APMC

HOT ideas

The legs problem — for all ages

Hot Ideas for this edition provided by Jenni Way, University of Western Sydney.

This is an example of a versatile multi-solution problem that can be used right across the primary years. The problem itself does not need to change — just our expectations for the solutions strategies, the sophistication of the recorded mathematics and completeness of the solutions.

The basic problem

Noah saw 16 legs go past him into the Ark. How many creatures did he see?

Variations

Any even number can be used, although, 2 legs allows only one answer and with 16 legs there are already 14 different solutions, so be cautious about using large numbers.

The context can, of course, be changed to something else, such as a farmer watching farm animals or a zoo keeper, or someone on safari and so on.

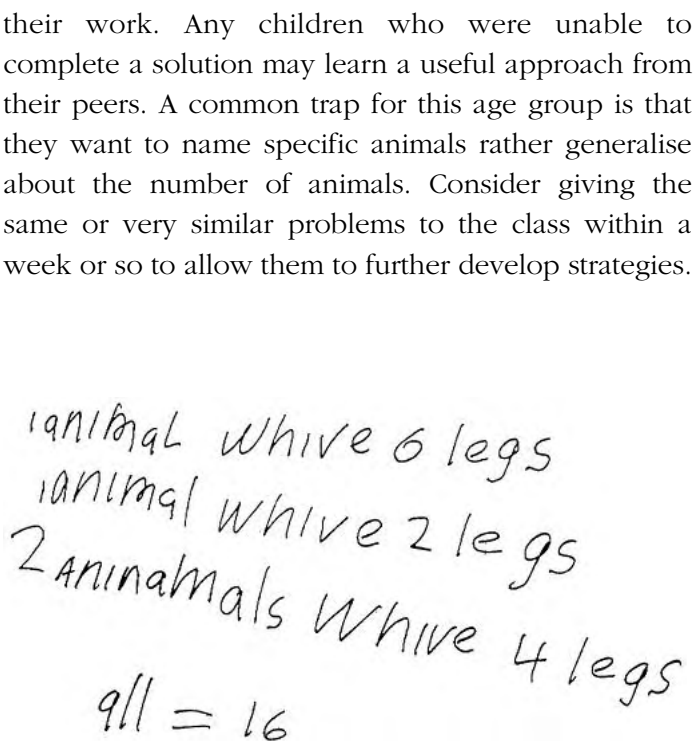
Which animals? How many legs?

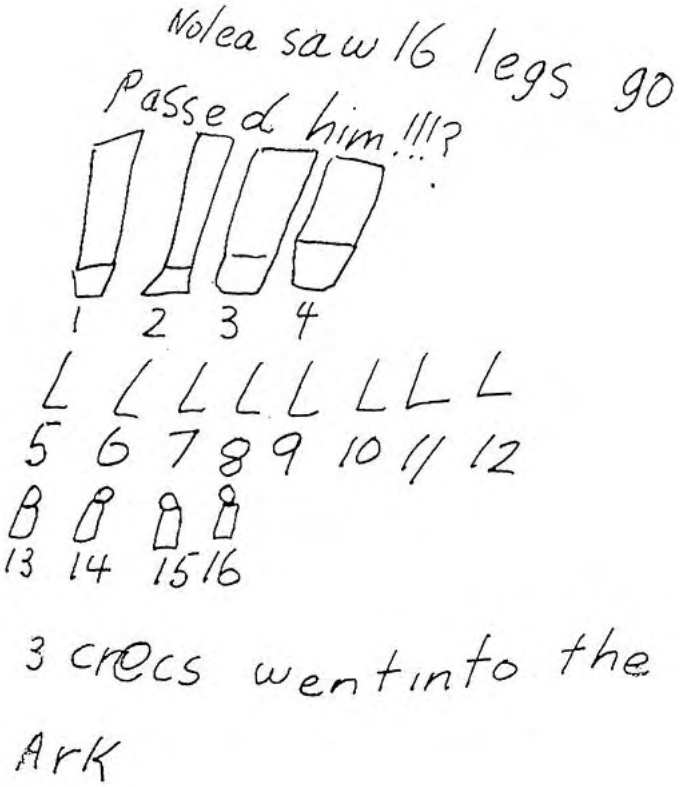
With all age groups, encourage discussion around the interpretation of the problem and of the range of possibilities. Take care to stay focussed on 'how many' creatures, not 'what' the creatures actually are. It is the groupings of legs that are of primary concern, not the animals that own them: groups of 2 legs (e.g., birds), 4 legs, 6 legs (insects) and 8 legs (spiders). Be prepared for a little zoological research (the Internet makes this easy) as children ask questions about creatures such as crabs and shrimp (crustaceans with 10 legs, and apparently barnacles have 12). Depending on the context presented in the problem, the class might decide that aquatic animals cannot be used in the solution. Usually the matter of centipedes and millipedes (arthropods) is raised and they have from 30 to over 300 legs, which probably makes them irrelevant. Often children will also suggest using creatures with no legs (e.g., snakes, fish) but most decide against it when they realise the implications for the number of solutions. Try to leave such decisions to the children themselves, as interpreting the context and setting parameters is an important part of the problem solving process.

Early years

Introduce the problem context through a simple story. Some plastic animals or felt-board animals could be used to help the children comprehend the situation and explore some possibilities. Hide two or three of these animals behind something, with just their legs showing. Ask the children to count the legs and talk about how many animals they belong to. Model the same example with some counters or sticks to represent the legs. Then pose the problem you want them to solve, perhaps limiting the number of legs to 8, 10 or 12. Encourage drawing and writing about the solution.

Figures 1 and 2 show the solutions from some 5–6 year olds. Notice how they have used a combination of words, invented symbols and numerals to express their thinking. It is well worth making time to share and discuss solutions so the children accept that there is more than one solution and many ways to record their work. Any children who were unable to complete a solution may learn a useful approach from their peers. A common trap for this age group is that they want to name specific animals rather generalise about the number of animals. Consider giving the same or very similar problems to the class within a week or so to allow them to further develop strategies.





Figures 1 & 2. Problem solutions from 5/6 year olds.

Middle primary

Present the problem and facilitate a discussion about what might be involved in working on a solution. Children in this age range often benefit from working in pairs, so consider allowing that option. Expect a recorded solution using mathematical language and symbols (addition and possibly multiplication). Encourage them to look for more than one solution. When most children appear to have found at least one solution it might be useful to form them into small groups to share and compare their answers.

Some or all children could, in pairs or small groups, publish solutions using clip art on computers. The children select an animal from clip art to represent a particular number of legs and then the animal is copied and pasted as many times as is needed to display the solution. This often leads to the discovery of either new solutions or some more systematic approaches to finding and displaying solutions. Figure 3 shows a solution to the '12 leg' problem by a pair of Year 3 children, using Microsoft Word clip art. They have used a mixture of notations, including multiplication, to express their solution.

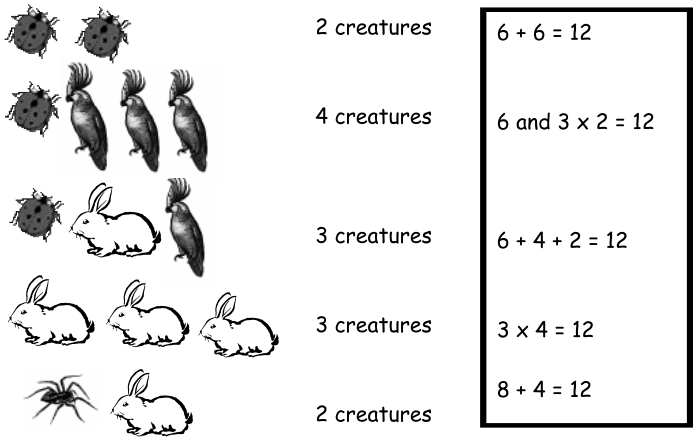


Figure 3. Solution to 12 leg problem published using computer clip art (Year 3).

Upper primary

Expect a more systematic approach and production of the full set of solutions, perhaps recorded as equations using multiplication and addition. This might be achieved by forming groups of student to share and compare the solutions and recording methods they have worked on individually. Encourage 'proof' of the full solution. Ask, 'How do you know you have found all the combinations? Encourage the creation of a table, perhaps as a whole class effort, that summarises the solution set.

Figure 4 is a table produced by three Year 6 students who worked on the 16 leg problem. They prepared it on the computer and used the 'Sort' function to order the various columns in ascending and descending order to look for patterns.

2 legs	4 legs	6 legs	8 legs	Creatures?
8				8
6	1			7
5		1		6
4	2			6
4			1	5
3	1	1		5
2	3			5
2		2		4
2	1		1	4
	4			4
1		1	1	3
	1	2		3
	2		1	3
			2	2

Figure 4. Table of all the combinations for 16 leg problem (Year 6).