Promoting repeating patterns with young children - More than just alternating colours!

Marina Papic explains how experiences in the early years of schooling should focus on identifying, justifying and transferring various patterns using a variety of materials if a sound understanding of patterning is to develop in students.

Patterning is an essential skill in early mathematics learning, particularly in the development of spatial awareness, sequencing and ordering, comparison and classification. This includes the ability to identify and describe attributes of objects and similarities and differences between them. Patterning is also integral to the development of counting and arithmetic structure, base ten and multiplicative concepts, units of measure, proportional reasoning and data exploration. The importance of early algebra and patterning is reflected in current curriculum frameworks in pre-school and primary years both in Australia and internationally.

This article describes some teaching/learning experiences from a recent intervention study on patterning with 4-6 year olds (Papic & Mulligan, 2005, 2007). It was designed to encourage children to 'see' the structure of patterns and to symbolise, represent and transfer patterns from one mode to another. The approach to repeating patterns differed from the teaching of patterning as 'alternating' colours or attributes. Rather, children were immersed in experiences that would allow them to observe the structure of units of repeat. Children were exposed to complex repetitions and to patterns as rotations and translations. They developed their skills in seeing similarity and difference between attributes of objects. Through these experiences children were enabled to observe more than one attribute of pattern at one time: colour, number, shape and orientation. The results of the study provide some direction for the teaching and learning of patterns and algebra in the transition to and in the first years of schooling.

Repeating patterns

Repeating patterns contain a discernable unit of repeat; that is, "the pattern has a cyclic structure that can be generated by the repeated application of a smaller portion of the pattern" (Liljedahl, 2004, p. 27). This 'smaller portion' can vary in the number and complexity of items depending on attributes such as size, shape, dimension and direction and is commonly referred to as the pattern unit, element, segment or part. For example, Δ O is a repeating unit or element of a pattern containingΔ O Δ O Δ O.

Patterns also provide a basis for early

algebraic thinking. Experience with repeating and growing patterns can develop functional thinking when students move beyond simple single data sets to seeking relationships between data sets (Warren & Cooper, 2006). Warren and Cooper (2006) advocate that patterns need to be presented in a variety of different modes such as with actions, music or geometric shapes. This enhances the potential for generalising patterns as a unit of repeat regardless of the mode. Similarly, patterns can be symbolised and represented in different ways, thus creating the potential for early algebraic thinking.

The intervention study

A descriptive study monitored the development of 53 preschoolers' mathematical patterning skills in two similar preschools, one of which implemented a 6-month intervention promoting patterning concepts. The intervention comprised three distinct components: structured individual and small group work on pattern-eliciting tasks, 'patternising' the regular preschool program, and observing children's patterning in free play. A framework of assessment and learning was devised where children were placed on an individual 'learning trajectory' and progressed through an increasingly complex series of tasks. The intervention provided on-going professional development for the teachers about the importance of pattern and structure in early mathematical learning. This assisted teachers in modifying the emergent curriculum to incorporate patterning.

Pre- and post-intervention assessment data and follow-up data provided evidence of the impact of the intervention on the growth of children's patterning skills. Intervention children outperformed non-intervention children across a range of patterning tasks and this trend was maintained 12 months after formal schooling. Intervention children readily identified the unit of repeat within a variety of patterns including linear, cyclic and hopscotch. Without exposure to growing patterns the intervention children, at the end of the first year of schooling, identified, extended, represented and justified triangular and squared number patterns.

The study found that some children may be able to copy and extend patterns, but they may not necessarily identify a pattern as a unit of repeat (Papic & Mulligan, 2005). Most non-intervention children recognised, copied, continued and created repeating patterns as an alternation of two or three colours, without identifying the pattern element or the number of repetitions. When the complexity of patterns increased this proved to be a less efficient strategy.

Exploring repeating patterns

1. Linear patterns

Repeating patterns are usually displayed in linear form such as a straight line. Linear patterns may extend in different directions and can be repeated infinitely. Simple repetitions such as ABABAB are a typical example of linear patterns where a pattern element is duplicated horizontally, vertically or obliquely. Table 1 shows examples of linear tower patterns made with different coloured blocks and provides examples of experiences to encourage children to identify the unit of repeat and the number of repetitions within the pattern.

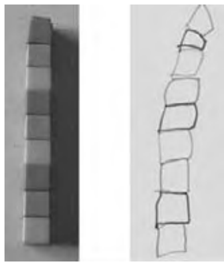
In order to assess whether children have an understanding of repeating patterns as a unit of repeat, children should be encouraged to justify the pattern and their strategy for solving the task. For example, the excerpt below shows dialogue between a teacher and a child after the child had drawn an ABBC tower pattern from memory:

Teacher: How did you know when to stop? How did you know when not to draw any more?

Child: Oh, because I had to do red, blue, blue again and then black and I had to put it three times.

The child's ability to abstract the unit of repeat and identify the number of repetitions was clearly evident and this also showed multiplicative thinking

Table 1: Tower pattern tasks

Tasks	Materials	Tower Pattern
Copy a 9 block ABCABCABC tower using blocks Ask children to identify the unit of repeat and the number of repetitions. Copy a 9 block ABCABCABC tower by drawing from memory. Continue an ABCABCABC tower pattern.	Various 9 block ABC tower patterns Unifix cubes: 15 of each colour Paper, textas – same colours as the Unifix cubes	
Create a repeating tower pattern using blocks. Ask children to justify their pattern. Remove the tower from view and draw the tower from memory. Ask children to explain how they remembered the tower	Unifix cubes: 15 of each colour Paper, textas – same colours as the Unifix cubes	
Identify screened element/blocks of an ABBCABBCABBC tower. Screen whole tower from view and ask children to make the same tower pattern using the blocks showing all the blocks, even the screened ones.	12 block 'red, blue, blue, black' tower pattern with the ninth and tenth block screened Unifix cubes: 15 of each colour	

2. Cyclic patterns

Cyclic patterns have no clear beginning or ending point. An example would be a simple repetition constructed as a border pattern. Table 2 shows examples of cyclic border patterns made with different coloured tiles where children are encouraged to identify various units of repeat and justify the pattern.

Observing children as they solve the border tasks can provide teachers with valuable insight into children's understanding of repeating patterns as unit of repeat as well as their counting strategies. In this intervention study, kindergarten children were shown an incomplete ABC ('red, blue, green') border pattern and asked to determine the number of greens required to complete the pattern. Intervention children outperformed non-intervention children because they were aware that the pattern element contained three colours and they needed only to count every third tile. Such a strategy would suggest a sophisticated understanding of pattern as repetition and reflect early multiplicative thinking. Many of the intervention children immediately identified every third position in the border by placing their finger on the square where the missing green tile needed to be placed. It appeared that these children visualised the pattern element accurately; some 'skip counted' every third position in the pattern, translating the repetition of colours into a number pattern of multiples.

3. Hopscotch patterns

Hopscotch Patterns explore children's ability to rotate a unit of repeat created with vertical and horizontal square tiles. These tasks investigate changes in orientation of the pattern and children's transformation skills. Table 3 shows examples of Hopscotch Patterns where children are encouraged to identify the unit of repeat in pattern presented in different orientations.

Table 2: Border pattern tasks

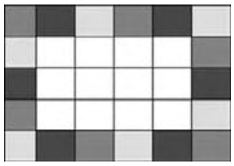
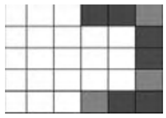
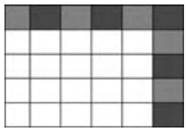
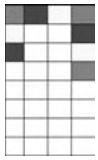
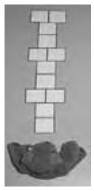
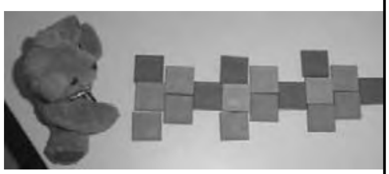
Tasks	Materials	Border Pattern
Copy the border pattern using cut out tiles. Identify that a border pattern does not have a clear start or finish: encourage children to identify the unit of repeat (e.g. red, blue, green) and then ask them if they can identify a different unit of repeat in the pattern (e.g. blue, green, red).	Completed border pattern, 'red, blue, green' 6 x 5 square grid 12 red, 12 blue, 12 green and 12 yellow tiles	
Using the square tiles complete the border pattern.	Incomplete 'blue, blue, red' border pattern 15 blue tiles, 15 red tiles, 15 green tiles	
Create own border pattern and justify the pattern.	Various square grids e.g. 6 x 5, 4 x 3, 8 x 3, etc. Square tiles of various colours – 15 of each colour	
Identify the number of blue tiles required to complete the 'red, blue' pattern.	Incomplete 'red, blue' border pattern	
Ask children if the 'red, yellow, blue' pattern could be completed exactly within the number of available squares. Ask them to justify their response.	Incomplete 'red, yellow, blue' (anticlockwise) border pattern	

Table 3: Hopscotch pattern tasks

Tasks	Materials	Hopscotch Pattern
Copy using square tiles a hopscotch template that is repeated three times. Copy the hopscotch pattern from memory using square tiles.	Hopscotch Pattern – 2 vertical, 2 horizontal tiles 50 Individual square tiles	
Copy by drawing the hopscotch pattern rotated by 180°. Copy by drawing the hopscotch pattern rotated by 180° from memory. Using square tiles, continue the hopscotch pattern rotated by 180°.	Hopscotch Pattern rotated by 180° Paper and pencil 50 Individual square tiles	
Create own hopscotch pattern and identify the unit of repeat. Encourage children to use more than one variable e.g., direction and colour.	Square tiles of various colours – 20 of each colour	

Growing patterns

Growing patterns increase or decrease systematically. They represent variation of one data set, where the relationship between successive terms within the pattern can be identified (Warren, 2005). Kindergarten children in the second year of the study had not been exposed to growing patterns and these tasks had not comprised part of the intervention. Nevertheless, many of the intervention children could construct, extend, represent and justify these patterns. They could continue a growing triangular number pattern "1, 3, 6", presented as a triangular dot pattern (see Figure 1) and continue a growing squared number pattern "1, 4, 9", made with square tiles (see Figure 2). In comparison, non-intervention children were unable to identify or extend growing patterns. Many saw the triangles and squares exclusively as items in simple repetitions in the same way as the simple repetitions that they were familiar with e.g. ABC repetitions.

Most intervention children who could successfully extend growing patterns could also justify the pattern. The following excerpt demonstrates one intervention child's justification of the pattern as growing systematically in two dimensions.

Teacher: Can you tell me what is happening each time we make the square bigger.
Child: Yeh, here it has one, then it has 2 and 2 lines and it's bigger. Then this one has three and three lines and then four and four lines.
Teacher: What do you mean four and four lines.
Child: See there's four in each line.
Teacher: So what would the next one in my pattern be?
Child: Umm … five and five lines

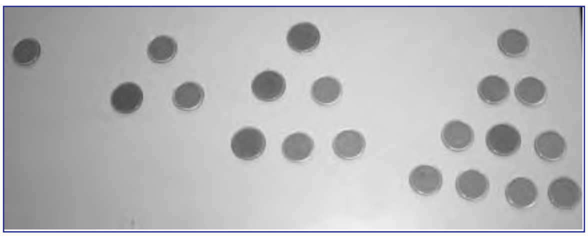


Figure 1: Triangular growing pattern

Results of the study suggest that intervention children's success on growing patterns could be attributed to their ability to identify both the structure of repeating patterns and their representation in different spatial forms (Papic & Mulligan, 2007).

Teaching implications

It may be taken for granted that exposure to patterning experiences using shapes, objects or pictures readily enables an understanding of pattern as unit of repeat. However, as the results of this intervention study show, this is not necessarily the case. If young children are to develop a sound understanding of repeating patterns as a repetition of units then experiences in the early years of schooling should focus on identifying, justifying and transferring various patterns, and using a variety of materials. Further, children should be encouraged to identify the element within repeating patterns and observe the number of repetitions. Other processes that promote more complex pattern formation include drawing from memory, viewing patterns from different orientations and extending a pattern in multiple directions. These processes are reflected in curriculum documents across Australia but more explicit direction could highlight the potential for patterning and how it is integral to other strands of the curriculum.

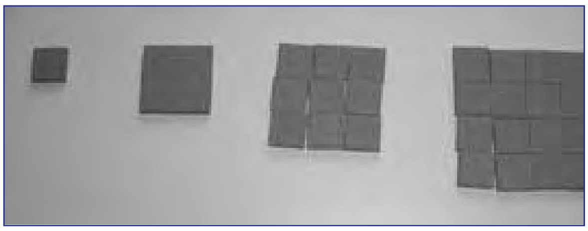


Figure 2: Square Tile growing pattern

Concluding comments

According to Warren and Cooper (2006), "children spend a great deal of time in the early years investigating repeating patterns" (p. 130). However, it may be that the child's development of repeating patterns without a sound understanding of the unit of repeat limits and possibly impedes the development of more complex patterns such as growing patterns. Commonly, when teachers are dealing with repeating patterns, the structure of the pattern is ignored or misinterpreted. Therefore, expecting children to observe other pattern structures such as growing patterns seems unreasonable. This study found that young children are capable of developing more than just repeating patterns. They can symbolise, abstract and transfer complex patterns. It is recommended that patterning skills in the early grades should extend beyond simple repetition provided that they focus on the unit of repeat. The application of other aspects of mathematics learning is potentially powerful and includes justifying and transferring various patterns to other modes, using a variety of materials.

References

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