An Investigation of K-8 Preservice Teachers' Concept Images and Mathematical Definitions of Polygons

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Prior to introducing and teaching the topic of polygons, a five-sided figure was drawn on the board (see Figure 1) and the class was asked, "What is the name of this polygon?" Responses included:

◆ *It isn't a polygon because it doesn't look like one, but it does have sides. I'd say it's just a shape. A boxy letter M.*

◆ *It has five sides but it can't be a pentagon because pentagons look like those school crossing signs.*

◆ *Five sides…Maybe a hexagon? But that's not a normal hexagon.*

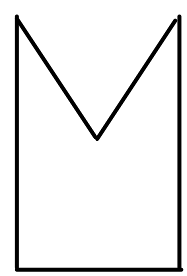

Figure 1
A non-traditional representation of a pentagon.

These answers may not be surprising had they been spoken by elementary level students; however, these responses were made by K-8 preservice teachers. The responses seem to indicate that these teacher candidates' mathematical definitions of a polygon seemed to conflict with their mental representations, that is, their concept images, of a polygon. In other words, the teacher candidates indicated that having a certain number of sides was a requirement for the definition of a polygon, yet the figure the teacher candidates

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were examining, despite having sides, did not match their concept image of a polygon.

In order to further explore K-8 preservice teachers' concept images and mathematical definitions of polygons, a study was carried out in which K-8 teacher candidates enrolled in an elementary mathematics content course were asked to sort, identify, and provide definitions of such shapes including triangles, quadrilaterals, and other n-gons. Of interest to the author was the collection of concept images these teacher candidates possessed of certain polygons. The results reported here focus on the teacher candidates' completion of those tasks involving triangles and hexagons.

Theoretical Framework

Concept Image

The notion of a student's "concept image," as defined by Vinner and Dreyfus (1989), is "The set of all mental pictures associated in a student's mind with the concept name, together with all the properties characterizing them" (p. 356). Further, a student's concept image is the result of his or her accumulated experiences with examples and nonexamples of the concept. Quite often there exists a gap or conflict between a student's concept image and the concept's definition as taught by the teacher. Despite teachers presenting definitions of mathematical concepts to students, quite often students do not use the definition when deciding whether a mathematical object is an example or nonexample but, instead, decide on the basis of their concept image of the object. When the set of mathematical objects considered by the student to be examples does not match the set of definitions of the objects, students' behavior may differ from what the teacher expects (Dreyfus & Vinner, 1982; Vinner & Dreyfus, 1989).

Related Research on Student's Concept Images in Geometry

Many researchers have investigated students' mathematical ideas, conceptions, and misconceptions as well as their development (Bishop, Clements, Keitel, Kilpatrick, & Laborde, 1996; Borasi, 1996; Grouws, 1992; Nesher & Kilpatrick, 1990; Smith, diSessa, & Roschelle, 1993). Their findings indicate that students build their knowledge of mathematical concepts and ideas in ways that often differ from what is assumed by instructors and those in the professional community. Thomas and Holton (2003) report that concepts in mathematics can be "rather diffuse and difficult to pin down…because they can take many forms, including processes, objects and statement…In addition to this there is yet another level of complexity associated with every concept, namely that they can be represented in a number of different ways" (p. 354). Consequently, it is widely accepted today that teachers be aware of and knowledgeable about students' mathematical understandings and learning, since such awareness and knowledge can "significantly contribute to various aspects of the practice of teaching" (Even & Tirosh, 2002, p. 219).

Clements, Sarama, and Battista (1998) reported that the elementary school students in their study possessed very limited and rigid ideas about what was, or was not, a triangle. When presented with a task in which they were asked to identify examples and nonexamples of triangles, a majority of these students believed that the only "true" triangles were either isosceles or equilateral. Some of the children expressed the belief that a triangle's base had to be horizontal; that is, the base of the triangle was parallel with the bottom edge of their worksheet, also referred to as gravity-based. Accordingly, these students ruled out one example of an equilateral triangle that was "pointing down," although "the students felt the need to explicitly state that it would be a triangle if you turned it" (p. 52). Consequently, the teachers of the children in this study "repeatedly expressed surprise" (p. 52) that their students demonstrated such limited and rigid ideas about triangles. In a follow-up activity facilitated by the teacher, these elementary students were able to collaboratively define a triangle as "a closed path with three line segments" (p. 54). However, it was these students' concept images, undoubtedly formed over the course of many years, that played an integral role in distinguishing examples from nonexamples.

Hershkowitz (1987) carried out a study involving students in grades 5-8 as well as preservice and inservice teachers in which she explored the evolution of misconceptions with age and instruction. The participants were presented with several shapes and asked to identify examples of quadrilaterals. Her findings indicated a strong improvement with age in identifying the non-standard examples of quadrilaterals (e.g., concave). However, in certain cases, some misconceptions had the same pattern of overall incidence for all of the participants. For example, when asked to identify right triangles, students, as well as preservice and inservice teachers, demonstrated difficulty in identifying those triangles with perpendicular sides not in the vertical-horizontal (that is, gravity-based) position, which is prototypical in textbooks. A somewhat surprising pattern reported by Hershkowitz included misconceptions that increased with age and instruction. For instance, subjects were asked to draw the altitude to one side of several supplied triangles. Analysis of the data revealed that the number of subjects who incorrectly constructed all altitudes inside the triangle increased with age and instruction.

Findings from other research studies conclude that students limit concepts to studied exemplars and consider inessential but common features as essential to the concept (Burger & Shaughnessy, 1986; Fisher, 1978; Fuys, Geddes, & Tischler, 1988; Zykova, 1969). Components of concept images were also identified. For example, students' predominant concept image of a right triangle was a right triangle in a gravity-based position. However, their concept image was less likely to include a similar triangle slightly rotated, and least likely to include a right isosceles triangle with a horizontal hypotenuse. Clements & Battista (1992) advocate the study of students' concept images since they may provide insight into and useful information about students' errors. For example, students who know a correct verbal definition of a mathematical concept, but also possess a specific concept image tightly associated with that concept, may have difficulty applying the verbal description correctly (Clements & Battista, 1989; Hershkowitz, Ben-Chaim, Hoyles, Lappan, Mitchelmore, & Vinner, 1990; Vinner & Hershkowitz, 1980).

Methodology

Participants

The participants in this study were seven students, attending a four-year, comprehensive public university located in central California, enrolled in a mathematics content course designed for K-8 teachers. The seven participants, all of whom volunteered to be a part of this study, were scheduled to enter a K-8 teacher credential program the following fall. Six of the seven participants were female and one was male. Five of the six female participants were traditional-aged students, while the other female and the one male participant were both in their late thirties, returning to school to obtain their K-8 teaching credential. The teacher candidates had all successfully completed the first two of the three required courses in the mathematics content sequence and had thus covered topics including set theory, number theory, real numbers, probability, and statistics. The third course in this three-course sequence in which the participants were currently enrolled, was devoted entirely to geometry. The participants were interviewed during the first week of this course prior to the introduction of the topic of polygons.

Data Collection

Each teacher candidate was videotaped individually for approximately forty minutes as he or she completed three tasks (see Figures 2, 3, and 4) in which the teacher candidates were asked to sort, identify, and provide definitions of such shapes including triangles and hexagons. In order to further probe the author's hypothesized potential conflict between the teacher candidates' concept images of polygons and their mathematical definitions of the polygons, more probing questions were asked during the interviews as deemed appropriate by the interviewer. At the end of each interview, the videotape was immediately transcribed. Then, the transcriptions and videotapes were analyzed for potential evidence of concept image conflicts and to gain insight into the concept images these teacher candidates possessed of certain polygons.

Task #1

In the first task (see Figure 2), the teacher candidates were given a sheet of paper containing a variety of triangles and were asked to identify which of the shapes, if any, was a triangle. Although originally designed as an "ice-breaker" question to allow the teacher candidates to become familiar with the nature of the subsequent tasks and questioning style of the interviewer and to allow them to become comfortable in front of the camera, this question, surprisingly, posed a problem for three of the seven teacher candidates. In fact, only four of the seven teacher candidates were able to confidently state that all of the shapes were examples of triangles, recalling their learned definition of a triangle as a shape "having three sides."

One of the teacher candidates who incorrectly answered this problem initially replied that all of the shapes were triangles "because they

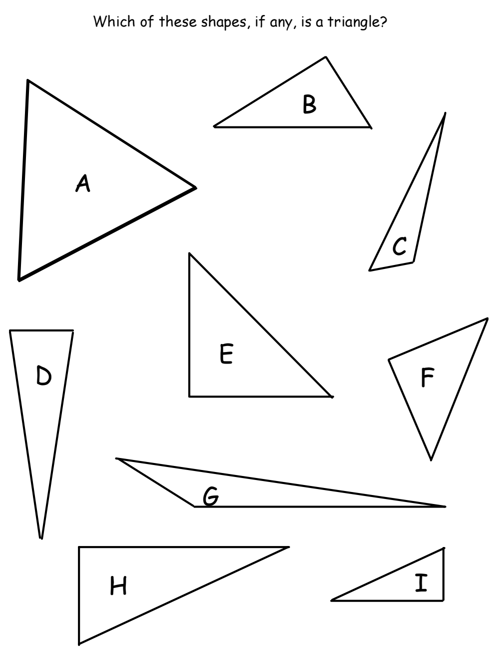

Figure 2
Task #1: Triangle Task

have three sides," a definition she stated learning "early on in elementary school somewhere." However, she quickly retracted her statement saying, "Wait, maybe not this one (points to G). It's kinda hard to tell without a protractor exactly." The interviewer asked her why she needed a protractor and she responded, "I remember something about a triangle having 180 degrees in it and those angles don't look like they would add up to 180." After a few more moments of consideration and expressing embarrassment over the fact that she "ought to know this," the teacher candidate decided that triangle G was not a triangle because, "Regardless of the angles, it's just so long and stretched." Thus, even though triangle G fit this teacher candidate's mathematical definition of a triangle as being a three-sided shape and whose angles sum to 180 degrees, she ultimately relied on her concept image of a triangle and thus excluded G as an example of a triangle stating, "It just doesn't look like a triangle."

Another teacher candidate, in an attempt to determine which shapes were or were not triangles, continually rotated the paper trying to view the triangles from different perspectives and orientations. More specifically, she would stop rotating the paper when the triangle she was viewing had its perpendicular sides in the vertical-horizontal position; that is, was gravity-based. She commented, "A triangle, as I remember it, has three sides. So, as I look at these, every single one has three sides. And so I would say that they are all triangles." However, in making this statement, the videotape recorded her shaking her head from left-to-right in an unsure manner as she appeared to lack confidence in her response. When the interviewer asked the teacher candidate if she was certain that all of the shapes were triangles, the teacher candidate then confirmed, although rather hesitantly, that all of the shapes were indeed triangles, despite the fact that two of the triangles, namely triangles C and G, "looked weird." When asked by the interviewer what she meant by this, she commented, "I think they are triangles but they are just not your typical triangles you see in school." Again, even though all of the triangles fit this teacher candidate's mathematical definition of a triangle as a shape having three sides, her demonstrated ambivalence and lack of confidence, captured on the videotape, appeared to indicate a concept image conflict, substantiated further by her comments that two of the triangles looked "weird" and were not "typical."

Upon viewing the triangles, a third teacher candidate also rotated the paper several times, viewing the triangles in different orientations, forcing some to be gravity-based. He first defined a triangle as "a three-sided figure containing at least one 90 degree angle." Using this definition, he then concluded that all of the shapes were examples of triangles except A and G (despite the fact that two other triangles, namely C and

D, should be excluded as well, using this definition). The interviewer then asked, "So what shape would you call A?" The teacher candidate paused, sighed as if frustrated, and then corrected himself stating, "No, wait, a triangle is just a three-sided figure. So I would say that they all are triangles, but I'm not sure about G." When asked what type of shape G was, he responded quizzically, "I think it's an obtuse triangle?" The interviewer responded with, "So it is a triangle then?" Again, seemingly frustrated, demonstrated by his sighs, the teacher candidate did not respond to this question but, instead commented, "If I get rid of the 90 degree stipulation and call a triangle a three-sided figure, then all of the shapes are triangles. But I'm still not sure about G." Despite fitting his definition of a triangle, the teacher candidate displayed uncertainty about triangle G as it did not match his concept image of a triangle, supported by his later claim that triangle G was "unfamiliar" in shape.

Task #2

In completing task #2 (see Figure 3), teacher candidates viewed the same worksheet as the one used in Task #1, but were asked to identify which shapes, if any, were right triangles. The rationale behind this question was to determine if teacher candidates would identify only those triangles "pointing right" as right triangles. All seven teacher candidates provided a correct, but in some cases, not very refined, mathematical definition of a right triangle, stating:

> *The bottom left angle which would be the one going up and sideways has a 90 degree angle.*
>
> *A right triangle has at least one 90 degree angle.*
>
> *A right triangle is 90 degrees.*
>
> *A triangle that has one right angle in the left corner.*

The one male teacher candidate defined a right triangle as "having an angle measur-

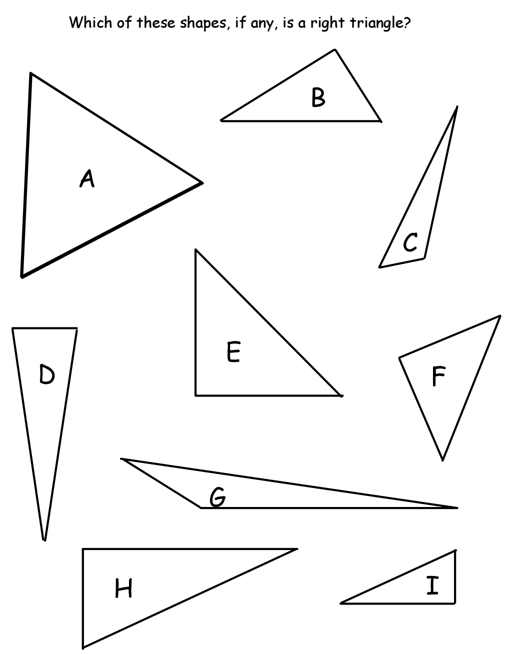

Figure 3
Task #2: Right Triangle Task

ing 90 degrees" but added the further, yet incorrect, stipulation that the "legs of the triangle are equal in length." However, after deliberating longer, he altered his definition, excluding the requirement that the legs of the triangle be congruent, and therefore he eventually provided a correct definition.

After providing a correct definition of a right triangle, six of the seven teacher candidates correctly identified all five right triangles, being triangles B, E, F, H, and I, with only one teacher candidate neglecting to identify B as a right triangle. In order to identify which triangles were examples of right triangles, as captured on the videotape, six teacher candidates repeatedly rotated their paper and then judged whether the triangle in question was a right triangle by orienting the triangle so it was gravity-based and therefore "pointing right." In other words, they oriented the triangle such that the right angle was located in the bottom left corner causing the triangle to appear to be pointing right. This orientation of a right triangle, which is generally the typical pictorial representation of a right triangle in mathematics textbooks, seemed to be the concept image held by these teacher candidates of a right triangle. Given that triangle E was in this aforementioned orientation when the paper was handed to them, the author argues that is why perhaps all of the teacher candidates first identified triangle E as an example of a right triangle before naming any of the other right triangles.

Two teacher candidates both used a similar strategy of first rotating the paper so that the triangle was gravity-based and therefore pointing right. Then they formed a right angle using their thumb and pointer finger and placed it on top of the right angle in the triangle, visually verifying its measurement was 90 degrees. While doing this, one teacher candidate commented:

> *When I look for right triangles, I look for an L shape and, growing up, when you were taught right triangles, that's exactly the first one you'd see (points to triangle E). You'd be shown, I guess, the letter L and so you would know that is 90 degrees of 360. As so that's why. This is just the traditional shape.*

This same teacher candidate continued on to say that triangle I is "a backward L. But it has the same kind of shape as the E does." The notion that right triangle, I, appeared as a backward L further substantiates this teacher's concept image of a right triangle being gravity-based.

A third teacher candidate retrieved a piece of paper from her backpack and used it as a straightedge to assist her in determining which angles were right angles. More specifically, as captured on the videotape, this teacher candidate first rotated the paper to orient the triangle to point right and then placed the corner of the retrieved piece of paper on top of each triangle's right angle as a means to verify whether the angle measured 90 degrees. However, after rotating the paper so that triangle B was pointing right, this teacher candidate did not apply her clever and rather precise technique of measuring right angles. Instead she looked at triangle B, paused a moment and stated, "It does not contain a right angle."

When completing this task, one teacher candidate first omitted triangle B from the list of right triangles but then changed her mind, stating:

> *B looks like it could be one. Actually, when I first looked at it from underneath, it doesn't look like it. Because if you look at the bottom, neither of the sides are perpendicular but if you flip it over then it does appear that there could be a 90 degree along top.*

This teacher candidate then affirmed that B was an example of a right triangle after rotating her paper in order to make triangle B gravity-based.

Thus, based on their comments and actions, it appeared as though these teacher candidates had developed a concept image of a right triangle as a gravity-based triangle that points right. This claim is supported by their rotating the paper so that the triangles' bases were horizontal and by describing right triangles as looking like "the letter L" and having an angle in the "left corner."

Task #3

In this next task (see Figure 4), teacher candidates were shown four shapes and asked which of these shapes, if any, is a hexagon. The rationale behind this question was to explore whether the teacher candidates might possess the concept image of a hexagon as being gravity-based as well as regular (having congruent angles and congruent sides), since a regular, gravity-based hexagon is the most typical pictorial representation of a hexagon in elementary level mathematics textbooks. Also, children are often exposed only to convex (all angles measuring less than 180 degrees) shapes (Reys, Lindquist, Lambdin, Smith, & Suydam, 2004).

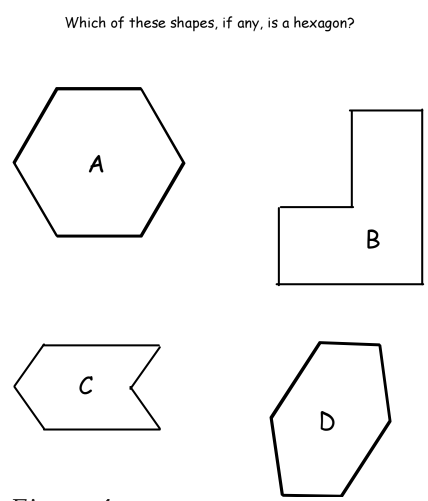

Figure 4
Task #3: Hexagon Task

Six of the seven teacher candidates correctly defined a hexagon as a "six-sided shape" and, without hesitation and with much confidence, first identified A as an example of a hexagon. Their comments included:

> *It is the basic shape you are taught in elementary school so that's why I went straight for that one (points to A).*

> *It's the most common representation of a hexagon. It's what I'm familiar with. [Hexagon] A is a traditional one.*

However, only one of the seven teacher candidates correctly responded that all four shapes were examples of hexagons, using her definition that a hexagon was "a six-sided shape."

Three teacher candidates selected A and D to be the only hexagons, all defining a hexagon as a "six-sided figure" but also stipulating that a hexagon be regular. Their comments included:

> *A hexagon has six sides that are congruent to each other.*

> *It's a six sided figure but I think all sides have to be equal, if I remember that correctly.*

> *It's got six sides and the angles have to be the same.*

These three teacher candidates appeared to use their mathematical definition of a hexagon to identify examples and nonexamples of hexagons; however, their working definition was incorrect, as a hexagon is any six-sided polygon, whether regular (congruent angles; congruent sides) or not.

Two other teacher candidates also identified hexagons A and D as the only examples of hexagons, but excluded hexagons B and C as these shapes appeared to conflict with their concept image of a hexagon:

> *I excluded these (points to B and C) almost automatically just because they didn't look like a traditional hexagon. I am not real certain how to define those because they are six-sided figures and that is the definition that I used.*

> *It's a six-sided figure (points to B), but if you think of a stop sign, that's a hexagon. But you never look at something like B and call it a hexagon, even though there are six sides to it.*

Despite defining a hexagon as a six-sided shape and acknowledging that B and C were indeed six-sided shapes after the interviewer asked them to count the sides in B and C, these teacher candidates still omitted B and C from the set of examples of hexagons. Based on their comments that B and C were not "traditional" hexagons, it appeared as though their concept image of a hexagon was that of a regular, convex, gravity-based hexagon.

Finally, one other teacher candidate, when beginning this task, admitted to not remembering what a hexagon was stating, “I know there is a rule about how many sides a hexagon has…polygon, hexagon, octagon has eight sides. I don’t remember for sure how many sides is in a hexagon.” Despite not being able to provide a verbal definition of a hexagon she, instead, relied on her concept image of a hexagon and subsequently claimed that A was an example of a hexagon but that “B and C are definitely not hexagons.” She continued:

> *It’s because it’s even length of sides, I believe, not just how many sides. For example, this side of B (points to B) is much longer than that side. Even if it had the right number of sides, B and C are not a mirror.*

Here, this teacher candidate seemed to indicate that not only was being regular a requirement for a hexagon, but also that symmetry was a component of the definition of a hexagon, based on her reference to “a mirror.” However, she excluded C as an example of a hexagon even though it is symmetric. When the interviewer confronted her with this, she admitted, “Maybe the sides don’t have to match.” After struggling with this question, the teacher candidate admitted, “I am trying to remember a more specific definition but it has been too long.” She then concluded that A and D were hexagons because a hexagon “looks something like A.” Thus, it appeared as though this teacher candidate’s concept image of a hexagon was limited to that of a regular, gravity-based convex hexagon.

Results

In task #1, although all seven teacher candidates defined a triangle as a three-sided shape, only four correctly identified all of the examples of triangles. Two teacher candidates excluded triangle G from the set of examples of triangles, calling its shape “unfamiliar” and stating its appearance was “so long and stretched.” Thus, these two teacher candidates relied on their concept image of a triangle, which appeared to exclude obtuse scalene triangles, to determine examples and nonexamples of triangles. Another teacher candidate indicated that shapes C and G, although triangles, looked atypical. Here this teacher candidate relied on her definition to identify examples of triangles but still demonstrated a reliance on her concept image of a triangle, which also seemed to lack examples of scalene triangles that were also obtuse.

In task #2, all seven teacher candidates provided a correct working definition of a right triangle and six correctly identified all of the examples of right triangles. Given that six of the teacher candidates rotated the paper when completing this task, orienting it such that perpendicular sides of triangles were in the vertical-horizontal position, it appeared as though these teacher candidates' concept image of a right triangle was limited to a gravity-based triangle pointing right.

In task #3, six of the seven teacher candidates defined a hexagon as a six-sided shape, yet only one teacher candidate correctly identified all of the examples of hexagons. Three of the teacher candidates, who defined a hexagon as a six-sided shape, also verbalized the additional (but incorrect) stipulation that a hexagon be regular, using such terms as "congruent" and "equal" sides or angles. These three teacher candidates then used their definition of a hexagon to identify examples and nonexamples. Two other teacher candidates, although correctly defining a hexagon as a six-sided shape, instead deferred to their concept image of a hexagon, which was that of a regular, convex, gravity-based hexagon, to determine examples and nonexamples. One other teacher candidate admitted to not recalling the definition of a hexagon and thus relied on her concept image of a hexagon, which included only regular, convex, gravity-based hexagons, to assist her in this task. Additionally, one teacher candidate referred to a hexagon as a stop sign when, in fact, a stop sign is an example of an octagon.

In completing all three tasks, it was noted that the mathematical definitions provided by the teacher candidates were, in most cases, not very refined. For example, all of the teacher candidates defined a triangle as a three-sided *shape*; however, a three-sided *polygon* is a more precise definition, as a three-sided shape is not necessarily a triangle (see Figure 5). The same argument and set of nonexamples holds for the teacher candidates' definition of a hexagon as a six-sided shape, as opposed to a six-sided polygon. Similarly, in task #2, it was clear to the interviewer and author that the teacher candidates knew what a right triangle was, although they did not articulate the definition very precisely. For example, a more precise mathematical definition of a right triangle would be, "A triangle with one right angle" in comparison to a definition given by one of the teacher candidates as, "A right triangle is 90 degrees."

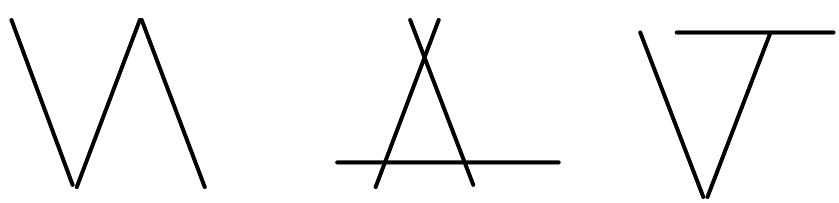

Figure 5
Examples of Three-Sided Shapes That Are Not Triangles

Conclusions

The data collected in this portion of the study indicated that the concept images of polygons held by these teacher candidates were limited in scope. In particular, the collection of concept images of triangles for three of the preservice teachers lacked examples of obtuse scalene triangles. The findings from Task #2 indicate that these preservice teachers' collections of concept images for right triangles were comprised of gravity-based right triangles pointing right, indicated by their responses and by the repeated turning of their worksheets. Thus, like the participants in other studies (Burger & Shaughnessy, 1986; Clements et al., 1998; Fisher, 1978; Fuys, et al., 1988; Hershkowitz, 1987; Zykova, 1969), it appeared as though these teacher candidates also had limited ideas; that is, concept images, of triangles. In regard to hexagons, it appeared as though the collection of concept images of hexagons for six of the seven preservice teachers was limited to those hexagons that were regular, convex, and gravity-based. Thus, as demonstrated by subjects in other studies (Burger & Shaughnessy, 1986; Fisher, 1978; Fuys et al., 1988; Zykova, 1969), these preservice teachers also considered inessential but common features (e.g., regular, convex, gravity-based, or symmetric) as essential to the concept.

Additionally, in all three tasks, there were instances when individual teacher candidates provided a correct mathematical definition of the shape in question but had difficulty applying their verbal description because their concept images, which were limited in scope to specific exemplars, were tied so closely to that concept. Other researchers (Clements & Battista, 1989; Hershkowitz et. al, 1990; and Vinner & Hershkowitz, 1980) reported similar findings.

Limitations of the Study

In analyzing the data for Task #3, it would be worthwhile to include an example of a regular hexagon that is not gravity-based (see Figure 6), to see if teacher candidates would identify this shape as an example of a hexagon. If they chose *not* to include such a shape, this would potentially provide more definitive evidence that their concept image of a hexagon is one that is not only regular, but also gravity-based. Although such a shape was not included, the author still asserts that being gravity-based was one component of the teacher candidates'

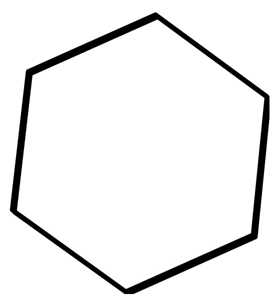

Figure 6
An Example of a Regular Hexagon That Is Not Gravity-Based

concept images of a hexagon, based on their dialog and responses during the interview.

Additionally, interesting data could potentially be generated if teacher candidates were asked not to *identify*, but instead, to *draw* examples of triangles (or hexagons, etc.) to see what pictorial representations they create and the order in which they create them. This collection of pictorial representations would, in fact, be the concept images they possessed of these polygons.

One other potential limitation to this study was the small sample size. Increasing the number of participants may yield more insight into the concept images held by K-8 preservice teachers.

Implications and Recommendations

As early as kindergarten, young children are introduced to the notion of shape and, by the end of first grade, students gain more familiarity with many fundamental polygons such as triangles, squares, and rectangles. Students first learn to recognize a shape by its appearance as a whole (van Hiele, 1986) or through such characteristics as “pointiness” (Lehrer, Jenkins, & Osana, 1998). Too often, when a polygon is introduced to a student for the first time, the student views a regular, convex, gravity-based figure. Students frequently do not recognize properties of shapes, as standard elementary curricula focus on recognizing and naming geometric shapes and using formulas in geometric measurement (Porter, 1989; Thomas, 1982). Consequently, students develop concept images of polygons, which tend to be limited in scope in terms of examples and nonexamples, but which tend to be more powerful than the mathematical definitions and set of properties that accompanying them. This was certainly evident in this study.

Ball (1990a, 1990b) argues that preservice teachers do not possess an understanding of the principles underlying mathematical procedures adequate for teaching and that their knowledge of mathematics is not sufficiently connected. She advocates that subject matter knowledge be a central focus of teacher education programs and that much more knowledge is needed about how teachers can be helped to increase and develop their understandings of mathematics in order to teach mathematics effectively. In the case of these seven preservice teachers the author contends that, based on their limited collections of concept images of triangles and hexagons, as well as their poorly refined definitions of certain mathematical terminology, they do not possess the understanding of mathematics nor the connected knowledge that Ball (1990a, 1990b) articulates in order to teach effectively.

Several researchers share the view that teaching is strongly influenced by a teacher's personal experiences as a learner (Zaslavsky, 1995; Stigler & Hiebert, 1999). Ma (1999) posits that "the quality of teacher subject matter knowledge directly affects student learning" (p 144). In the case of these preservice teachers, their personal experiences as learners resulted in them developing a narrow collection of concept images of particular polygons. The author questions whether the K-12 teachers of these preservice teachers lacked the deep, connected understanding of mathematics articulated by Ball (1990a, 1990b), resulting in these teachers imparting limited exemplars of polygons to the teacher candidates. Unless the preservice teachers in this study receive some type of mathematical "intervention" perhaps during their mathematics methods course or student teaching experience that would broaden their repertoire of concept images of polygons, the author asserts that this same limited collection of concept images would again be imparted to their future students once these teacher candidates enter the classroom as certified K-8 educators.

Mathematics educators, namely, those teaching mathematics content courses, need to break this cycle. In order to broaden the scope of students' concept images, students should view polygons that are not always "traditional" in appearance; i.e., regular, convex, and gravity-based. By allowing students to experience polygons that are irregular, asymmetric, convex as well as concave, and those that are not necessarily gravity-based, students will develop a more well-rounded concept image of a polygon and, more importantly, one that is in harmony with the mathematical definition. Additionally, instructors of mathematics content courses should encourage students to verbalize and describe their collection of concept images of polygons and provide their definitions as well, as a means to assess the breadth and depth of their conceptions, to clarify misconceptions, and to then assist students in developing a deep, connected understanding of concepts.

In its *Principles and Standards for School Mathematics* (2000), NCTM remarks that "Students need to see many examples of shapes that correspond to the same geometrical concept as well as a variety of shapes that are nonexamples of the concept" (p. 98). Providing activities where students apply transformations such as flips, slides, and turns will assist them in creating mental images; that is concept images, of shapes in various orientations. Clements et al. (1998) recommend using technology; in particular, Logo, to facilitate student learning about geometric figures and their properties, while Petty and Jansson (1987) advocate using a rational sequence of examples and nonexamples to facilitate concept attainment.

A primary goal of a mathematics content course is to provide K-8 teacher candidates with a strong foundation in mathematics. Given that a majority of mathematics content courses focus on the study of Euclidean geometry, instructors of these courses need to explore their teacher candidates' concept images of polygons and expand their repertoire of concept images, as a means to build more and stronger connections between mathematical definitions and teacher candidates' concept images. This recommendation not only applies to the study of polygons, but to all K-8 topics taught in a mathematics content course. In summary, mathematics educators need to provide teacher candidates with experiences that will enable them to develop that strong, connected foundational knowledge needed to teach K-8 mathematics. By providing such experiences it is hoped that the cycle can be broken.

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