not sure. Anybody think they know what that means, Margaret?

S: I think it is like 25 days old.

T: Okay. Okay. Cathy?

S: One fourth of a year old.

T: Cathy says a ¼ of a year old. Do you think a month? Margaret, do you think that's close?

S: I think it is like 3 months.

T: Oh, how do you get 3 months, Sienna?

S: Because 3 months times 4 equals 12 months.

T: Great, great. So you think that 25⁄100 of a year is the same as ¼; how do you know it's the same as ¼, Cathy?

S: Because 25⁄100 and ¼ are equivalent fractions. I can multiply one by 25 to get 25 and 4 by 25 to get 100.

T: Good, good. Anybody else get to ¼ differently than she did? Okay. . . . (videotaped dialogue, November 8, 2002)

This sequence demonstrated a set of probing and guiding questions where the teacher moved between probing what students knew and then guiding them to answers as the discussion proceeded and included several students.

Teachers who initiated more open-ended questioning techniques were able to engage students with probing and guiding discourse that allowed students to validate or broaden their understanding of the question(s) being answered. Also, teachers who used multiple questioning techniques, such as open-ended and cloze questions, appeared to promote students' initial understanding.

Deciding when students demonstrated understanding was difficult to determine from extant video. Two of the issues in determining student understanding were that many of the analyzed segments seemed to provide for prolonged interaction when compared to other segments and there was a diversity of interpretations of student verbal responses. Three indicators were used to judge student understanding: (a) they engaged in successively more revealing questions; (b) they offered more than comments such as, "I understand" or "okay" to terminate the sequence; and (c) they summarized or generalized the idea back to the teacher without teacher prompting. Although students might acknowledge understanding after asking one question, this response was not necessarily indicative of understanding at any significant level; therefore, we opted for a more rigorous demonstration. We used E_1, E_2, E_3, G_2, and J designations to show which paths led to evidence of initial understanding. Interactions ending with a coding of E_1, E_2, or E_3 were generally long, taking 3–7 minutes to complete, and some were interrupted with other students asking questions. Typically, the ending of an interaction was similar to the following:

> I see when you double the top number 3 [numerator] and the bottom number 4 [denominator] you are not really multiplying by 2 because the next number in the sequence would really be $\frac{6}{8}$. But if you double again you

will have 12⁄16. I would miss one equivalent fraction. (videotaped dialogue, February 15, 2005)

Student-Initiated Questions

Students also initiated questions that were cloze in nature. With A_1 questions, the students asked a "what" question in which the student-teacher discussion also did not lead to rich discussion or deeper interactions. The interaction was initiated by the student, but the response by the teacher was limited and procedural in nature. In our study, there were no "how" or "why" prompts asked by the student or the teacher. For example,

S: What does "b" mean?
T: Oh, it means to make a table like this.
S: Oh, okay. Do I have to write down all the answers and stuff?
T: Yes, just like this.
S: Okay. (videotaped dialogue, March, 19, 2003)

Procedural questions initiated by the students received a label of H. The response of the teacher was used to distinguish between H_1, H_2, H_3, and H_4. If students asked the teacher why or how something happened or just stated that they did not understand the question and the teacher restated the students' statement without providing supplementation, a label of H_1 was coded. The following example illustrated an H_1 student-generated question and explanation:

S: Where did we get "lbs." to stand for pounds?
T: I honestly do not know, but that is a good question—a good mathematician kind of question. Why don't you find that out for me?
S: Ooh, can I have extra credit?
T: Yes, some. But I expect your answer on my desk in the morning.
S: Okay. (videotaped dialogue, November 17, 2002)

If the student asked a question, but the teacher ignored, redirected, or deflected the question, then an H_2 label was coded. For example,

> S: I can't use reds?
> T: [shaking her head] not with the blue . . . because this doesn't fit.
> S: I know. I know. But six of these go into that, and then . . .
> T: So you're gonna use it for a hexagon?
> S: Yeah.
> T: Okay [nods and walks away] . . . okay. (videotaped dialogue, April 25, 2005)

It is not until the student began to engage in a series of cloze questions from the teacher that an H_3 label was coded. The following example illustrated a student-generated question with the teacher responding with a series of cloze questions:

> S: Miss [referring to the teacher], how did we get this answer? [Several students talk to each other about their answers.]
> T: What are the first two numbers that you have?
> S1: I got positive 1.
> S2: I got negative 15?
> S3: The signs are the same.
> T: Are the signs the same or different?
> S2: Signs different.
> T: So, what are you going to do? What did you get?
> S1: I got negative 14, now.
> S3: Oh, I see . . . never mind.
> T: Okay. (videotaped dialogue, March 18, 2003)

A label of H_4 was coded if the student asked a "how" or "why" question about the content at hand, and the teacher provided a counterexample or a new, similar example with limited explanation:

The teacher showed the class that .03 = 3%. A student then asked,

S: What if the 3 is in the tenths place?
T: Okay, let's see. Read this number 0.3
S: Three tenths.
T: What do we need the number out of to know the percent?
S: Out of 100.
T: How can we get this number to be out of 100?
S: See how many times 10 will go into 100.
T: Good. So, now what does it look like we are finding?
S: Equivalent fractions?
T: Good! (videotaped dialogue, November 17, 2002)

In general, only dialogue-terminating responses by the teacher in response to student-generated questions led to H_1, H_2, H_3, or H_4 coding.

The teacher's response also was vital in encouraging or discouraging rich discussions. When students used "why" prompts such as, "Why something happened," the teacher generally provided an explanation as can be seen in this example of the label I_1. The following episode is within the context of simplifying fractions:

T: Yes, Alyssa, you have your hand up?
S: You said that 6/9 gets smaller. I thought it gets bigger.
T: 12/18, 6/9? We went . . .
S: 6/9.
T: Our unit went down. Our unit was 18 when we had our triangles. The pieces are getting bigger, but our unit overall is being reduced.
S: Oh.
T: Did everyone understand that? She was confused. She said, "Okay, I thought that when I divided it into

> fewer pieces that it got bigger." Well, yes, the pieces are bigger, but I am talking about the overall unit here. We had 18 whole units, but then we divided it up. And then we went down to 9. Then we went down to 3. So we were still reducing it. But yes, our pieces did get larger, but our total unit went down. We went from 18. Then, we went down to 9. Then we went down to 3. (videotaped dialogue, October 8, 2002)

If the teacher also used additional questions to check for student understanding, a label of I_2 was applied. If the researchers could tell from student responses captured on video that the student was beginning to understand, a label of J was used. The following example was labeled I_2 leading to J even though the initial student "question" was a statement:

> [the equation $d = 55t$ is written on the board at the front of the class]
> S: I'm lost.
> T: Where did you get lost at?
> S: The beginning.
> T: Okay, she rode the same speed . . . for 110 miles . . . for 2 hours. How fast did she go?
> S: 55.
> T: Okay, so we're going to write that down. Let's write that down so we can think about what we have got. . . . So 55. . . . Okay now, the next one says. "Suppose she keeps the same speed." . . . What's the speed she is going again?
> S: 55.
> T: 55 . . . but for the whole trip. "Then complete the table for the distances driven." Oh my gosh. In 1, 2, 3, 4, and 5 hours. So, in one hour, how far did she go?
> S: You did 55 . . . divided by . . . hours? I don't know.
> T: Well, how fast is she going?
> S: 55 miles per hour.

T: Miles . . .
S: Per hour.
T: Per hour. So if she drove 1 hour . . .
S: She drove 55 miles.
T: Right.
S: So in 2 hours, she drove 55 plus 55 which is 110.
T: Or 55 times . . .
S: Two.
T: Two. Okay, so then, . . .
S: 55 times three. . . .
T: Right, so then when we look at it, we want to say, if I want to know how far she went, I just take 55 and times it by [T is at the board pointing to the equation $d = 55t$]. . . .What's t? . . . What was I multiplying 55 by?
S: Hours.
T: So I am going to use t for . . .
S: Hours.
T: Time. Time in hours, yes? (videotaped dialogue, March 19 2003)

Persistent questioning originated from both teacher-generated and student-generated questions and led to a level of discourse perceived to be at a deeper conceptual mathematical level. Typically, verbalized persistent questioning emanated from the teacher, whereas students often used intonation to indicate their questions rather than explicitly asking questions. Even nonverbal questions often prompted the teacher to provide more in-depth responses, deeper and more robust explanations, and more persistent inquiry into student understanding. When teachers engaged in persistent questioning, they often explored incomplete answers to queries. The following example illustrates this point:

S: [shrugs shoulders] I'm not sure . . .
T: Elise, How did you get your answer?
S: I divided.

T: What did you divide?
S: I divided the two numbers?
T: What did you want to know?
S: I wanted to know the smallest fraction?
T: Do you mean the reduced term?
S: Yes, I wanted to go backwards to find the starting fraction?
T: So, explain what you did.
S: I divided the 6/8 by 2?
T: Really? How did you do that?
S: First, I divided the 6 by 2 and then divided the 8 by 2 and then put the 3 over the 4.
T: What fraction is that?
S: Three fourths. (videotaped dialogue, October 15, 2002)

Conclusion

Many researchers, teachers, and teacher education programs operate under the assumption that discourse benefits student learning; if students are talking about mathematics, they must be learning about mathematics. Although theoretical arguments for this assumption are strong and some research does exist to support this assumption, definitions of discourse should be clarified (Hiebert & Wearne, 1993). For example, Brown and Kane (1988) found that students who verbally elaborated on an idea, with or without coaching, outperformed students who were provided an explanation of the idea. Theoretical arguments for the discourse-learning connection are based on social-constructivist and social-cognitive perspectives (Cobb, Yackel, & Wood, 1992; Hatano, 1988; Pimm, 1987).

Because classroom discourse is frequently initiated by questions, a strong relationship between questioning and learning also may be frequently assumed. Martin and Pressley (1991) provided some support for this assumption. They found that when students were asked "why" questions, learning was enhanced even when students were not able to successfully answer. Although their findings are somewhat incongruous with the findings of this study, Good, Grouws, and Beckerman (1978) found a positive relationship between factual teacher questions and student achievement. As documented in our research study, although detailed mathematical interactions do not ensure quality mathematical learning, these interactions, especially those emphasizing mathematical reasoning, afford students who listen and participate the opportunity to learn rather than just memorize.

Rich, meaningful discourse between teacher and student in a mathematics classroom is a complex, yet important, objective for educators to understand and implement in the classroom. Hicks (1995–1996) suggested that the analysis of classroom discourse provides one possible means through which educators across disciplines can explore how teachers and children collectively and individually construct disciplinary knowledge.

In this study, teacher talk was dominant and student talk was mainly a response to teacher questioning, emphasizing the need for further research on how better to provide students with the skills and mathematical competence to ask and engage in rich mathematical discourse with teachers. Based on the questioning paths and complexity of the discourse when students initiate and persist in their questioning, our findings seem to support previous findings (Corwin, Storeygard, Price, Smith, & Russell, 1995; Dillon, 1988; Kazemi, 1998; King, 1994) that students need the opportunity not only to hear what the teacher is teaching, but actually converse and articulate their own understanding of the content being presented. Conceptual understanding of the content coupled with an ability to engage in rich mathematical discourse through a probing, guiding, and interactive dialogue is a goal many teachers should strive to attain. It is reasonable to extrapolate that when this goal is attained, student achievement in advanced academics will follow.

As shown in the DSTCP map that resulted from this study, the paths between teacher and student can be initiated from several different inception points. The map created from this study could be refined and confirmed through further research with other participants to further unpack the nature of discourse associated with teacher and students' questioning in mathematics classrooms across broader mathematical curricula. However, these paths clearly address a research niche identified by previous researchers (Hicks, 1995–1996; Kazemi, 1998; King, 1994; Lampert, 1990; Redfield & Rousseau, 1981) and provide a framework that can link both teacher and student outcome measures as well as teacher evaluation and development protocols used in every school district (Ralph, 1999a). Stryker (1987) asked an important question to which researchers and practitioners may want to pay close attention and attempt to answer: "The proper question is not whether human social behavior is constrained or constructed; it is both. The proper question is under what circumstances will that behavior be more or less heavily constrained, more or less open to creative constructions" (p. 93). The implications that these and related results have for professional development opportunities could be important for any mathematics teacher wanting to make the most of classroom discourse and for those who wish to understand the importance of student-posed questions in the teaching and learning process for advancing academic success for all students. Given a confirmed version of the map and associated pathways, practitioners could be guided to include in their questioning and response practices more characteristics that have been shown to lead to rich, meaningful discourse. Imagine a classroom in which teachers engage discourse on the student's level of interest without preconceived notions about question composition or how much must be completed by the next class session. For a more discursive classroom to exist, professional development should extend teacher's understandings of how students seek assistance and include how more successful teachers address student questioning. During professional development opportunities, teachers could view and analyze videos using the DSTCP map and reflected upon their questioning techniques. These teachers might become action researchers who improve classroom discourse practices and subsequent student understanding of mathematics.

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