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Looking for Attributes of Powerful Teaching for Numeracy in Tasmanian K-7 Classrooms

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This paper reports on the development and use of a classroom observation reflection tool designed to measure the extent to which pedagogies acknowledged in the literature as contributing to effective teaching of mathematics for numeracy are present in classrooms. The observation schedule was used in conjunction with a record of classroom activity to examine numeracy pedagogies in a sample of Tasmanian classrooms from Kindergarten to Year 7. Low levels of intellectual challenge in highly socially supportive classrooms were typical.

From 2002 the Tasmanian Department of Education (DoET) began implementing a new curriculum framework. It was underpinned by recognition of the importance of thinking and a commitment to the development of deep understandings that span traditional discipline based subjects. It aimed to focus upon the essential outcomes of education (DoET, 2002). The description of the key element Being Numerate in the *Essential Learnings Framework* 1 included a definition of numeracy that emphasised the ability to apply mathematics to everyday life and acknowledged the importance of affective outcomes to this end. At the same time, the importance of more abstract ideas for some academic and vocational pathways was recognised (DoET, 2002).

The study reported here was conducted in 2003 and 2004, and hence in the relatively early stages of Tasmania's curriculum reform. It provides a snapshot of the nature of mathematics teaching for numeracy in K-7 classrooms at that time, and presents a classroom observation reflection tool that may be useful in assessing the extent to which pedagogies recognised as effective in teaching mathematics for numeracy are present in mathematics classrooms. The tool is innovative in that it links generic effective pedagogies with those specific to mathematics teaching. The study is timely because the picture it reveals of teaching for numeracy in Tasmanian schools early in the curriculum reform process will be available for comparison with that produced by subsequent studies perhaps using the instrument developed here.

Effective Pedagogies

Of the many factors that impact students' achievements, the most potent relate to teachers and their practice (Sanders, 2000). Although it is difficult to specify precisely the teacher actions that are effective, research has provided important insights into the characteristics of effective pedagogies and these have underpinned efforts to improve the quality of teaching in Australia and internationally. In Queensland, pedagogical reform was initiated in conjunction

with curriculum reform and was encapsulated in the *Productive Pedagogies* (Goos & Mills, 2001). This was followed by the development of the *Quality Teaching Framework* in NSW (Ladwig & King, 2003). Important aspects of both can be traced back to the work of Newmann and associates on authentic instruction (e.g., Newmann, Marks, & Gamoran, 1996; Newmann & Wehlage, 1993). Newmann et al. (1996) were concerned that calls for student-centred learning not be implemented at the expense of intellectual quality. They identified three dimensions of authentic pedagogy: construction of knowledge through higher order thinking; disciplined inquiry, by which students come to share in the means of inquiry and communication characteristic of disciplines; and value, either personal or professional, beyond the school context.

The *Productive Pedagogies* comprise four aspects, two of which concern teachers' practices that support cognitive aspects of students' learning ('Intellectual quality' and 'Connectedness'), and which most clearly relate to the work of Newmann and colleagues. The remaining two relate to the social support for students' achievement ('Supportive classroom environment' and 'Recognition of difference') aspects of which were present but less prominent in the authentic pedagogy literature. Intellectual quality is also consistent with the teaching for understanding tradition (Blythe, 1994; Murdoch, 1996; Wiggins & McTighe, 1998) that was influential in the Tasmanian reform.

Effective Mathematics Pedagogies for Numeracy

The preceding discussion described general findings concerning effective pedagogies. The frameworks developed from them were intended to be applicable across subject areas, including mathematics. However, in parallel with these generic developments there have been a number of projects that have examined effective teaching for numeracy in primary schools. Some of these have included a focus on the professional learning of teachers along with the development of diagnostic tools, and improved understandings of the development of students' conceptions and the teaching practices that appear to support it (Clarke et al., 2002; Mulligan & Busatto, 2004; NSW Department of Education and Training, 2001; Young-Loveridge, 2005). Others have focussed explicitly on examining teaching with a view to identifying effective pedagogies (e.g., Askew, Brown, Rhodes, Johnson, & Wiliam, 1997; Department of Education Science and Training, 2004; Mulligan & Busatto, 2004; NSW Department of Education and Training, 2001). The findings of both groups of studies are consistent with the model of quality mathematics teaching presented by Sullivan and Mousley (1994) on the basis of survey responses of mathematics educators and experienced teachers of the subject. Sullivan and Mousley (1994) regarded building understanding as the overarching aim of mathematics teaching that the other five components of their model (organising for learning, nurturing, engaging, communicating, and problem solving) are instrumental in achieving. They shared Newmann et al.'s (1996) concern that reforms to teaching, aimed at achieving an active role for learners, should not result in the potentially useful activities, such as working in groups and using materials, becoming ends in

themselves. Rather, their value needed to be recognised as residing in their potential to assist students to build understanding.

Other researchers (e.g., Askew et al., 1997; Beswick, 2007; Watson & De Geest, 2005) have recognised that the beliefs of teachers about such things as the nature of mathematics, the capacities of their students to learn mathematics, and their role as teacher, influence their practice. This fact, along with Sullivan and Mousley's (1994) reminder that the over-arching goal of mathematics pedagogy must be to build understanding, demands caution in prescribing effective pedagogies. In this study the challenge was to design a classroom observational tool that described effective pedagogies for numeracy in terms that were sufficiently broad to encompass understandings that teachers' purposes and not simply particular practices are what matter, but also specific enough to be reliably inferred from observation. To this end, parallels between the mathematics education literature on effective teaching, and New Basics curriculum (Education Queensland, 2000) and Tasmania's Essential Learnings (DoET, 2002) were useful. These are outlined briefly in the following section.

Linking General Effective Pedagogies with Effective Pedagogies for Numeracy

Intellectual quality is consistent with endorsements in mathematics education literature of practices that promote higher order thinking (Grouws & Lembke, 1996; NCTM, 2000). Similarly, the importance of helping students to make connections between various aspects of mathematics (Askew, 2001; NCTM, 2000), mathematics and other disciplines (Askew, 2001; Sawada et al., 2002), existing understandings and new material (Grouws & Cebulla, 2000; Swan, 2001), and between particular mathematical examples and more generalised conclusions (Jones, Tanner, & Treadaway, 2000; Romberg, 2000), is a major theme in the mathematics education literature.

The socially directed elements of the *Productive Pedagogies*, Supportive classroom environment and Recognition of difference, are also reflected in the mathematics education literature. Supportive mathematics classroom environments are characterised by autonomy for students with respect to their thinking (Grouws & Lembke, 1996; Sullivan & Mousley, 1994), high expectations (Ollerton, 2001), explicit classroom norms (Cooney, 1999), and engaging tasks that motivate students (Askew, 2001; Grouws & Lembke, 1996). Recognition of difference is reflected in the literature concerning affective and cultural aspects of teaching and learning mathematics (Gellert, Jablonka, & Keitel, 2001; Ollerton, 2001). These elements were also consistent with DoET's (2003b) *Supportive School Communities* initiative that formed part of the context in which the classrooms observed in this study were operating.

Notions of effective teaching for numeracy have also been influenced by ideas originating from examinations of effective literacy teaching. For example, Hill and Crevola's (1999) Early Literacy Research Project was influential in the instigation of the Early Numeracy Research Project (Clarke et al., 2002). In

Tasmania, Hill and Crevola's recommendations are represented in professional learning materials developed for literacy interventions but often applied to numeracy as well (DoET, 2003a). Because of their widespread use in Tasmanian schools and their resonance with the *Productive Pedagogies*, elements from DoET's *Flying Start* materials (2003a) were incorporated in the classroom observation reflection instrument developed, along with descriptors of these elements and those derived from the *Productive Pedagogies*, designed to more clearly define the relevant ideas in the context of mathematics teaching for numeracy. The resulting Classroom Observation Reflection Chart (CORC) used in this study highlights Attributes of Powerful Teaching (APTs) that were thus formulated from these and the *Productive Pedagogies*, and interpreted and operationalised in light of the literature on effective mathematics teaching. The numeracy CORC with references to the literature is provided here as an appendix. The CORC provides a means of assessing the extent to which each of these APTs is present in classrooms.

Research Questions

The study reported here was part of a larger one that focussed on both literacy and numeracy teaching and is described in broad terms elsewhere (Andrew, Beswick, & Swabey, 2005; Andrew, Beswick, Swabey, Barrett, & Bridge, 2005). The focus here is on the design and use of an observation reflection tool related to numeracy teaching (the CORC) and the snapshot of mathematics teaching for numeracy in a sample of Tasmanian K-7 classrooms produced principally by the use of this tool. The two research questions that formed the study's focus were:

1. How might the Classroom Observation Reflection Chart be useful for describing the extent to which the APTs are present in teaching for numeracy?
2. Which pedagogies, as described by the APTs, are evident in the numeracy repertoires of a sample of Tasmanian K-7 teachers?

Method

Participants

The 20 schools where the classrooms observed were located comprised 13 government, three Catholic, and four independent schools representing a spread across grade levels (K-7), socio-economic contexts, geographic locations, and school size. Data for each class in relation to these aspects are presented in Table 1. Schools located outside of the Hobart or Launceston metropolitan areas are classified as rural. School size was based on primary school enrolments (some schools included secondary grade levels as well as primary) with 'Small' referring to schools with fewer than 200 primary enrolments, 'Medium' indicating 200-400 primary enrolments, and schools with more than 400 primary enrolments designated 'Large'. Each Tasmanian school is assigned an Economic Needs Index (ENI) reflecting socio-economic and geographic factors. The higher the ENI of the school the more disadvantaged it is considered to be. In Table 1,

High, indicates an ENI greater than 69.9 and, Low, indicates an ENI less than 40.0. The classes are listed in ascending order of grade level.

The schools in which the classrooms were located were selected by the management team of the larger study (comprising personnel from each of the government, Catholic, and independent school systems) and each participating teacher was a volunteer from within a selected school.

Table 1
Characteristics of Participating Classes

Grade level of class	Sector	Rural/Urban	Size	ENI
K/Prep	Government	Urban	Large	Low
Prep	Government	Urban	Medium	Low
P/1/2	Government	Rural	Small	Moderate
1	Catholic	Urban	Medium	Low
1	Government	Urban	Medium	Low
1	Independent	Urban	Large	Low
1/2	Catholic	Rural	Small	Moderate
2/3	Government	Urban	Small	High
3	Independent	Urban	Medium	Low
3	Independent	Urban	Medium	Low
3	Independent	Urban	Large	Low
4	Government	Rural	Small	Low
4/5	Government	Urban	Medium	Low
4/5	Government	Rural	Medium	Moderate
3/4/5/6	Government	Urban	Large	High
5	Catholic	Urban	Small	High
5	Government	Rural	Medium	High
5/6	Government	Rural	Medium	High
5/6	Government	Rural	Medium	Moderate
6/7	Government	Rural	Medium	High

Note. ENI + Economic Needs Index.

Instruments

Classroom Observation Reflection Chart (CORC). This chart was designed to allow an observer to record evidence of the presence or otherwise of each of the APTs. The indicators (shown on the appended version) were derived from a review of the literature on effective teaching for numeracy. Recommended pedagogies, framed as questions beginning, "Does the teacher …?" were aligned with each of the dimensions of the APTs. The wording of the indicators was refined in response to trialling of the CORC prior to its use in the study.

The CORC was completed immediately after each observation period and was intended to enable the observer to make holistic judgements regarding the extent to which each of the APTs were evident during the observation period. A reflective tool was chosen because the features to be observed necessitated a degree of inference that could not be made on the basis of isolated or particular incidents. Rather, the contexts of the lessons and the purposes of the teacher, which often became evident only some time after particular incidents, were relevant and the intention was to record the overall nature of the teaching observed. As described already, the APTs comprised the *Productive Pedagogies* (Education Queensland, 2002) and elements derived from the Tasmanian *Flying Start* materials (DoET, 2003a). Difficulty with operationally distinguishing the *Productive Pedagogies* of Deep knowledge and Deep understanding led to these elements being combined in the APTs. Indicators derived from the mathematics education literature were provided for each element in an attempt to operationalise each in the context of mathematics teaching for numeracy. A similar set of indicators, against the same APTs, was also developed for literacy. The observers had, for reference, the versions of the CORC (numeracy and literacy) with indicators from the literature. Elements sourced from the *Flying Start* materials are italicised in the appended CORC. A version with the indicators column blank was developed to enable the observer to record examples of the relevant teacher behaviours that contributed to the judgement made in relation to each APT. In this version the column for references was replaced by a narrow column in which the overall assessment of each element as 'High' or 'Low' was recorded. High signified frequent or multiple evidencing of the relevant pedagogy while Low referred to sporadic, one-off, or lack of evidence. The two-level scale was also designed to maximise inter-rater reliability (Trochim, 2006).

Classroom Activity Record (CAR). Detailed notes focussing on the actions of the teacher were made during each observation period. The CAR included a detailed record of the classroom routines, tasks used, times allocated to various tasks and phases of lessons, groupings of students, and questions and prompts used by the teacher. This information was useful in contextualising and interpreting data derived from the use of the CORC.

Post observation interviews with teachers. Each of the teachers was interviewed following the observation in their classroom. The questions relevant to the focus of this paper concerned the typicality or otherwise of the teaching observed and its place in the teachers' overall programs.

Procedure

Once drafts of the instruments had been designed, the researchers (authors) trialled their use in three different classrooms in a non-project school and minor modifications were made. The researchers then independently used the revised instruments in relation to a whole morning's teaching in a single classroom, again in a non-project school. The results were compared and the few discrepancies moderated. The three research assistants who conducted the

observations all had experience of working in classrooms. They all participated in two days of intensive training and moderation focussed particularly on ensuring that they and the researchers had shared understandings of the numeracy and literacy indicators of the CORC and how they might be evidenced in teachers' practice. As part of this process, the researchers and research assistants all independently observed three teachers in the same school over the course of a morning such that one researcher and one research assistant were in each classroom at any given time. Each completed a CORC independently following the observation of each teacher. The results obtained by all observers were moderated such that the researchers were confident that the research assistants were using common bases in making their judgements concerning the extent to which each of the APTs were evident. The inclusion of examples in the CORC served the dual purpose of adding to the richness of data collected and providing a check on the interpretations of the indicators that were being made. In order to ensure that concern about being identified did not influence teachers' decision to volunteer, lessons were not video-taped. In addition, videotaping was not necessary because the aim of the observations was to form holistic judgements about the level of each of the APTs rather than to analyse particular behaviours in detail. It was, nevertheless, important that the observers could justify their judgements by reference to examples and the records made on the CAR and the CORC were sufficient for this purpose.

The research assistants were assigned approximately equal numbers of schools and they negotiated convenient times for the observations with each of the schools and teachers concerned. Each observation period was approximately 4 hours from the start of the school day until lunch time. Mornings were chosen because it was anticipated that most literacy and numeracy teaching would occur in that period. One CAR and one CORC were completed for each 4-hour period and hence the ratings on the CORC refer to an overall assessment of all of the teaching observed, principally literacy and/or numeracy. The observers were careful to record examples of practice that were representative of the curriculum areas in which the relevant pedagogies were evident, and that contributed to the rating of each element of the APTs. Having rated each of the elements under each of the five broad categories of APTs (i.e., *Intellectual quality, Supportive classroom environment, Recognition of difference, Connectedness*, and *Strategic instruction*), the observers then made an overall High or Low judgment in relation to each of the five categories.

Completed CORCs were sent to the research team throughout the observation period allowing these to be monitored. The researchers and research assistants maintained frequent contact for the purpose of ongoing moderation, throughout the several weeks in which observations were conducted.

Results and Discussion

In this section overall ratings for each of the APTs as recorded using the CORC are first considered. The results specifically related to numeracy are then discussed.

Overall Evidence of the Attributes of Powerful Teaching

Table 2 shows the percentages of High ratings obtained for elements in each of the five categories of APTs, and the sum of the percentages of High ratings for all of the APTs, and for the pairs of factors related to cognitively oriented pedagogies (i.e., *Intellectual quality and Connectedness*) and to socially oriented pedagogies (i.e., *Supportive classroom environment* and *Recognition of difference*). The classes are arranged in descending order according to the percentage of High ratings for all APTs.

It is evident from Table 2 that 50% or more of the APTs were evident at a High (sustained or frequent) level in six classrooms. It is also apparent that, with the exception of the grade 5 class at the top of the table, the socially oriented pedagogies were very much more evident than those concerned with Intellectual quality or Connectedness. Indeed, in more than half (12 of 20) of the classrooms none of the elements in these categories were observed at High levels.

Table 3 shows the percentages of High ratings for each of the elements of the five categories of the APTs. The classes are also divided into grade level bands that could be described as upper primary, middle primary and early childhood.

These data indicate that the majority of observed teachers used pedagogies that contributed to Supportive classroom environments but that almost all other elements of the APTs were only rarely evident in more than half of the classrooms. In particular, pedagogies contributing to Connectedness were seldom observed and the Intellectual quality was typically low.

Time Allocation for Numeracy

Because the CORC was used to make an overall assessment of the levels to which each of the APTs were evident in a substantial block of teaching that was not restricted to numeracy, the time devoted to this area of the curriculum is relevant to the extent to which the data presented in Tables 1 and 2 can be seen as applying to teaching for numeracy. As indicated in Table 1, one class was not observed during numeracy time and hence is not included in the following discussion.

The time devoted to mathematics/numeracy teaching is significant in terms of its implications for the status of numeracy in Tasmanian classrooms. In the 19 classrooms, time spent on numeracy during the 4 hours of observation ranged from 10 minutes to 80 minutes with an average of 47 minutes. The grade 2 class that spent 80 minutes ostensibly on numeracy in fact had some children engaged in reading activities and at various times other individuals left the classroom for violin lessons. Numeracy occupied the first part of the morning in just two schools. In two other classes a brief initial period was devoted to automatic response and in one the teacher spontaneously used the attendance registering routine to engage the students in some brief mental computation, but the main numeracy focus occurred later. The composite 3/4/5/6 class had no distinct numeracy time. Instead the small groups of students took turns to work on a maths game on a computer while the rest of the class were involved in other non-

Table 2
Percentage of High Rating for Various Categories of APTs

Classes	All APTs[a]	Intellectual quality(IQ)[b]	Connectedness (C)[c]	IQ + C	Supportive Classroom Environment (SCE)[d]	Recognition of Difference (RoD)[d]	SCE + RoD	Strategic Instruction[e]
5/6	75	67	100	167	100	40	140	100
Prep	65	17	75	92	100	80	180	67
6/7	60	67	0	67	80	80	160	67
3	55	67	25	92	60	60	120	0
3	55	50	25	75	80	60	140	0
3	50	50	25	75	80	40	120	100
K/Prep	40	33	0	33	80	40	120	67
3/4/5/6	30	0	25	25	40	60	100	67
5/6	30	0	0	0	80	40	120	100
1	25	0	0	0	40	60	100	33
1	25	0	0	0	80	0	80	67
2/3	25	0	0	0	60	40	100	100
4	20	0	0	0	60	20	80	0
4/5	15	0	0	0	40	20	60	33
P/1/2	10	0	0	0	20	20	40	0
4/5	10	0	0	0	40	0	40	100
5[f]	10	0	0	0	0	40	40	0
1	5	0	0	0	0	20	20	0
5	0	0	0	0	0	0	0	0
1/2	0	0	0	0	0	0	0	0

Note. [a]n = 23. [b]n = 6. [c]n = 4. [d]n = 5. [e]n = 3. [f]Only literacy teaching was observed in this class.

Table 3
Percentages of High Ratings for Elements of the APTs by Grade Bands and Overall

	Grade Bands			
Elements of APTs	5-6/7 (n=6)	2-4/5 (n=7)	K/P-1/2 (n=7)	Overall
Intellectual quality	33.3	42.9	0.0	25.0
Higher-order thinking	16.7	42.9	0.0	20.0
Deep knowledge/understanding	33.3	14.3	14.3	20.0
Substantive conversation/*question*s	16.7	28.6	0.0	15.0
Knowledge as problematic	0.0	0.0	0.0	0.0
Metalanguage	33.3	42.9	28.6	35.0
Constructive self critique	33.3	14.3	0.0	15.0
Supportive classroom environment	66.7	85.7	71.4	75.0
Student direction/*choice*	16.7	14.3	14.3	15.0
Social support/*Positive High Expectation*	66.7	85.7	42.9	65.0
Explicit quality performance criteria	50.0	71.4	42.9	55.0
Self-regulation	50.0	71.4	71.4	65.0
Motivate/celebrate	66.7	57.1	57.1	60.0
Recognition of difference	50.0	42.9	28.6	40.0
Cultural knowledge/ context-honouring	50.0	14.3	14.3	25.0
Inclusivity/*all engaged*	83.3	42.9	42.9	55.0
Narrative	16.7	28.6	14.3	20.0
Group identity/*Community of Learners*	50.0	71.4	71.4	65.0
Active citizenship	16.7	14.3	14.3	15.0
Connectedness	16.7	0.0	14.3	10.0
Knowledge integration	16.7	28.6	28.6	25.0
Background knowledge	16.7	14.3	14.3	15.0
Connectedness to world	33.3	0.0	14.3	15.0
Problem-based curriculum/*inquiry learning*	16.7	0.0	0.0	5.0
Strategic Instruction	50.0	57.1	42.9	50.0
Explicit/Expository	50.0	42.9	42.9	45.0
Modelling	66.7	57.1	57.1	60.0
Purposive monitoring	50.0	42.9	0.0	30.0

mathematical activities. The teacher of this class explained that this strategy made behaviour management easier in a group in which 9 of the 20 students had major behavioural difficulties. One grade 3 class began with 25 minutes devoted to a mapping activity which was then followed up in a PE session that incorporated map reading. These were built upon in the classroom after the break. In 12 classes all of the numeracy time observed was later in the morning, typically after a break. All but one teacher, whose numeracy lesson was curtailed by a non-routine school assembly, described the morning as typical in terms of both time allocation and teaching approaches.

In comparison with literacy teaching, which occupied the initial part of the school day in all cases where numeracy did not, numeracy time was both less and less well-guarded from interruptions. Hill, Hurworth, and Rowe (1998) reported that Tasmanian primary school principals estimated the average weekly time devoted to specific teaching for numeracy to be 5.5 hours. This corresponds to a daily average of 65 minutes which is considerably more than that observed in this study in most classrooms. In fact only 5 of the 19 classes focussed on numeracy for 65 minutes or more and the average for the remaining 14 was just 34 minutes. If the classes in this study are representative of Tasmanian classes, then either the amount of time devoted to numeracy in Tasmanian classrooms has declined since 1998 or the principals involved in the 1998 survey (Hill et al., 1998) over-estimated the amount of time that was being devoted to numeracy teaching in their schools. It is relevant in relation to the first of these possibilities that the principals surveyed in 1998 noted the greater financial and professional support available for literacy compared with numeracy. When asked to nominate two changes that had altered numeracy teaching 19% of the responses referred to changes considered to have had a negative impact. Of these, approximately one fifth (4% of the total responses) perceived increased competition between numeracy and literacy for both time and funds with numeracy losing out in terms of both (Hill et al., 1998). It is also possible that the emphasis on trans-disciplinary inquiry emphasised in the *Essential Learnings* frameworks (DoET, 2002) meant that numeracy was being taught at other times of the day as an integral part of broader learning experiences. Although it is not possible to dismiss this possibility on the basis of the data gathered in this study there was some evidence (referred to below) that numeracy was not a prominent feature of the integrated curriculum as it featured in these classes and there was no evidence that links were made between such studies and the numeracy concepts that were addressed in numeracy time.

Attributes of Powerful Teaching in Numeracy

In this section data from the CORC relating to teaching for numeracy are discussed in relation to each of the APTs in turn. In each case the discussion is organised around common themes that were evident across classes.

Intellectual Quality

Table 2 shows that the overall level of each aspect of Intellectual quality was low in most classrooms. Teaching for numeracy tended to be dominated by procedural teaching, use of closed tasks, and devoid of attempts to promote deep conceptual understanding. Classroom talk was dominated by the teacher with little attention paid to the role of specific language in mathematics or to the development of students' abilities to regulate their own learning.

It should be remembered that an overall Low rating did not mean that there was nothing positive in terms of Intellectual quality observed. For example, in a grade 1 class where numeracy occupied just 25 minutes of the entire morning and hence contributed relatively little to the Low rating for Intellectual quality that was given, the teacher demonstrated considerable skill in using incidental opportunities and encouraging the students to think about the relevant mathematics. This was the class in which registering attendance was used to prompt some mental computation. The teacher asked, "If we have 6 away and there are usually 30, how many are here?" Later a game of "what's my number?" was played with the teacher drawing students' attention to the possible use of a 1-100 chart to help them to think about what constituted a 'good question' in this context.

Mental computation. In 5 of the 19 classrooms numeracy time began with timed mental mathematics. In each case there was no discussion of strategies, observable attempts to remediate difficulties, or clear connections made with subsequent tasks. In each case the students marked their own work and in two classes then plotted their results on graphs which constituted a record of their own progress over time. The latter practice aligns with Constructive self critique (Luke et al., 2003) which is one of the aspects of Intellectual quality included in the APTs and sourced from Tasmania's *Flying Start* materials, however the potential of mental computation to contribute to the development of number sense by way of flexible strategies for computation underpinned by sound understanding of numeration and operations was untapped. McIntosh, Reys, and Reys (1993) emphasised the importance of these aspects and described number sense as a person's general understanding of number and operations along with the ability and inclination to use this understanding in flexible ways to make mathematical judgements and to develop useful strategies for handling numbers and operations. In contrast with this, the mental computation practices observed justify Maclellan's (2001) concern that mental computation is prone to becoming routine and reduced to drill-and-practice exercises.

Procedural knowledge. There were several examples of lessons in which the emphasis was on formal symbolic recording and procedural knowledge rather than on the development of conceptual understanding. For example, in an introductory lesson on addition, problems were presented in formal symbolic form. The K/Prep students were required to use counters to solve them, to write down the answers and also to draw counters to show how they had performed each calculation. A grade 2 class was learning to apply the standard algorithm to

calculations involving multiplying 3-digit numbers by 2-digit numbers. The teaching focussed on the procedure, explaining the use of zeros in the algorithm in terms of the adding zeros 'rule' for multiplying by multiples of 10. There is compelling evidence that most children of this age are not conceptually ready to learn standard algorithms and that premature teaching of them is likely to have deleterious effects on subsequent conceptual understanding (Clarke, 2005). A similarly procedural approach to the same topic was observed in a grade 5 classroom. At this grade level it is at least possible that the students have had the pre-requisite understandings and knowledge i.e. the meaning of multiplication, place value, and knowledge of basic multiplication facts.

Reflection on learning. Classroom activity was typically organised so that students moved from one activity to another (usually unrelated) without any requirement or opportunity to reflect upon the mathematics that they had encountered. Plenary sessions, which can be used to this end, occurred at the ends of just three lessons and in only one of these instances was it used to highlight important mathematics. In this case the overall Intellectual quality of the teaching was rated High. The grade 6/7 students had been engaged in a skip counting by 0.1 activity and the teacher used questions that demanded more than recollection of facts and sought multiple responses from the students. For example, the question, "Why did I decide to play this game (skip counting) today?" was used to focus the students' attention on the mathematical purpose of the activity and, "What was the hardest part?" led to a discussion of place value and the relative difficulty of skip counting by 0.1 across whole numbers. Interestingly, one of the other teachers who conducted a plenary at the end of a numeracy lesson used a question very similar to the second of these but crucially did nothing with the responses. This question had been preceded by just one student being invited to explain how he had added together the prices of two grocery items. Although the student's response could have stimulated a useful discussion of different approaches and their relative merits the teacher simply moved to the next question. These examples illustrate that the value of 'good' questions depends upon the teacher's ability to use them effectively to stimulate students' thinking.

Use of manipulatives. Similarly, 'good' materials are not sufficient for effective teaching and do not automatically result in high Intellectual quality. Although manipulatives were used in almost all of the early childhood classrooms observed it was difficult to discern the extent to which they were being used to facilitate the development of conceptual understanding as opposed to simply obtaining answers. However, the absence of a clear focus on mathematical ideas that was evident in the majority of classrooms at all grade levels suggests that the latter may well have been the case in many instances. The literature is replete with reminders of the crucial role of teachers and the need for them actively to mediate the use of both tasks and materials (e.g., Askew et al., 1997; Clarke et al., 2002; DEST, 2004; Grouws & Cebulla, 2000; Swan, 2001). In relation to the maintenance of high level mathematical thinking, Henningsen and Stein (1997) stated:

> Not only must the teacher select and appropriately set up worthwhile mathematical tasks, but the teacher must also proactively and consistently support students' cognitive activity without reducing the complexity and cognitive demands of the task. (p. 546)

Similarly, Grouws and Cebulla (2000) make the point that teachers should be "knowledgeable in their (manipulatives) use" (p. 27). The examples described above suggest that maintaining the focus of mathematics teaching on building understanding (Sullivan & Mousley, 1994) is a difficult undertaking. No use of manipulatives was observed in classrooms beyond grade 3 despite recommendations that their use extend well beyond the early years (Grouws & Cebulla, 2000).

Missed opportunities. There were many unexploited opportunities to raise the Intellectual quality of numeracy lessons. For example, a Prep class used blocks to measure cut out paper "giants" that they had made in a previous lesson but there was no explicit focus on the mathematical ideas inherent in the activity or even on exactly what attribute (length) of the giants they were measuring. Rather than highlighting one or more of the following: the importance of aligning the first block with the start of the length to be measured; using units of consistent length; avoiding overlap of units or gaps between them and the implications for the measure of allowing gaps or overlaps (Willis, 2005); the purpose of the lesson appeared to be the completion of a worksheet. A grade 5/6 numeracy lesson included a similarly mathematically rich task that initially required students to draw as many shapes as possible of a given area. However, the task was almost immediately limited to rectangles with integral side lengths. Although the potential for students to learn about the limitless variety of shapes that a given area can have was thereby lost there was still the potential for this idea to be explored in the context of rectangles, and for the relationship between the areas and perimeters of rectangles to be explored. In fact the students had just 10 minutes to work on the task before they were presented with the formula for calculating the area of a rectangle and were told that they would be taught how to find the areas of irregular rectilinear shapes the following day.

Supportive Classroom Environment

It is evident from Tables 2 and 3 that the majority of classrooms were characterised by most of the elements that contributed to a supportive classroom environment and all but one of these elements were rated High in more than half of the classrooms observed. The one exception was Student direction/Choice which was rated High in just three classrooms. Teachers were careful to provide respectful and encouraging environments for their students and to this end much of the teachers' talk related to behavioural expectations.

Impact on Intellectual Quality. Considered in conjunction with the relatively low levels of Intellectual quality observed it seems that the emphasis in many classrooms was on smooth relationships, perhaps at the expense of learning. Smith (2000) described a teacher who, motivated by a desire to help her students experience success, consistently reduced the intellectual demands of tasks rather

than allowing students to struggle with challenging ideas in ways likely to lead to deeper learning. Cooney (1999) identified a similar concern for their students' success and enjoyment of mathematics among preservice teachers, with similar results. If teachers' concern with creating Supportive classroom environments does indeed militate against the Intellectual quality of their pedagogy then it is likely that this is particularly true in relation to mathematics in primary classrooms. Schuck (1999) linked primary preservice teachers' dislike of mathematics and limited confidence to teach it with their determination to make mathematics fun for their students. She argued that this contributed to these teachers devaluing mathematics content knowledge. There is evidence that primary teachers remain lacking in confidence with respect to their teaching of mathematics (Watson, Beswick, Caney, & Skalicky, 2006) and this may be a further driver of an emphasis on a Supportive classroom environment at the expense of Intellectual quality.

Recognition of Difference

The most prominent aspect of Recognition of difference was Group identity/ Community of learners. This element includes a degree of overlap with Supportive classroom environment and in view of the preceding discussion its relative strength is not surprising.

Task differentiation. Table 4 shows the numbers of classrooms in the same grade level bands as in Table 3 in which the major part of the time devoted to numeracy was spent working as a whole class group, in groups with unrelated tasks, and in groups working on related tasks. Group work involving differentiated tasks is one way in which teachers can cater for differing student needs but the use of unrelated tasks in these contexts makes it much more difficult to engage in whole class discussions of mathematical ideas and hence more difficult for teachers to make sure that all students participate in the kinds of mutually educative exchanges of ideas, explanations and justifications of their thinking that would enhance the intellectual quality of their endeavours.

In at least two of the classes, group work on unrelated tasks was clearly not a means of differentiating the curriculum because the students rotated through all of the activities in the course of the lesson. Similar rotations may have operated in other classrooms over longer time frames than the observation period. Very few examples of open-ended tasks with which students could engage at a level appropriate to them were observed although another teacher indicated that he made frequent use of such tasks. The use of open-ended tasks is widely recognised as effective in terms of inclusion (Clarke, 2000; Ollerton, 2001) and allows for meaningful whole class discussion of the common important mathematical focus of the activities. Askew (2001) pointed to evidence that whole class teaching may be preferable to other formats if the quality of teaching is high but also more damaging if this is not the case. He stressed that it is the quality of interaction between teacher and students, rather than the organisational style that is of key importance.

Table 4
Group and Task Organisation by Grade Level Band

Grade Level Band	Whole class on same task	Groups working on unrelated tasks	Groups working on different related tasks	Total
Early childhood (K/P-1/2)	6	1	0	7
Middle primary (2-4/5)	2	4	1	7
Upper primary/middle (5-6/7)	3	2	0	5
Total	11	7	1	19

Connectedness

Most aspects of Connectedness were rated Low in most of the classrooms observed. The use of groups working on unrelated tasks was not helpful in this regard particularly where students rotated through a series of such tasks. One teacher explained that her mathematics teaching had a different focus for each day of the week, with Problem solving on Monday, Number on Friday and the remaining days shared among Space, Measurement and Chance and data. Although there are many possible connections between these strands, this approach seemed, unsurprisingly, to result in a fragmented curriculum.

Connections between aspects of mathematics. Missed opportunities in terms of making connections among various aspects of mathematics also contributed to generally low Intellectual quality such as in the activity that involved students drawing rectangles with the same area without any explicit highlighting of the fact that these rectangles could have different perimeters. Similarly, one class played a place value dice game before moving into groups working on different activities. Two of these — a rounding worksheet, and focussed teaching on the multiplication algorithm — related to place value but although the teacher mentioned place value in relation to the algorithm no explicit links with the game just played were made. One of the few instances in which links were specifically made was in a grade 6/7 class where students changed decimals to fractions and wrote about the ways in which they had done this. This was the same class referred to earlier in relation to Intellectual quality, in which the students skip counted by 0.1 and subsequently engaged in a substantive conversation about the relevant mathematics.

Connections between mathematics and its uses. Links between mathematics and its uses were evident in five lessons. In all but one of these the link was inherent in the activity but not the subject of any explicit discussion. The five activities were as follows: the exploration of number combinations making 10, framed in terms of bears and dwarfs from stories with which the children were familiar; the use of a street directory in one of a series of unrelated activities in a grade 2 classroom; the use of grocery catalogues in an addition exercise in a grade 3

classroom; map reading and making in another grade 3 classroom; and the experience of one high support needs student who was taken to a nearby supermarket. Although these activities provided Connectedness to the world, most were quite routine, and used in ways that made few intellectual demands of students.

The one exception to this was also the only example of a rich task that made links not only to 'real world' contexts, but also across strands of the mathematics curriculum, and across learning areas. The grade 3 mapping task briefly mentioned in relation to Intellectual quality involved students creating a bird's eye view of their playground and required them to use and develop skills, knowledge and understandings in both the measurement and space strands. The physical education teacher also used ideas related to mapping the playground after which the class teacher facilitated discussions in which various aspects of the task were connected. The teacher used thought provoking questions that facilitated the students making connections between various aspects of the tasks. These included, "On a map would we have right and left?" and "If you had something that covered more than one point on the grid, what would you do?" Students' questions were often answered with questions designed to prompt them to think through the ideas for themselves. Opportunities for further connections to be made were provided in subsequent activities in which some groups of children engaged in work which related to coordinate systems in other contexts, while others worked on using a scale (1 cm = 1 m) to draw their playground maps. The distinguishing feature of this lesson was not the nature of the tasks, rich as the tasks were, but rather the way in which the teacher exploited the tasks through the use of many of the APTs.

This lesson was also one of very few that embodied a Problem solving or Inquiry based approach. Other examples included: a Prep/1/2 class in which students explored number combinations making 10; a game of "What's my number?" in a grade 1 class; and another grade 1 lesson in which the teacher initiated a discussion with the question: "What do we know about 10?" Nevertheless, in the majority of cases tasks were closed and students were expected to follow specified procedures.

Connections with students' prior knowledge. The use of a single observation period made it difficult to discern links made with prior learning. The activities just listed almost certainly provided the teachers concerned with information about their students' thinking which they may well have used in subsequent lessons. However, these sorts of tasks were rare and there were no other instances in which the teacher clearly sought to uncover students' existing understandings and to build upon them. When asked about the place of the observed lessons in their overall program teachers responded in a variety of ways referring to the structure of their day, their weekly planning or longer term program and not always mentioning numeracy specifically. Nevertheless, five teachers clearly indicated that they worked from quite structured predetermined programs based on a curriculum document in one case and on a school program in another. Just two teachers explicitly mentioned that they altered the pace and/or content of their teaching depending upon the students' progress.

Connections between mathematics and other curriculum areas. Few links between mathematics and other curriculum areas were observed. Many of the teachers indicated that they devoted the mornings to literacy and numeracy and used integrated units in their afternoon lessons. In the one classroom in which work on an integrated unit was included in the morning session the students were studying endangered species. The unit was framed around a series of questions that had been suggested by the students and then refined by the teacher and linked to curriculum outcomes. Although this teacher exhibited High levels of many of the APTs, including those related to Intellectual quality the potential of the unit to incorporate important mathematics was not exploited with just two of the 11 questions relating to numeracy.

Strategic Instruction

The indicators relating to strategic instruction were drawn from the *Flying Start* materials designed principally for literacy and were difficult to interpret in relation to numeracy. The mathematics education literature contains many references to explicit teaching (Clarke, 2000; Clarke et al., 2002; DEST, 2004; Jones, Tanner, & Treadaway, 2000; NCTM, 2000) and to modelling (DEST, 2004; DETYA, 2000; Doig, McRae, & Rowe, 2003; Grouws & Lembke, 1996) but it is crucial to note what should be made explicit and modelled. Rather than prescribing particular procedures and teaching them in an expository style, the consensus among mathematics educators such as those referred to is that problem solving processes and strategies should be made explicit and modelled. That is, rather than modelling procedures and having students mimic them, teachers should model the mathematical thinking and positive attitudes to mathematics that are the essence of numerate behaviour. Approximately half of the classrooms observed in this study featured High overall levels of Strategic instruction and High levels of the various aspects contributing to it. These resulted principally from literacy teaching or from the modelling of positive attitudes to mathematics. In at least five classrooms explicit/expository teaching and modelling of procedures without attention to conceptual understanding was observed and most of the instances of these APTs related to literacy. It is possible that the greater emphasis on literacy compared to numeracy that was noted by Hill et al. (1998) and reflected in the time allocation for literacy observed in these classrooms, had led to transfer of these strategies to numeracy without a thorough understanding of the limits of their applicability in this context.

Conclusion

In this section findings in relation to each of the research questions are presented. We end with a discussion of the implications of the study.

Research Question 1: How might the Classroom Observation Reflection Chart be useful for describing the extent to which of the APTs are present in teaching for numeracy?

The relatively small proportion of the observation periods devoted to teaching for numeracy meant that the CORC ratings disproportionately reflected the pedagogies used in other learning areas, principally literacy. Nevertheless, the inclusion of illustrative examples meant that a sense of the nature of teaching for numeracy in each of the classes was captured. The numeracy indicators provided against each of the APTs were especially useful in allowing observers who were not experts in teaching for numeracy to unpack the meaning of each of the APTs in this context. It is likely that the instrument would be of use as a professional learning tool because the indicators make more explicit what is meant by generic pedagogies in the specific context of teaching mathematics. It may also have value in assisting preservice teachers to focus their attention on salient aspects of teaching that they observe.

The relative emphasis on literacy teaching (as evidenced by the time devoted to it) in contrast to numeracy is also very telling in terms of the comparative importance attached to the two areas and perhaps also in relation to teachers' confidence to teach each area. This was captured via the CAR and hence it seems that the CORC might best be used in conjunction with another tool such as the CAR that allows a running record of events to be kept. Given that the CORC is a reflective instrument, requiring judgements about the presence of APTs to be made holistically after a substantial period of observation, the use of two instruments is not problematic.

Research Question 2: Which pedagogies, as described by the APTs, are evident in the numeracy repertoires of a sample of Tasmanian K-7 teachers?

It would be unreasonable to expect that any teacher would demonstrate all of the APTs at High levels in any one period of teaching. Rather, the APTs would ideally be part of the repertoire of teachers to be called upon as appropriate. Nevertheless, the overall low levels of many that were observed, and particularly of Intellectual quality and Connectedness suggest that typical teaching for numeracy in Tasmanian K-7 classrooms treated mathematics as a series of essentially disconnected skills rather than as an integrated web of ideas with implications for all of the curriculum and life beyond school, and with tremendous possibilities for engaging students in challenging and powerful ways of viewing and thinking about the world.

In a description of the relationship between numeracy, mathematics and the Essential Learnings curriculum, the DoET (2006a) lists elements of mathematics which it describes as core concerns that should be reflected in the mathematics programs of schools. These are:

1. Questioning, conjecturing, collecting and organising data, seeking and seeing patterns, generalising, justifying and communicating possible solutions to problems —including the use of technology.
2. Connecting new understandings to old, developing a mathematical point of view, valuing the processes of mathematisation and abstraction, and having the predilection to use them.
3. Applying mathematics in a variety of contexts.
4. Developing competence with the tools of the trade and using those tools in the service of the goal of understanding structure – mathematical sense making, including the notion of mathematical certainty (proof).
5. Reflecting on the use of mathematics, by themselves and with others, to solve problems, influence and inform.
6. Exploring the imaginative, aesthetic and cultural aspects of mathematics.

All of these elements are consistent with the APTs and, in fact, their achievement requires that teachers have the APTs, including those related to Intellectual quality and Connectedness prominent in their pedagogical repertoires. The results of this study suggest that in 2004 there was some distance between the aims of the new curriculum in terms of helping students to develop deep understandings and higher order thinking skills, and the capacities of many K-7 teachers to foster those skills and understandings.

Implications

In referring to a set of features of effective teaching proposed by Hattie (1992, cited in Ingvarson, Beavis, Bishop, Peck, & Elsworth, 2004) and consistent with the APTs used in this study, Ingvarson et al. (2004, pp. 5-6) pointed out that,

> While these attributes appear to be generic, in reality the capacity to implement them necessarily depends on a deep understanding of what is being taught. It is very difficult for teachers to run lively and effective discussion, in which they respond to and build on students' ideas, and provide timely and appropriate feedback, without deep knowledge of the mathematics being taught and how students learn it.

The kinds of teacher knowledge referred to by Ingvarson et al. (2004) resonate with Shulman's (1987) content knowledge, pedagogic content knowledge, and knowledge of students and their characteristics. Shulman (1987) recognised the need for teachers to have discipline specific as well as generic knowledge and the mathematics content knowledge of teachers remains of interest to researchers (e.g., Hill, Rowan & Ball, 2005; Watson, Beswick, Caney, & Skalicky, 2006). The CORC, and particularly its inclusion of indicators that operationalise each of the APTs in the context of teaching mathematics for numeracy, has potential to be of use to researchers interested in observational studies of how teachers of mathematics implement apparently generic pedagogies.

In the Tasmanian context it would be of interest to examine the extent to which the APTs may have become more prominent in the repertoires of K-7 teachers since the study reported here was conducted. At the time of the study

six of the schools involved reported a focus on professional learning related to the new *Essential Learnings* curriculum (Andrew, Beswick, Swabey, Barrett, & Bridge, 2005) and just two reported a numeracy focus in their professional learning activities. A major emphasis of the professional learning that accompanied the implementation of the ELs was on equipping teachers to plan, teach and assess for deep conceptual understanding and to develop their students' thinking abilities. If it achieved its objectives it would be reasonable to expect that the APTs are more prominent now than at the time of this study. However Kennedy (1998, cited in Ingvarson et al., 2004) concluded from a review of professional development programs that those which were most effective in terms of improving student outcomes focussed on deepening teachers' knowledge and understanding of content and how their students learned it. In Tasmania, recent changes to the curriculum (DoET, 2006b) have resulted in a greater focus on traditional disciplines including mathematics in the language of the curriculum but this has yet to be matched by a similar shift towards discipline specific professional learning.

In relation to Intellectual quality, and to a somewhat lesser extent Connectedness, it also appears that much mathematics teaching was conducted without a clear purpose in terms of the particular mathematical understandings that need to be fostered, or that particular tasks and activities afford the opportunity to address. Rather, in many classrooms the purpose appeared to relate primarily to smooth social functioning. Although very little learning is likely in a chaotic and acrimonious environment, order and supportiveness should not be treated as ends in themselves. This is, of course, analogous to Newmann et al.'s (1996) concern that student-centred teaching should not be interpreted in ways that reduce the intellectual quality of students' activities. Sullivan and Mousley's (1994) placement of building understanding as the over-arching objective of quality mathematics teaching is crucial, but teachers' ability to do this is likely to be mediated by their own knowledge of both the mathematics they teach and relevant mathematics specific pedagogies. Teachers need to be urged to be purposeful in their teaching of mathematics and also supported to identify appropriate mathematical purposes and pedagogies to achieve them.

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Appendix: The Classroom Observation Reflection Chart

Intellectual quality

Higher-order thinking

Does the teacher….

Ask questions that require more than recall of informationor facts? Require children to interpret, generalize, explain, hypothesise, justify procedures, opinions and solutions? Recognise and reward all students for powerful thinking and reasoning? Provide all children with challenging mathematics? Provide non-routine problems requiring students to invent solution strategies?

Deep knowledge or understanding

Does the teacher….

Plan and deliver content consistent with curriculum guidelines? Not stop with the answer but get at the thinking behind it through questions such as "Why?" and "Explain?" Solicit multiple approaches to the same correct answer. Help children to develop autonomy by using logic, reasoning and proof as verification rather than relying on the teacher or textbook for answers? Solicit and address (challenge) children's non-standard or limited ideas? Emphasise learning to reason and constructing proofs as part of understanding maths? Give attention to developing the meaning of numbers, operations and number relationship before asking students to learn basic facts and algorithms? E.g. by facilitating the development of flexible mental computation strategies. Give attention to developing the meaning of numbers, operations and number relationship before asking students to learn basic facts and algorithms? E.g. by facilitating the development of flexible mental computation strategies. Use technology to develop understanding rather than as a means of finding or verifying answers? Refrain from presenting steps or rules for performing calculations or solving problems? Encourage the use of manipulatives to promote sense making rather than as tools for obtaining answers? Maintain an explicit focus on mathematics?

Substantive conversation or questions

Does the teacher….

Use teacher led classroom conversations to develop important concepts and skills? Explain connections between mathematics concepts or skills and other concepts or skills? Use plenaries to draw out or bring together important content from the lesson? Encourage children to share their thinking and to question others' ideas? Require children to justify their answers and procedures? Do more asking and listening and less telling? Discourage reliance on the teachers (or text or calculator…) as the source of correct answers?

Knowledge as problematic

Does the teacher….

Accept and value children's alternative methods of performing calculations or solving problems? Encourage or require children to understand others' viewpoints? Communicate or encourage valuing of intellectual rigour, constructive criticism and challenging of ideas? Make explicit mention of the tentativeness of knowledge (even in maths)? Utilise open-ended problems and contestable outcome activities like estimation?

Meta-language

Does the teacher….

Facilitate explicit discussion of what it means to think or work mathematically? Use and name or compare or evaluate problem-solving strategies and processes in any kind of lesson? Develop and name problem-solving strategies? Explain and model the use of mathematical terms?

Constructive self critique

Does the teacher….

Help students to devise, explain and evaluate their choices of strategies for problem solution or storage and retrieval of facts and processes? Use paired or group process to explore possible solutions? Help students to maintain records of their concept and skill development? Ask students to write about their own mathematical understanding and learning? Model self-correction and openness to peer and student correction?

Supportive classroom environment

Student direction or choice

Does the teacher….

Give children frequent opportunities to create estimation and mental mathematics techniques? Value non-standard but meaningful methods developed by students? Not prescribe the use of specific procedures or methods? Make children's existing conceptions explicit and build on these? Allow for a range of modes of communication of understandings? Allow the focus and direction of the lesson to be determined by ideas originating from the students? Allow timing to be determined by the students? Provide explanations at the students' request? Make decisions about which children need what kind and how much practice?

Social support or Positive High Expectation

Does the teacher….

Provide all students with challenging mathematics? Encourage and support all students? Recognise and reward all students for powerful thinking and

reasoning? Create a climate of respect for what others have to say? Convey high expectations of all students by assisting or expecting all students to develop conceptual understanding of mathematical ideas and the procedures? Expect all students to engage in high level thinking?

Explicit quality performance criteria

Does the teacher….

Communicate clear expectations of student behaviour and task requirements (without prescribing methods or strategies)? Make explicit mention of what is valued in terms of mathematical thinking?

Self-regulation

Does the teacher….

Design lessons that engage all students? Stimulate children's interest, curiosity and excitement, and sustain engagement? Promote and model metacognition and ways to monitor one's own thinking? Help children choose an appropriate calculation technique? Communicate valuing of effort, persistence and concentration?

Motivate or celebrate

Does the teacher….

Stimulate children's interest, curiosity and excitement, and sustain engagement? Design lessons that engage all students? Recognise and reward all students for powerful thinking and reasoning?

Recognition of difference

Cultural knowledge or context-honouring

Does the teacher….

Recognise and make explicit the culture and value laden nature of mathematics? Include and value mathematics from non-Western cultures?

Inclusivity or all engaged

Does the teacher….

Provide open-ended questions that allow children to respond at an appropriate level for them? Stimulate children's interest, curiosity and excitement, and sustain engagement? Design lessons that engage all students? Provide all children with opportunities to engage with challenging and important mathematics? Use multiple assessment techniques?

Narrative

Does the teacher….

Share personal accounts of learning/using mathematics? Use stories of mathematicians and the development of mathematics? Attend to students'

affective states and adjust instruction accordingly? Provide opportunities for children to reflect on their learning using various kinds of writing? (e. g. think boards, journals…) Encourage/model positive attitudes to mathematics? Encourage or model perseverance and flexibility?

Group identity or community of learners

Does the teacher….

Create a climate of respect for what others have to say? Engage students as a community of learners? Work with the children to solve mathematical problems together?

Active citizenship

Does the teacher….

Negotiate clear social norms that facilitate the participation of all students in a community of mathematical inquiry? Create a climate of respect for what others have to say? Accept and value children's alternative methods of performing calculations or solving problems? Encourage or require children to understand others' viewpoints?

Connectedness

Knowledge integration

Does the teacher….

Develop or highlight connections between mathematical ideas or topics or strands? Engage students in rich tasks that cross strands or topics? Emphasise that most calculations can be completed using prior knowledge and simpler calculations? Make explicit the connections between mathematics and other subjects? Make links between logical processes and the role of intuition and aesthetics in mathematics?

Background knowledge

Does the teacher….

Make children's existing conceptions or knowledge explicit and build on these? Elicit and recognise children's intuitive or commonsense understandings of mathematics and build on these?

Connectedness to world

Does the teacher….

Routinely use data and graphs as problem solving contexts? Routinely provide examples of how mathematics is used in the world? Whenever possible make historical connections? Whenever possible make connections to other subject areas?

Problem-based curriculum or inquiry learning

Does the teacher….

Promote a disposition to formulate, represent, abstract and generalise in problem situations? Introduce concepts and skills through problem solving or reasoning experiences that require higher level thinking and significant challenge, rather than delaying problems until students have mastered a procedure? Engage children in problems that require them to mathematise rather than simply apply previously mastered procedures? Allow the use of technology in problem solving situations? Use problem solving as a means of teaching both concepts and skills?

Strategic Instruction

Explicit/Expository

Does the teacher….

Conduct transparent problem solving including the articulation of process or strategy? Encourage students to evaluate each others' thinking and be open to discourse around possibilities?

Modelling

Does the teacher….

Model positive attitudes including perseverance and flexibility in solving problems? Engage in adult-level numerate behaviour with and around children? Encourage children's families to engage in exploration and enjoyment of maths?

Purposive monitoring

Does the teacher….

Carry forward knowledge about children's skill and concept growth from episode to episode? Maintain records of use to long term and shared assessment, including with families? Engage the learner as a partner in assessment?

Personalised /tailored learning

Does the teacher….

Look to the individual and shared experiences of students as sources of topic or unit development? Seek out effective avenues in learning style/learning orientation/ mode for all individuals and groups in the class? Encourage students to explore their own 'best practice'?

Note. The wording used for many of the indicators are cited directly from Charles (1999). A version with references is available from the first author.