LONGER IS LARGER — *Or is it?*

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outlines some student misconceptions about decimal fractions, examines some tasks and suggests teaching strategies.

Why might a child have no trouble ordering this first set of decimals but have difficulty with the second set?

First set	0.4	0.9	0.3	0.7	
Second set	0.4	0.9	0.3	0.7	0.10

Results from a large scale study of students' misconceptions of decimal notation (Steinle & Stacey, 2004) indicate many students treat decimals as another whole number to the right of the decimal point. This "whole number thinking" leads some students to believe, in the context of comparing decimals, that "longer is larger"; e.g., 0.45 is larger than 0.8 because 0.45 has more digits. This misconception (one of several major misconceptions) appears to be the most prevalent and is likely to be persistent beyond Year 10.

These results and results from a research project (Roche & Clarke, 2004) indicate some students misunderstand the decimal numeration system and that some students use a rule to compare decimals possibly to the detriment of their conceptual understanding. This rule provides a quick fix to students unable to compare decimals accurately while continuing to encourage the "whole number thinking" misconception. Also, students who are not dependent on this rule (and successful in a decimal comparison task) are more likely to be able to solve more difficult tasks involving the relative size of decimals.

A research project

In 2004, I interviewed 48 students from Years 3 to 6 using a range of tasks, where the mathematical focus was decimal knowledge and understanding. During the analyses of these interviews I was able to follow the progress of students who were successful on a decimal comparison task. The task included nine decimal pairs, and the students were asked to say which was larger and why. Patterns of errors suggested some students held the misconceptions outlined by Steinle and Stacey (1998). The decimal comparison task was implemented in an interview situation rather than a pencil and paper test, and I was able to identify two strategies used by students who achieved no more than one error on the decimal comparison task.

Strategy 1. Some students used fractional language and benchmarking strategies to compare the decimals. For example: "0.567 is greater than 0.3 because five tenths is greater than three tenths, or 0.567 is more than a half and 0.3 is less than a half", or "0.87 is greater than 0.087 because 87 hundredths is greater than 87 thousandths".

Strategy 2. Other students used a rule by which zeros are added to the shorter decimal to equalise the length of the two decimals ("annexing zeros") and then the decimals were compared as whole numbers. For example: "0.37 is greater than 0.217 because 370 is greater than 217.

Stacey and Steinle (2004) categorised students who had few errors as "apparent experts", stating that this coding "does not necessarily imply that a student is truly an expert with respect to decimal understanding. [Also] students who can accurately follow correct (or nearly correct) procedures for comparing decimals will score highly, whether or not they understand why those procedures work" (pp. 541–542).

Results from my interviews indicated that students who might be catergorised as "apparent experts" but who used the rule to extend uneven decimals (strategy 2), were unable to perform successfully on two other tasks related to the relative size of decimals. These tasks involved ordering a set of 12 decimals (see Figure 1) and a benchmarking task (see Figure 2).

Twelve number cards are arranged randomly in front of the student who is asked to order them from smallest to largest.

0	0.01	0.10	.356	0.9	1	1.2	1.7	2	1.70	1.05	.10

Figure 1. A task involving ordering a set of twelve decimal numbers.

The most common error on the task in Figure 1 was placing 0.9 before 0.10. In fact, 13 out of 16 fifth-graders and 4 out of 6 sixth-graders ordered the set with what they may believe as "point nine before point ten", indicating the decimals were being treated as whole numbers. No "apparent experts" who used the rule to extend decimals (strategy 2) were successful on this task.

The benchmarking task (see Figure 2) also proved difficult for students who used strategy 2 to compare the relative size of decimals.

In this task the student is presented with two cards and is asked which of these numbers (pointing to the string of numbers) is closest to this (pointing to 0.18).

0.18	0.1	0.2	17	0.15	2

Figure 2. A task involving benchmarking the size of decimals.

Common errors for this task were:

- "Seventeen because seventeen is closest to eighteen"; and
- "0.15 because zero point fifteen is closest to zero point eighteen"; indicating the students were reading and treating the decimals as whole numbers.

While few students achieved success on these tasks, those who did were students who used fractional language and benchmarking strategies in the decimal comparison task (see Strategy 1).

If Strategy 2 (annexing zeros) is used to the detriment of the students' understanding about the relative size of decimals, what key ideas might be of benefit to teachers who aim to promote and assess a conceptual understanding of decimals?

The importance of ragged decimals

I found comparing, ordering and benchmarking tasks to be useful for assessing a student's understanding of the relative size of decimals. However, if these tasks and classroom activities only include equal length decimals, then students are able to solve them using whole number thinking, thus possibly hiding a misconception.

While a task like that in the first set, at the start of the paper, may appear suitable for a Year 4 class, it can successfully be solved without an understanding of the value of these numbers, by ordering them as 3, 4, 7 and 9. Success using this strategy may affirm a students' misconception that decimals work like whole numbers. Unlike decimals of equal length, "ragged" decimals (i.e., decimals of unequal length) require the students to confront the place value structure of the decimal system.

Simply changing the task slightly (by adding 0.10 to the set, as in the second set at the start of the paper) may uncover more about a student's understanding or misunderstanding about decimals and may challenge their misconceptions.

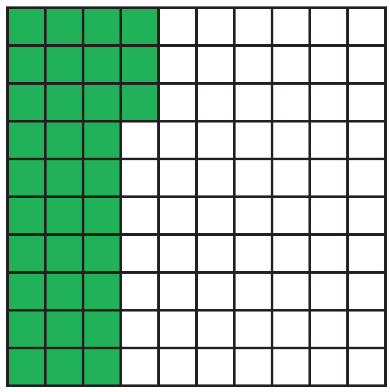
The resultant errors may provide an opportunity for the class to discuss the place value of each digit. Also, students could be required to show evidence of proof by comparing representations of these numbers.

The importance of representations

While tasks that require students to interpret certain representations of decimals or shade pre-divided regions may be common place, these tasks, if they stand alone, may also affirm "whole number thinking" for decimals.

When students are taught to identify decimals/fractions by counting shaded units (as the numerator) and counting shaded and unshaded units (as the denominator) the task becomes tied to activities of counting and matching (Carraher, 1993). Both tasks shown in Figure 3 can be achieved by a whole number counting strategy and are relatively easy (Brown, 1981). Neither task challenges a student's misconception and may affirm that

If the largest square is 1, what decimal does the shaded part show?



or

Shade in four tenths(or 0.4).

Figure 3. Tasks involving interpreting decimals from representations.

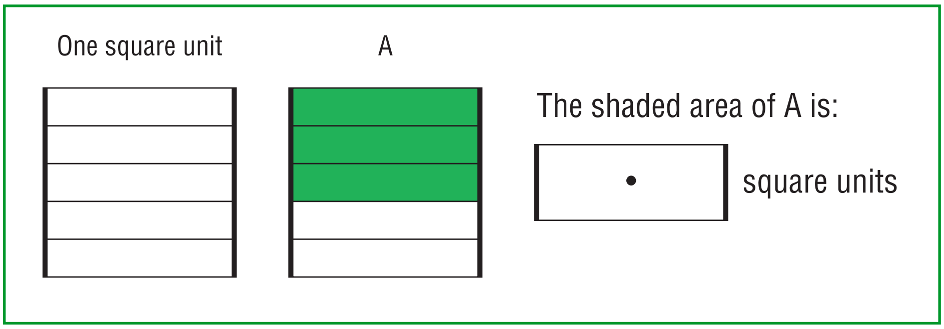


Figure 4. Swan's written task for interpreting decimals with perceptual distracters.

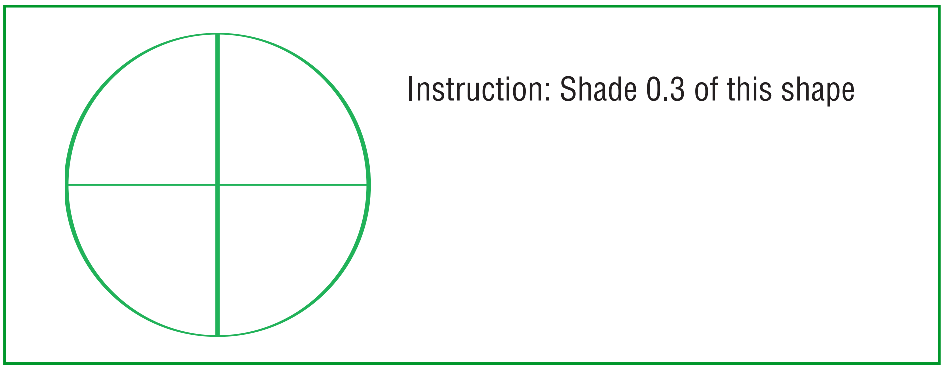


Figure 5. A task involving interpreting decimals.

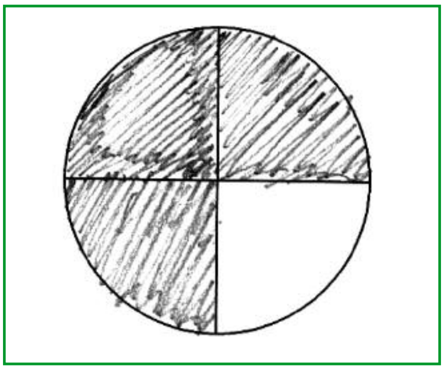


Figure 6. Some results of the task shown in Figure 5.

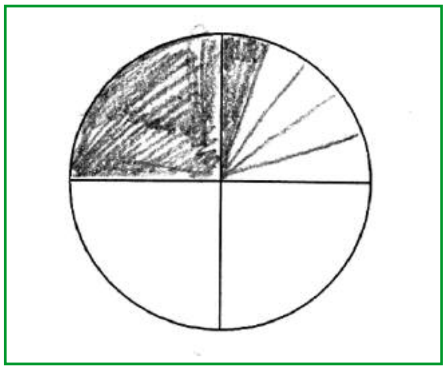


Figure 7. Violet's solution for the task in Figure 5.

decimals work like whole numbers. Some studies (Behr & Post, 1981; McIntosh, Reys, Reys, Bana & Farrell, 1997; Moss & Case, 2002; Swan, 1983) used items with perceptual distracters (visual information not consistent with the task) to determine whether the student could interpret decimals when the connection to the base ten system was not explicit.

An example of a task with a perceptual distracter is shown in Figure 4 (Swan, 1983, p.16).

The task shown in Figure 5 was given to some students as part of my research project, and while it is more difficult than regions that are divided into factors or multiples of the denominator, results confirm that many students appear to use whole number counting strategies in an attempt to solve this. On the other hand, it provided an insight into a student that demonstrated a flexibility between fractions and decimals and an appropriate understanding of the size of 0.3 (see Violet's solution, discussed later).

Nine out of fifteen students given this task shaded three quarters (see Figure 6), indicating they possibly viewed this task as a counting task and that possibly 0.3 represented "three", not "three tenths".

As shown in Figure 7, Violet a grade 5 student, indicated that the quarter shaded is 0.25, and one fifth of 0.25 is five hundredths. Therefore, twenty-five hundredths and five hundredths is equal to three tenths or thirty hundredths (0.25 + 0.05 = 0.3 or 0.30).

When assessing a student's understanding of the decimal notation system, representations with perceptual distracters seem useful. However, when teaching, it may be beneficial to use representations as a method of proof or self-checking of task solutions. In this way, representations are a means to a solution rather than a static task of interpreting a number. The *decimat* (Wright, 2004) or Linear Arithmetic Blocks (LAB, Helme & Stacey, 2000) may provide a useful alternative to the hundred square. Used in conjunction with ordering and benchmarking tasks, the decimat and LAB allows the student to "see" the place value of each digit in the decimal.

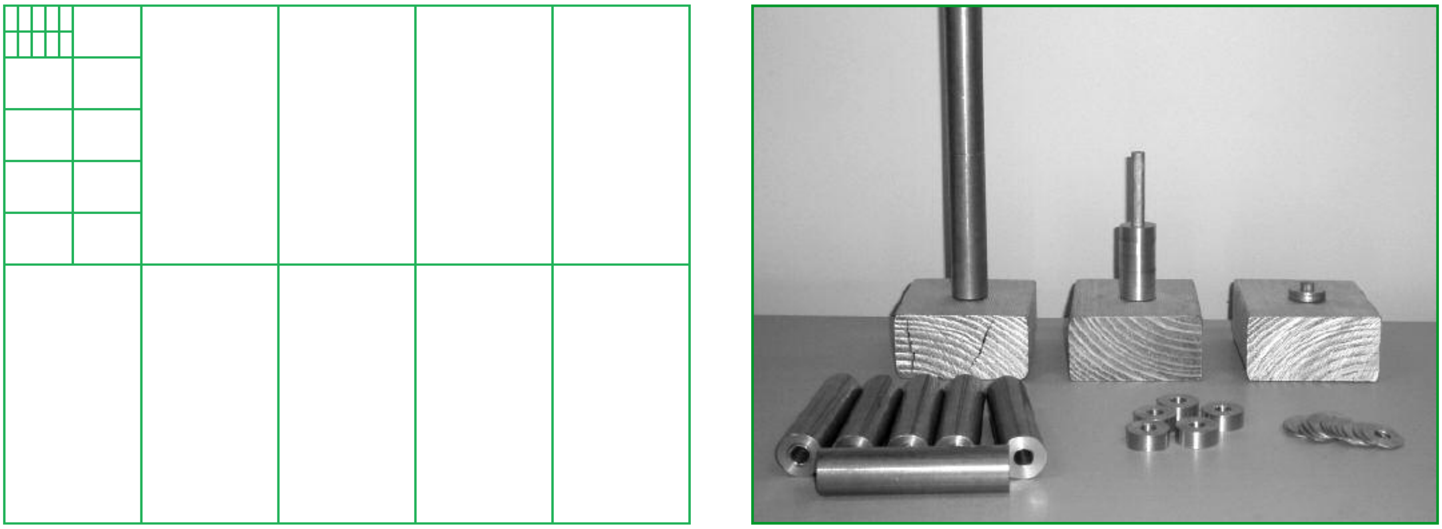


Figure 8. Representations for decimals: (a) a decimat (b) LAB.

Here are some number cards and some blanks that could be any number. Could you use these cards to show me what "two tenths" looks like?

.	0	2	7	1

Continue with these numbers
(a) two tenths
(b) seven hundredths
(c) 27 hundredths
(d) 2 thousandths
(e) 702 thousandths
(f) ten tenths
(g) 27 tenths
(h) one and 17 tenths
(i) 712 hundredths

Figure 9. A task involving "writing" a decimal fraction.

The importance of fractional language

When considering the results of the decimal comparison task from an interview, we might conclude that the ability to describe decimals using fractional language may be an indication of a more conceptual understanding about decimals. In order to determine whether the students who participated in my research project could write a decimal given the decimal verbally in fractional language, I asked the question shown in Figure 9.

Table 1 shows the results collected from the task shown in Figure 9. The fraction represents the proportion of correct responses (e.g., 12/18 means 12 out of 18 were correct responses). The last row shows the most common errors.

Two-thirds of fifth graders and most sixth graders could successfully record decimals in the tenths and hundredths. Most sixth graders could also record decimals in the thousandths, but examples that required regrouping and renaming (e.g., 27 tenths and one and 17 tenths) were not surprisingly more difficult, with just under one half of fifth and sixth graders being unsuccessful at writing "ten tenths" as a decimal.

The most common errors indicate that these students were not familiar with which place represents the value stated and appeared to "write" the decimal number using whole number knowledge (e.g., seven hundredths is .700) or were simply converting given numbers from one form to another (e.g., two tenths is 2.10 and ten tenths is 0.10).

Conclusion

The research discussed in this paper indicates the importance of identifying children who develop "whole number" thinking with respect to decimals and to make sure that the learning opportuni-

Table 1. Results for "Writing" decimal fractions

Task No.	a	b	c	d	e	f	g	h	i
Correct response	**0.2**	**0.07**	**0.27**	**0.002**	**0.702**	**1.0**	**2.7**	**2.7**	**7.12**
Grade 3	1/6	0/2	*	*	*	*	*	*	*
Grade 4	3/9	2/7	0/4	*	*	*	*	*	*
Grade 5	12/18	12/18	12/18	6/16	3/10	7/13	4/6	1/5	1/2
Grade 6	8/9	8/9	6/9	8/9	6/7	5/8	2/7	2/2	2/3
Most common error(s)	2.10	0.700 7.10 7.100	.027 27.00	.2000 2.000	0.7002	0.10	0.27	1.17	70.12 .712

* This question was not given to the students.

ties we provided do not in fact reinforce this misconception.

Implications for teachers that have emerged from this research include the following:

- Avoid rules and tasks that encourage whole number thinking, such as adding zeros to compare unequal length decimals and using equal length decimals in classroom activities.
- Encourage the use of fractional language to describe decimals (i.e., 2.75 is "2 and 75 hundredths" rather than "2 point 75").
- Use representations such as the decimat or LAB in conjunction with comparing, ordering and benchmarking tasks to provide a self checking strategy to solutions.

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