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Using CRA to Teach Algebra to Students with Math Difficulties in Inclusive Settings

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The importance of algebra instruction has increased in the United States in the past few years. Thus, in most states, middle school students are required to take Algebra 1. Middle school students with math difficulties in inclusion algebra settings may require a different instructional approach. The purpose of this research was to compare student achievement in solving linear algebraic functions across two procedural approaches: a multisensory algebra model using a concrete-to-representational-to-abstract sequence of instruction (CRA) and a repeated abstract explicit instruction model. Out of 231 students who participated, the students who learned through the CRA model scored significantly higher on the post- and follow-up test. The success of the CRA model was consistent for students with a history of low, medium, and high math achievement. Implications of this model and possibilities for future research are discussed.

Keywords: Algebra, Math Disabilities, Inclusion, Concrete-to-Representational-to-Abstract Sequence of Instruction

As a response to the low U.S. showings in international comparisons, there has been an increase in the number and difficulty of mathematics course requirements for high school graduation in recent years (National Center on Educational Statistics [NCES], 1996, 1999). As a result, several states now require students to pass end-of-year or graduation tests that show knowledge in algebraic understanding (Ysseldyke et al., 1998), considered the gateway to abstract thought. Not only are many school districts and states beginning to require four years of math at the high school level, but the formal introduction to Algebra 1 occurs earlier in the curriculum than before. Most sixth- and seventh-grade students are being taught prealgebra content while others are enrolled in an Algebra 1 course.

As algebra-based standards continue to shift to lower grade levels, it may be necessary to change how teachers present content. Moreover, with a potentially wider range of student achievement in inclusive settings, teachers must use curriculum and instruction that is beneficial to students with histories of both high and low math achievement. To help teachers and students alike, district administrators have tried several approaches. A possible approach is to embed separate algebra programs (e.g., Algebra Tiles; Hands-On Equations) within the math program. However, district administrators have difficulty knowing which algebra programs to choose and how to supply training.

The National Council of Teachers of Mathematics Principles and Standards for School Mathematics (NCTM; 2000) and most state standards call for students to

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explore math through hands-on means in order to help build math problem-solving and higher-order thinking. Although many algebra programs claim to help students learn initial equations through hands-on instruction, such programs are typically limited in terms of the skills addressed. Even with introductory algebra skills it is difficult to locate math manipulatives that accurately address effective stepwise procedures for a variety of linear functions. Few secondary-level math teachers use concrete objects (Howard, Perry, & Conroy, 1995), suggesting that teachers may be concerned that the manipulative objects included in some algebra programs and textbooks do not accurately represent a concept or there is an unclear connection between the manipulation of the objects and the procedures used in abstract problem solving.

Nevertheless, many students learning secondary-level math benefit greatly from interacting with properly designed concrete materials (Demby, 1997; Goodson-Epsy, 1995; Kraayenord & Elkins, 2004; Maccini & Hughes, 2000; Zawaiza & Gerber, 1993). For example, interactions with concrete materials increase the likelihood that students remember stepwise procedural options in math problem solving, because they allow students to encode and retrieve information in a variety of sensory options: visual, auditory, tactile, and kinesthetic. Students can create portable concrete kits by connecting the concrete materials in class to pictorial representations on most homework and testing situations. Thus, when a student confronts a difficult math problem, he/she is able to create drawings similar to the use of concrete manipulations.

Teaching students through the use of concrete objects, pictorial representations, then abstract numerals, is called the concrete-to-representational-to-abstract sequence of instruction (CRA). CRA is a three-stage learning process where students learn through physical manipulation of concrete objects, followed by learning through pictorial representations of the concrete manipulations, and ending with solving problems using abstract notation. Teaching students through these three learning stages has been shown to be beneficial to secondary students with difficulties in such skills as fractions (Butler, Miller, Crehan, Babbitt, & Pierce, 2003; Jordan, Miller, & Mercer, 1999); word problems (Hutchinson, 1993; Maccini & Hughes, 2000); and advanced linear functions (Witzel, Mercer, & Miller, 2003). Nevertheless, few published research investigations have examined the use of the full CRA sequence of instruction with algebra students with and without learning disabilities combined in inclusive settings.

Transitioning from Arithmetic to Algebra

The reason for increased concern with algebra comes from the uniqueness of algebra equations. There are conceptual differences between the approach to an arithmetic problem compared to an algebra problem (Lee & Wheeler, 1989). Students may confuse arithmetic procedures due to the new algebra notation (Stacey & MacGregor, 1989). While algebra often requires meaningful manipulations of symbols, it also involves manipulations of equations that have little or nothing to do with the purpose of the equation (Phillipp & Schappelle, 1999). This disconnection appears because the apparently arbitrary symbols in algebra can be manipulated with little regard to the numerals in the original equation. An effective curriculum is one that can help students acquire the new symbol system as well as teach the proper steps to solving algebraic problems (Stacey & MacGregor).

Even though algebra evolves from arithmetic, a learning gap exists. In other words, knowledge of arithmetic is necessary but not sufficient for algebra success (Witzel, Smith, & Brownell, 2001). Even if the student understands the abstractness of algebra, a lack of fluency in arithmetic will interfere with learning algebra. For example, a student may understand how the systems of equations $2x + 8y = 12$ and $5y – 3x = 7$ may intersect at one point, but the same student cannot solve the system of equations numerically if he or she cannot solve the arithmetic aspects within this problem. While arithmetic is important, success in arithmetic does not guarantee success in algebra. Using the same example, if the student is confused by the notation and does not know why the two equations come to a single answer, it is unlikely the student will realize that he or she should or could compute them together. For teachers, arithmetic knowledge must be reinforced throughout Algebra 1.

Multisensory Algebra Model

The CRA algebra model used in this study was designed to portray concepts for solving for variables with multiple coefficients, fractions, and even exponents while supporting continued operational development. Different from other CRA models, this model represents more components of linear functions. For example, instead of a colored block representing 1X, the model represent the coefficient of one and the unknown X separately. (See Figure 1 for a description of a linear function using this model.)

Capable of generalized use to many algebra skills, the purpose of this model is to allow multiple uses within an algebra curriculum. The CRA model is considered multisensory because it relies on not only visual and auditory interactions with content, but also kinesthetic and tactile, through the use of hands-on manipulations of objects and matching of pictorial drawings. The purpose of this study was to compare the benefits of this multisensory CRA algebra model to those of traditional abstract instruction with middle school students in inclusion settings.

Methodology

A pre-post-follow-up design with random assignment of clusters was employed. Students were clustered by class and divided into two groups across each teacher, a treatment class and a comparison class. The objective for both groups was to improve prealgebra skills. The dependent measure, number of correct answers out of 27 possible on an algebra assessment, was analyzed for both groups before instruction, immediately after instruction, and three weeks following the conclusion of instruction.

For the purpose of this study, conceptual knowledge was defined as the ability to incorporate previously learned declarative and procedural knowledge to answer novel problems (Bottge, 2001). For example, students scaffolded procedural knowledge of solving inverse operations (e.g., $X + 3 = 5$) to solve two-step inverse operations (e.g., $2X + 3 = 5$). If one step is taught thoroughly and effectively, students should be able to generalize their understanding.

After 19 lessons covering five math skills, the two groups of students were compared on multiple-step linear functions with the variable on both sides of the equal sign (e.g., 3X – 4 = X + 8) using an assessment instrument standardized to tenth-grade local students who completed prealgebra and algebra with an A or B letter grade. One of the instructional groups of students learned through a CRA model, the other learned through repeated abstract instruction. Both groups were taught the same number of lessons using scripted lessons that followed the same instructional steps. Additional comparisons were made on performance per instructional groups according to students' previous year's math achievement scores.

Study Participants

Six general education math teachers and 358 students from four middle schools in a southeastern United States urban county participated in this research. Four teachers individually taught eight mathematics classes for sixth graders, and two teachers taught four mathematics classes for seventh-graders. Each teacher taught one class using the CRA method and one class using traditional abstract instruction. A coin flip decided which of each teacher's two possible math inclusion classes would be the treatment CRA class. The six sets of teachers were trained individually on the algebra program in a one-day session with weekly meetings during instruction. Follow-up meetings consisted of the teacher demonstrating upcoming lessons with the manipulative sets.

Every class included students with and without learning disabilities. Due to the transient environment of the region, of the original 358 students, data from only 231 students were usable for analysis. Student data were eliminated if test data from the previous year were missing or if the district did not have a student's complete educational history. Student groups were compared according to statewide achievement score stanine, grade level, age, how many students with learning disability labels had goals in mathematics, and socio-economic status as determined by those provided free or reduced-cost lunch. See Table 1 for student demographics per instructional group.

Table 1

Demographics of Each Instructional Group with Achievement Stanine Means and Standard Deviations

	Descriptive Statistics									
	N	Achievement stanine	Mean age	General education	SLD math	Free/reduced lunch	Male	Female	6th grade	7th grade
Abstract	123	5.56 (*SD* = 1.82)	12.15	100	23	40	51	72	74	49
Multisensory	108	5.05 (*SD* = 1.56)	12.18	82	26	42	47	61	47	61

Algebra Construct

The CRA algebra model was used to teach students a series of algebra skills from reducing simple two-statement expressions to solving linear functions with unknowns on both sides of the equal sign. The unit plan consisted of 19 lessons and five math skills: reducing expressions, solving inverse operations, solving inverse operations with negative and divisor unknowns, solving linear functions with unknowns on one side of the equal sign, and solving linear functions with unknowns on opposite sides of the equal sign (i.e., 27 + 7W = −2W − 18). The reason why this last step was chosen was that it is one step beyond any published algebra manipulative material; thus, it has never been used as a construct. Three lessons were allotted to reducing expressions and all other skills were allotted four lessons. The treatment group was taught one concrete lesson followed by one pictorial representation lesson for every math skill. On third and fourth days, each math skill was taught at the abstract level. By comparison, the abstract group was taught using abstract notation every day of the study. All groups across the school district began and ended instruction on the same days.

Pretest condition. Teachers who participated not only wanted a statistical comparison, they also wanted to learn more about the differences of each instruction first-hand. Using a coin flip, the researcher randomly chose one math class for the teacher to teach using CRA instruction and the other class to be taught using the abstract-only traditional methods. Therefore, each teacher or team of teachers taught one traditional abstract class for every multisensory CRA algebra class.

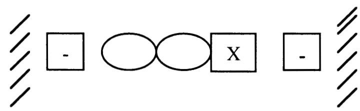
Common procedures. Mercer, Jordan, and Miller (1994) found that most math textbooks follow explicit instruction guidelines. Even though textbooks have evolved in the past 10 years, the format within textbook lessons has remained similar. In addition, explicit instruction has long been the accepted means to math instruction for students with disabilities (Bryant, Hartman, & Kim 2003). Specific to algebra it believed that explicit instruction allows students to see the connection from arithmetic to algebraic functions (McConnell & Bhattacharya, 1999).

To match what is considered best practice as well as standard within current math instruction, lessons were designed around explicit instruction guidelines. Components of each lesson in both groups consisted of an advanced organizer, description of activity, description of activity and modeling steps, guiding students through steps, and allowing students independent practice. Teachers taught both instructional groups during 50-minute class periods on the same day. (Each lesson consisted of the full 50 minutes.) Since it was expected that concrete lessons would cover fewer problems, teachers were informed to complete as much of the class period would allow. For sometimes rapidly moving abstract lessons, teachers were to include more independent practice problems if time remained following their script. Plans called for each group to cover the same problems and same corresponding worksheet.

Treatment group. Students in the treatment group worked in the same classroom setting that they had used throughout the year, but now their teacher taught using the CRA model. Since the students had minimal prior experiences with algebra, they were introduced to algebraic thinking through CRA. Each treatment lesson included four steps: (a) introduce the lesson, (b) model the new procedure, (c) guide students through procedures, and (d) begin students working at the independent level. These four steps were used for instruction at the concrete, representational, and abstract stages of each concept. Teachers taught the concrete lessons using manipulative objects, the representational lessons using pictures, and abstract lessons using Arabic symbols. (See Figure 1 for a sample of concrete and representational sets of reducing expressions.)

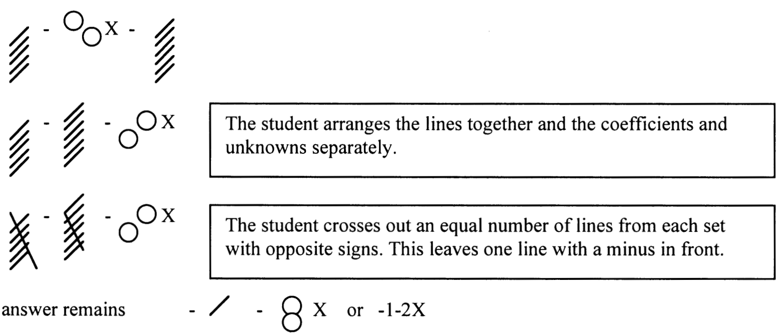
Comparison group. The comparison instructional techniques included introducing a lesson, modeling a procedure, working with students through guided and independent practice, and assessing their knowledge. Since the students had minimal

Step 1: A **concrete** representation of 5 – 2X – 6 must use manipulative objects. For this problem it would appear in the following order: five small sticks, a minus sign, one coefficient marker, an X, a plus symbol, a large stick, an equal line, and three small sticks. Manipulating the objects leaves the answer as a minus sign with one stick remaining and a minus sign followed by two cups of X.



Step 2: A pictorial **representation** would closely resemble the concrete objects but could be drawn exactly as it appears below. A student solves representational problems exactly as she would solve them concretely.

For example,



Step 3: An **abstract** problem is written using Arabic symbols as displayed in most textbooks and standardized exams. Students in the comparison group used this format for problem solving during each lesson. The multisensory group only used this format after concrete and representational manipulation. To solve abstract problems, students write each step to solving the problem. For example,

$5 - 2X - 6$
$^{+}5 - 6 - 2X$
$^{-}1 - 2X$

Figure 1. Concrete, representational, and abstract examples of a reducing expressions problem.

prior experiences with algebra, the abstract approach of instruction introduced them to the abstract thinking associated with algebraic concepts. Students in this group also worked in the same classroom setting that they had throughout the year. The teacher instructed each lesson using examples of abstract equations. (See Figure 1 for a sample of how an abstract equation appears. See Table 1 for demographics of students in each instructional group.)

Materials. The materials used in the study were developed to determine the difference in acquisition and retention of algebraic understanding by students with and without learning disabilities in math. Students in both the treatment and the comparison groups were tested using the same instrument for the pretest, posttest immediately following instruction, and three-week follow-up after instruction had ended. Treatment and comparison groups received the same questions and equations on daily learning sheets to guide them and their teachers through instruction. The treatment group had different instructional steps for both concrete and representational instruction.

Assessment Tools

To determine student gains, a test instrument was developed to measure students' acquisition and retention on the final skill of a five-step instructional set: solving linear functions with unknowns on both sides of the equal sign. After creating an initial pool of 70 items and elimination of seven items by an expert review of experienced high school math department chairs, a 63-item pool of questions on solving linear functions with unknowns on both sides of the equal sign were distributed to 32 tenth-grade students who had completed prealgebra and algebra with an A or B grade. The 27 items that remained carried a medium difficulty level between 37.5% and 62.5% accuracy. Thus, a score of 13 to 14 correct would mean that the student was performing on par for solving algebraic linear functions with tenth-grade students who had completed a full year of prealgebra and a full year of algebra with an A or B in the same school region.

A single test form was used for pretest, posttest, and follow-up. Pretest measures were obtained one week prior to implementation of the treatment. Posttest measures were obtained five weeks later upon completion of the last day of the treatment, and follow-up measures were obtained three weeks after treatment had ended. The tests did not have a time limit per se, except that the test had to be completed by the end of the class (50 minutes). No student used the entire class period on any of the measures. Students were not instructed on any of the equations between the posttest and the follow-up. Test items were scored as correct or incorrect by teacher comparison to an answer key. All test scores were checked by the author.

Treatment Fidelity

To ensure that the sequence of instruction components was used consistently throughout the treatment and across comparison groups, teachers were trained in both instructional approaches in a one-day session with weekly instruction of upcoming lessons. For fidelity purposes, a checklist was used during an observation of each teacher four times during instruction. The teacher was observed on delivery of the explicit instructional components. The lead author observed each teacher during a concrete lesson, a representational lesson, and two abstract lessons. Every teacher who participated in the study completed every required scripted component during the observations.

Results

The statistical results favored the treatment group who learned through multisensory algebra over the comparison group who learned through traditional abstract explicit instruction. Both the treatment and the comparison group showed improvement from the pretest to posttest and follow-up tests. For the group of students who participated in multisensory algebra, those who scored above average on statewide achievement tests scored at least as well on this assessment as students who passed algebra with a C or better. For the group of students who participated in traditional abstract instruction, only those who scored in the 9th stanine of the statewide achievement tests scored at or better than the average of students who completed and passed algebra with a C or better.

Overall Differences

The interaction between test occasion (pre-, post-, and follow-up) and instructional group (Multisensory Algebra and Abstract) yielded a significant difference, $F\,(5, 687) = 44.60$, $p < 0.01$. Calculations for effect size, $r^2_{pb}(687) = 0.245$, showed that 24.5% of the variance of scores was accounted for by the interaction of test occasion and instructional group. Since three follow-up tests were analyzed, the Bonferroni correction procedure ($\alpha = 0.05 / 3 = 0.017$) was used to maintain a 95% confidence level. See Table 2 for average scores on the pretest, posttest, and follow-up tests for both treatment and comparison groups.

Table 2

Pretest, Posttest, and Follow-Up Means and Standard Deviations within Each Instructional Group

	Pretest	Posttest	Follow-Up
Abstract	0.57 (*SD* = 1.12)	5.36 (*SD* = 5.75)	5.51 (*SD* = 5.97)
Multisensory	0.18 (*SD* = 0.53)	8.26 (*SD* = 7.65)	7.96 (*SD* = 7.84)

Treatment and comparison. Both the treatment and comparison groups showed significant improvement in students' ability to solve linear functions, $F\,(2, 687) = 100.77$, $p < 0.01$. Additionally, there was a significant difference between scores per instructional group, $F\,(1, 687) = 15.12$, $p < 0.01$. Examining instructional differences across each test revealed where such differences existed.

One significant difference was found on the pretest where the Abstract group outperformed the Multisensory group, $t(178) = 3.49$, $p < 0.001$. However, the Multisensory group outperformed the Abstract group on the posttest, $t(196) = -3.22$, $p < 0.01$, and the follow-up test, $t(198) = -2.64$, $p < 0.05$. Thus, while the Abstract group outscored Multisensory algebra students on the pretest, the latter showed more acquisition on the posttest and retention on the follow-up test.

Stanine differences. Notable differences were found between achievement in each form of instruction according to past math achievement. At every math achievement stanine, students who learned through multisensory algebra outperformed their peers who learned math through traditional abstract instruction. (See Table 3 for details on posttest and follow-up test outcomes per stanine.)

When posttest scores were aggregated by low (stanines 1–3), medium (stanines 4–6), and high (stanines 7–9), obvious score differences were present. This representation of the data showed that although multisensory techniques might be thought of as remedial, they also benefited students with high math achievement. Figure 2 represents a comparison of scores per stanine group.

Considering the grade difference. Unbalanced absenteeism and class sizes between the Abstract and Multisensory groups raised questions in the analysis. Since the final scores included more sixth-grade students from the Abstract group and more seventh-

Table 3
Posttest and Follow-Up Means and Standard Deviations per Achievement Stanine

Posttest									
Stanine	1	2	3	4	5	6	7	8	9
Abstract	–	1.14	1.78	3.19	5.32	5.50	7.14	7.54	11.83
		(2.04)	(3.07)	(3.76)	(5.61)	(4.00)	(6.48)	(7.90)	(7.96)
	n = 0	*n* = 7	*n* = 9	*n* = 21	*n* = 22	*n* = 24	*n* = 21	*n* = 13	*n* = 6
CRA	0	4.50	4.00	4.79	6.12	10.97	14.42	23	20.00
		(7.72)	(4.72)	(5.02)	(5.22)	(7.66)	(9.26)		(4.39)
	n = 1	*n* = 4	*n* = 14	*n* = 19	*n* = 25	*n* = 29	*n* = 12	*n* = 1	*n* = 3
Follow-Up Test									
Stanine	1	2	3	4	5	6	7	8	9
Abstract	–	0.57	2.56	3.29	5.77	5.46	6.95	7.62	13.17
		(1.51)	(3.36)	(3.41)	(6.68)	(5.02)	(5.98)	(6.92)	(8.93)
	n = 0	*n* = 7	*n* = 9	*n* = 21	*n* = 22	*n* = 24	*n* = 21	*n* = 13	*n* = 6
CRA	2	3.50	3.00	3.37	6.00	11.38	15.42	9.00	21.33
		(7.00)	(3.53)	(3.72)	(5.86)	(7.65)	(9.43)		(6.43)
	n = 1	*n* = 4	*n* = 14	*n* = 19	*n* = 25	*n* = 29	*n* = 12	*n* = 1	*n* = 3

grade students from the Multisensory groups, it was important to test for grade-wise differences that might account for the differences in posttest scores. Use of a one-way ANOVA revealed no significant difference between posttest scores of 6th and 7th graders who participated in abstract instruction, $F(1, 121) = 0.85$, $p > .05$. In addition, use of another one-way ANOVA showed no significant difference existed between posttest scores of 6th and 7th graders who participated in the multisensory instruction, $F(1, 106) = 0.04$, $p > .05$. In summation, the grade level at which students were introduced to the algebra skills using these procedures did not affect the posttest scores; therefore, grade level was not a significant factor in the test scores.

Discussion

The findings from this quasi-experimental study provide insight into algebra education for middle-school students in inclusive settings. Not only do the statistical

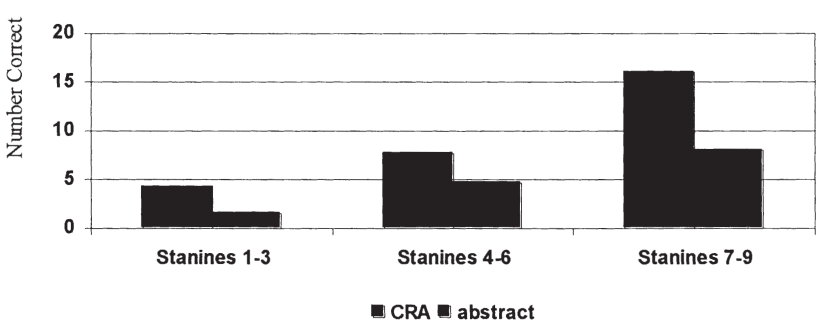


Figure 2. Mean posttest score differences per achievement stanine grouping.

analyses support CRA instruction for middle-school students who need remediation in math, they also support the use of CRA techniques for students with a history of high math achievement. This is different from research on early numeracy skills where researchers found that students with math difficulties rely more on concrete manipulations than students who are normally achieving (Jordan, Hanich, & Kaplan, 2003). Increasing student interactions in class and allowing students to experiment and examine algebra concepts may allow students of all levels to develop and remember procedural steps more accurately. Additionally, the use of this model involved students performing basic operations to help solve for variables. Although a few teachers complained before the instruction began that such operations may bore normally or higher achieving students, computing basic facts with manipulative objects may have helped students who had poor computational knowledge and skills. Regardless of the reason, interacting with the algebra concept using these manipulative objects and representations resulted in higher student performances immediately following instruction as well as three weeks after instruction ended.

Weaknesses and Future Research

The design and results of this study had some weaknesses. For example, high standard deviations in the treatment group's scores show that the model does not have an immediate effect for all sixth- and seventh-grade students with this content. Also, the high standard deviations in both groups' posttest and follow-up scores indicate that students enter algebra with different cognitive maturity towards abstractness. Oddly, this maturity is not easily identified as an age issue for the students in this study, since 6^{th-} and 7^{th-} grade students in both groups performed similarly. Additional research must focus on student variables that affect performance in algebra, such as cognitive maturity and previous instruction.

Another possible reason for the variable results for each group in the posttests may be the curriculum sequence that influenced this study. Few studies have identified effective algebra curriculum sequences for textbook writers to follow. Thus, school districts have adapted by setting pacing guides to match state standards regardless of the sequence of skills presented in textbooks.

A possible weakness in the analysis of the outcomes of this research was examining the results by comparing students' previous year's achievement stanines. Algebra is too unique in its concepts to assume we can predict students' success by their achievement in arithmetic (Witzel, 2003). The assumption was that if a student is poor in arithmetic, he will struggle in algebra. However, knowledge of arithmetic does not equal success in algebra. Research must continue to explore what variables lead to success in algebra over arithmetic.

Implications

The apparent success of the CRA model shows promise for inclusive settings where students are highly varied in their math abilities. The varied success across students illustrates how the model may benefit not only students who are struggling in math but also those who have a successful history in math. However, the success of this program should be viewed cautiously until reasons for the high standard deviations can be discovered and controlled.

The success of the model used in this study is founded on increased interactions with the concept and use of generalized arithmetic steps. Teachers should design classrooms to increase student interactions with new concepts that reinforce prior knowledge. Many teachers perform this function through warm-ups, guided practice using white boards, and open discourse with discussions of the relevance of each math skill. Teachers should continue to explore hands-on and pictorial approaches that effectively represent math procedures that lead to productive and efficient math skills connected to conceptual understanding.

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