Diversions
with John Gough

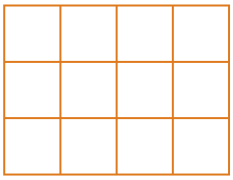
Deakin University • jugh@deakin.edu.au

Fixing misconceptions: Length, area and volume

Originally published in (2004) *Prime Number, vol. 19*, no. 3, pp. 8–14.]

Consider the following situations involving perimeter, area, volume and mass, and the children's misconceptions. What can teachers do to help them to understand correctly?

MISCONCEPTION: Andi says that the area you can make with 12 tiles (or some similar rectangle) is always bigger than the corresponding perimeter, because working out area uses multiplying, but working out perimeter is just adding lengths.



INTERVENTION: Find some rectangles whose perimeter is greater than area, and ask Andi to reconsider in the light of these counter-examples.

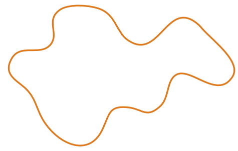
This strategy generalises to almost any situation where a student has a reason for thinking something, but the reason does not always work: show counter-examples — cases where it *does not* work.

Your challenge, as the teacher, is to try different rectangles to find some suitable counter-examples.

Once you know you already have some counter-examples, you could challenge Andi to investigate a variety of rectangles, and their respective areas and perimeters, and see if Andi stumbles upon some counter-examples without you having to rub Andi's nose in your own contrary cases.

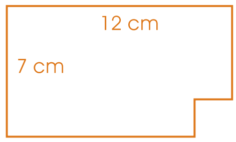
You can also point out that perimeter of rectangles includes multiplication. Andi may not understand the algebraic formula $P = 2 \times (L + W)$, but may be helped to see there is a doubling of the sum of length and width; obviously doubling is like multiplication — in fact it *is* multiplying by two. Since this shows that *both* perimeter and area involve multiplying, maybe the initial argument is not as solid as Andi thinks.

MISCONCEPTION: Brynne says that the area inside a fixed-length loop of string is always the same because the length is always the same.



INTERVENTION: Ask Brynne to make different shapes, preferably rectangular, for easier counting of unit-squares contained in the loop. Certainly the perimeter of the shapes is always the same because the loop does not (cannot) change in length, but Brynne should easily find different areas.

MISCONCEPTION: Collen says that the area in this "lopped" rectangle is 84 square cm.



INTERVENTION:

1. Use a red pen to draw the outline of the missing rectangle in the corner of the larger figure. Ask Collen to calculate the area of the red rectangle. If the dimensions are not provided initially, use any plausible dimensions for this red rectangle — in particular, the red rectangle must be smaller in size than the larger figure, i.e., smaller than 12 × 7.
2. Redraw or re-interpret the figure so that it represents floor-space, e.g., an area of a room that is to be tiled (1.2 m × 0.7 m) or a large hall or backyard (12 × 7 m).
   Explain that the missing section of the rectangular room cannot be tiled because this represents the rear of a heater unit, or a built-in cupboard, or window recess (or the hall has an indented section, or there is a garage in the backyard, etc.). Ask Collen to calculate the number of tiles needed, or otherwise calculate the area of the tileable real-space.
3. Draw a sketch plan, or even a precise centimetre-square grid; or use a geoboard (a rectangular array of nails, using rubber-

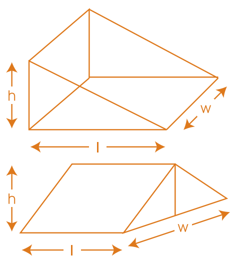
bands to loop around corners of figures — a geoboard is an invaluable aid for teaching area and perimeter, because it allows easy construction of geometric figures, and easy counting of units of length and units of area). Mark the centimetre scale along the 7 cm side, and along the 12 cm side, and draw the square grid pattern over the figure. Count the number of unit squares of area. (Similar concrete experience can be obtained using matchsticks for units of length, and square tiles for units of area — obviously the unit matchstick lengths will be the same as the length of the edge of the unit square.

MISCONCEPTION: Darill says that if the area of a rectangle is 50 square cm, then the perimeter is always 30 cm.

INTERVENTION:

1. Ask Darill what about other rectangles, with smaller "round" or "product" numbers; e.g., what about a rectangle of area 12 square centimetres or 16 square centimetres?
2. Whatever the rectangle area, expressed in square units, take that many square tiles, and ask Darill to make the rectangle, and work out the perimeter. Then ask Darrill if it is possible to make the rectangle any other way? If necessary, prompt about trying different width and length. What is the perimeter then?
3. Similar work with diagrams can save the time needed for fiddling with tiles, especially when the area is a "large" number.

MISCONCEPTION: Edan says that the volume of a "wedge" shape or a "pup-tent" is found by using the formula:

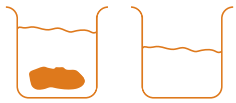


volume = length × width × height.

INTERVENTION:

1. Explain a slightly odd situation. Some campers are practising setting up their tent at home: because it is raining and they do not want the tent to get wet, they set the tent up inside the house. When they do this, they find (coincidence of coincidences) that their tent just fits inside the room. What is the volume of the room?
What is the volume of the tent?
What is the difference between the volume of the room and the volume of the tent?
2. Make a model of the tent out of firm cheese (easy to cut and is cheap and reasonably solid). Cut it in half: bisect the tent by slicing vertically along the line of the uppermost tent ridge, down to the centre axis of the tent floor. Pick up the two halves, each a right-angled triangular prism, and fit them together to make a new rectangular prism. What is the volume of the prism?
3. Ask Edan to make a cardboard model of the tent, so it is fully enclosed, except for one end of the tent — the tent-flap door(s). Calculate the expected volume of the model of the tent, using the drill-formula $V = L \times W \times H$. Use a measuring jug and dry sand to fill the jug to this volume. Pour the calculated volume of sand into the model of the tent.

MISCONCEPTION: Fern says that to find out the volume of a rock we can put it in a measuring jar of water, and the heavier the rock the more water it "displaces."



INTERVENTION:

1. Try several examples: sandstone, granite, basalt, marble, limestone, brick, concrete, etc. Make sure that some of the examples have different densities, but that they are all denser than water, i.e., will sink completely. (Related activities can use materials that are likely to float, such as different kinds of wood, polystyrene, rigid foam, and so on — but floating and sinking is a science-concept challenge, also.)
2. Find some solid pieces of metals such as aluminium, iron, lead — ask a hardware store or plumber or other handy person to help — and try these. Find their individual masses. Try to make sure that their volumes are reasonably similar. Then find their particular displacements, hence their actual volume.
3. Find different examples of glass — ordinary bottle-glass, and lead crystal — and do the same as before. Similarly for dense kinds of plastic, e.g., melamine, bakelite, polyvinyl, etc.