To build a dog run

The relationship between perimeter and area

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explores the relationship between perimeter and area with students from Grades 4 to 6.

Perimeter and area are important real-world concepts in their own right, and their relationship certainly is no less important. This relationship can be intuitively challenging for both children and adults. For example, it is not uncommon for students and even adults to believe — at least at first thought — that a fixed perimeter, such as a given amount of concrete curbing, yields the same area no matter how you shape it into a closed figure.

In this article, I describe students' attempts to make sense of this concept through explorations with colour tiles and subsequent decisions about what dimensions they would give a rectangular dog run (pen) with a specified amount of fencing. The students who did this activity were in a multi-age classroom that housed Grades 4 to 6. Preparation for this activity includes helping students achieve conceptual understanding of perimeter and area, including how to find each in units or square units and being able to provide real-world examples of their use (e.g., framing a picture or painting a wall).

In my lesson, I first asked students if they thought the area of a shape would always be the same no matter how I arranged a fixed amount of perimeter. In other words, if I had a specific amount of fence to make a garden, would the garden's amount of space — its area — be the same no matter what type of rectangle I arranged the fence into? I took students' answers, which varied, without evaluation and told them we would be doing some activities to help them find out if their answer was right.

On the overhead projector, I used colour tiles to have the class guide me in creating rectangles with a perimeter of 8 units, where the side of one colour tile was considered to have a length of 1 unit. The students had colour tiles at their desks so they could create rectangles before reporting them to me. As expected, the discussion arose as to whether or not a square is a rectangle, and we confirmed that it is by recalling the definition of a rectangle and comparing a square to it (this should be prior knowledge).

As a class, we determined that only two rectangles could be made with a perimeter of 8 units: a 1 × 3 and a 2 × 2. We discussed the "one-by-three" terminology as an example of one way to name rectangles, as well as the meaning of width, length, and dimension as they pertain to rectangles. (One potential difficulty here is mistaking a 2 × 2, for example, as having a perimeter of 4. This might be addressed explicitly.) With both rectangles still displayed on the overhead, I asked students to name the area for each. The areas of 3 and 4 square units showed students that the shape of different rectangles does yield different areas for a fixed perimeter. We completed the Perimeter-Area Recording Sheet (Figure 1) on a transparency on the overhead.

After this example was finished, students were ready to explore the concept on their own. They were asked to complete Perimeter-Area Recording Sheets in a manner similar to that shown in Figure 1 for perimeters of 12 and then 20 units. They worked collaboratively using colour tiles. After drawing the rectangles and filling in the chart for each established perimeter in turn (see student example in Figure 2 for a rectangle of perimeter 20 units), the students were asked to tell which rectangle gave the largest area for that perimeter and which gave the smallest. They were also asked to look for patterns and describe any they noticed. At this point some students saw patterns and relationships and some did not.

José wrote, "I saw that the skiner [skinnier] rectangles have less [area]."

Perimeter–Area Recording Sheet

Perimeter of Rectangle = _____

Drawings of All Possible Rectangles

Dimensions and Area for All Possible Rectangles		
Length (units)	Width (units)	Area (square units)

Figure 1. The perimeter-area recording sheet.

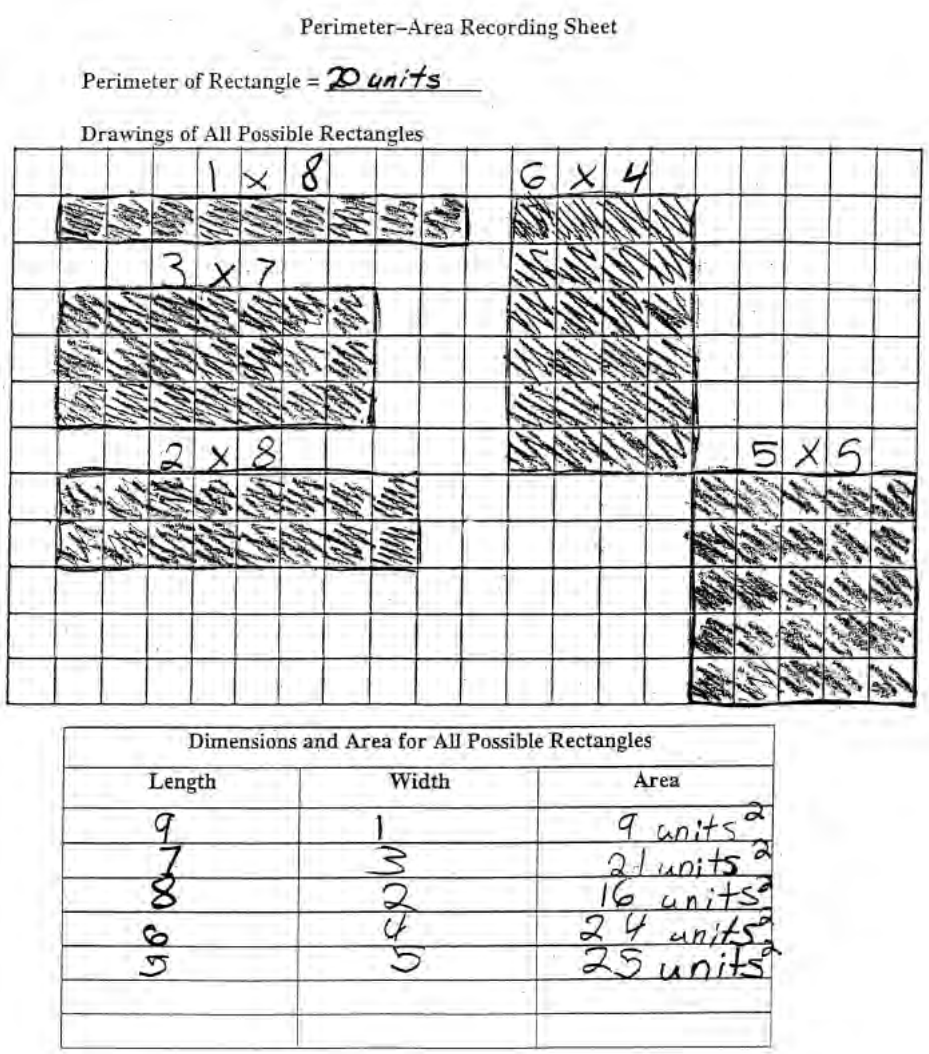


Figure 2. One student's perimeter-area recording sheet for a perimeter of 20 units.

Angela wrote, "I think the biger [bigger] fat ones have more area."

Chantel put it all together with, "The bigger, fat rectangles have more square units than the smaller, skinnier ones."

No students noticed that a rectangle's length plus width — the two dimensions recorded on the chart — equals half the perimeter (it would be worthwhile to discuss why this is true), but some students began to border on similar concepts. Beatriz wrote, "I noticed that if you add the length and the width it equals 10" [for all rectangles of perimeter 20 units]. Bernadette said for a rectangle of perimeter 12 units, you start at 1 and "have to stop" at 6 units and that the same idea was true for 20 units, stopping at 10 units. It seemed for her that "stopping at" meant you could go up to but not include the number that was half of the perimeter when listing rectangle dimensions. Kenzi said, "Every time you take a unit away from the length the width gets bigger," and Chantel made the particularly astute comment that "the less number difference between the 2 dimensions, the bigger the area will be". The square yields the greatest area for a given perimeter (when forming a rectangle), and Chantel is indeed right that the closer two adjacent sides are in length, the greater the area will be; the reverse is also true.

Next, the students were asked to apply their learning to the following problem, given access to colour tiles and graph paper:

> You have 28 metres of fencing to build a dog run in your backyard for your cocker spaniel, which is a medium-sized dog. Each fence piece is one metre, so each side of the dog run will be in whole metres. You have decided to use a rectangular shape that will stand by itself under some trees (not along any fence or building). Use any method you like to find all possible rectangular dog runs you could build. Draw them on a piece of graph paper.

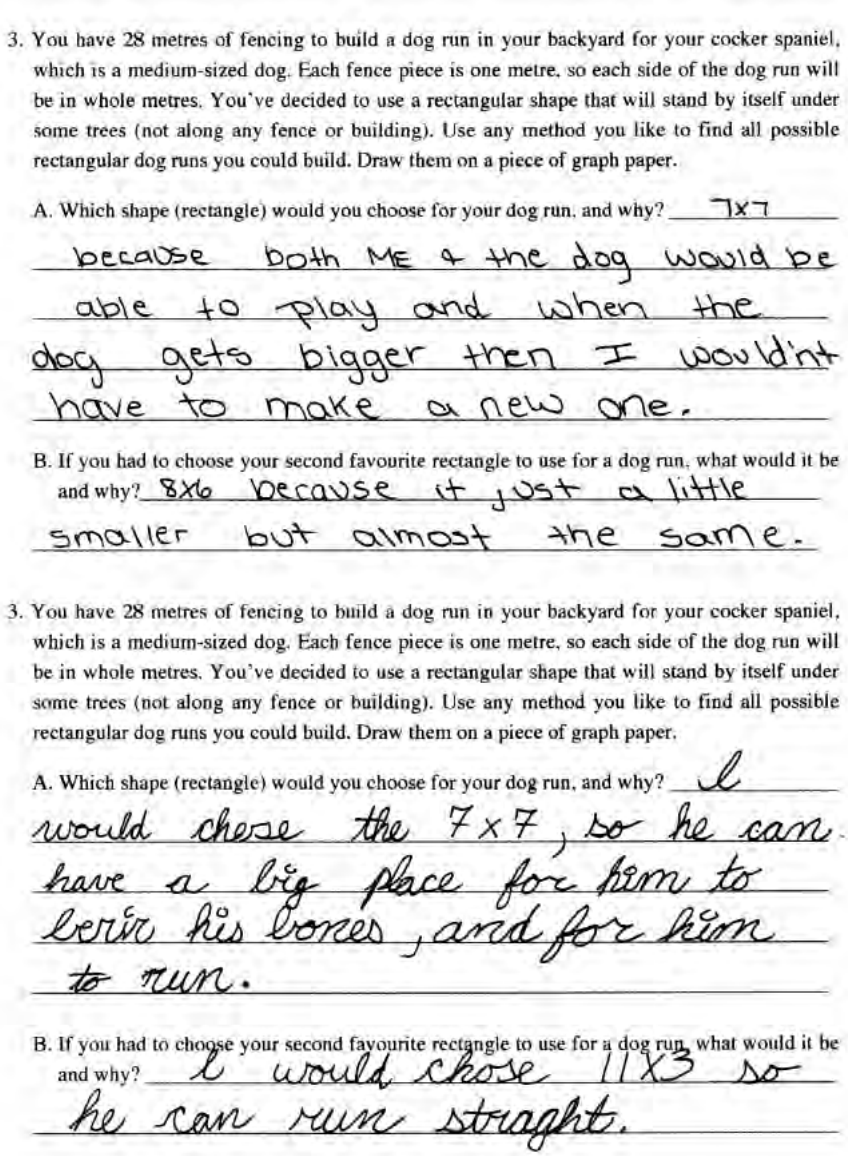
After drawing all possible rectangles on a piece of graph paper, the students answered the following questions on a worksheet:

- Which shape (rectangle) would you choose for your dog run, and why?
- Which rectangle would be your second favourite for a dog run, and why?
- If you had a black labrador (a large dog), which rectangle would you choose? Tell why.
- If you had a terrier (a small dog), which rectangle would you choose? Tell why.

The students had some good real-world discussions at this time, which formed the basis for their decisions. Some student responses to these questions are shown in Figure 3. Most students not only seemed to gain greater conceptual understanding of perimeter and area, but they also seemed to understand the relationship of these two and what that means in a real-world situation. Possible dog runs for 28 metres of fencing might be sketched on the playground to get a real-scale visual of the students' explorations.

With more time, the students could extend these concepts. Some or all of the following challenge questions might be posed:

- If you could use part of your house or fence as one side of your dog run (up to 20 metres), what dimensions would the other three sides be, and why (still using 28 metres of fencing)?



3. You have 28 metres of fencing to build a dog run in your backyard for your cocker spaniel, which is a medium-sized dog. Each fence piece is one metre, so each side of the dog run will be in whole metres. You've decided to use a rectangular shape that will stand by itself under some trees (not along any fence or building). Use any method you like to find all possible rectangular dog runs you could build. Draw them on a piece of graph paper.

A. Which shape (rectangle) would you choose for your dog run, and why? 7x7 because both ME & the dog would be able to play and when the dog gets bigger then I wouldn't have to make a new one.

B. If you had to choose your second favourite rectangle to use for a dog run, what would it be and why? 8x6 because it just a little smaller but almost the same.

3. You have 28 metres of fencing to build a dog run in your backyard for your cocker spaniel, which is a medium-sized dog. Each fence piece is one metre, so each side of the dog run will be in whole metres. You've decided to use a rectangular shape that will stand by itself under some trees (not along any fence or building). Use any method you like to find all possible rectangular dog runs you could build. Draw them on a piece of graph paper.

A. Which shape (rectangle) would you choose for your dog run, and why? I would chose the 7x7, so he can have a big place for him to berir his bones, and for him to run.

B. If you had to choose your second favourite rectangle to use for a dog run, what would it be and why? I would chose 11X3 so he can run straght.

Figure 3. Two students' rectangle choices for their dog run.

- If you could use part of your house and fence to form two sides of the dog run (up to 15 metres each), what dimensions would the other two sides be, and why (still using 28 metres of fencing)?
- If you could make any shape dog run you wanted from your fencing (a free-standing dog run without using part of the house or fence as sides), what would it look like and why? (Include a drawing on graph paper.) How much area does this dog run have?
- Do you think any shape has more area for a specific perimeter than a square? If so, what do you think it is, and why?
- True or false: doubling the length of one side of a rectangle will double the area. Tell why you think your answer is correct (draw a picture to help, if needed).
- True- or false: doubling two adjacent sides of a rectangle will double the area. Tell why you think your answer is correct (draw a picture to help, if needed).
  [The term adjacent will need to be reviewed with students prior to answering this item.]

This activity lends itself to numerous adaptations and variations. Other challenging explorations might include having the side of each colour tile or grid square equal 2 or 3 metres, or using fixed perimeters that are not multiples of four (i.e., rendering squares impossible when whole units are used). Students might be asked to do similar activities using increasingly abstract visual representations. Geoboards and dot paper, for example, do not show internal lines marking off square units, and shapes with neither internal lines nor pins/dots are even more abstract. In order to extend the real-life applications, students might be challenged to find ways that knowing perimeter-area relationships may be useful to them or adults in the real world.

Another important way that this concept can be extended is to reconsider the initial decisions for the rectangular dog run by assuming the shape does not have to be a polygon (i.e., have straight sides) and that the fencing is bendable and may be curved. In this case, the shape that yields the greatest amount of area for a fixed perimeter is a circle. Students might be shown a picture of a round house that may be found in some regions, particularly those with limited resources, and ask what potential benefits such a design affords. A good activity that explores this concept may be found in Chapter 8 of *The Multicultural Math Classroom: Bringing in the World* (Zaslavsky, 1996). The study of perimeter-area relationships using various types of shapes not only encourages important mathematical explorations, but it also lends itself well to authentic applications beyond the classroom.

Reference

Zaslavsky, C. (1996). *The Multicultural Math Classroom: Bringing in the World.* Portsmouth: Heinemann.

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