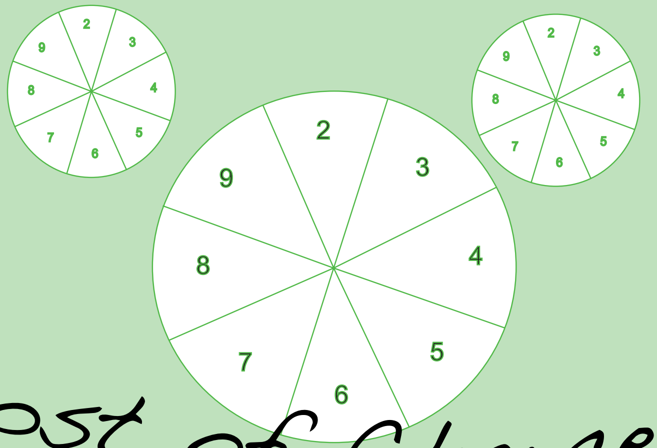

Making the most of Chance

Monica Baker and Helen Chick reveal the potential of a basic spinner game to prompt consideration of important concepts of chance, when developed further by the teacher.

We all know that teachers don't have unlimited time, lots of experience, and deep understanding of all the mathematics they teach. To solve this problem teachers often use textbooks, and the accompanying teacher's resource books, as sources of activities and advice about how to help students learn mathematics.

The activity that prompted this article came from such a resource. We had the privilege of observing two teachers using this activity in their Grade 5 classrooms. Both teachers prepared stimulating lessons, in which students had opportunities to explore and extend their understanding of Chance as the teachers drew out interesting concepts in the game. Unfortunately the teacher's resource book gave no details about the variety of important probability ideas that the classes could have explored, which restricted what the teachers covered in their lessons.

In this article we will explore those ideas, highlighting the concepts the activity can develop, and suggesting variations that make the concepts more accessible.

The activity

The "Two Spinners Game" from Nelson Maths for Victoria Teacher's Resource Year 5 (Feely, 2003, p. 173), presents two spinners divided into equal parts labelled with the numbers 1-9 (see Figure 1). Students spin both spinners, and if the sum of the two numbers is odd, player 1 wins a point; if the sum is even, player 2 wins a point. The winner is the first to ten points.

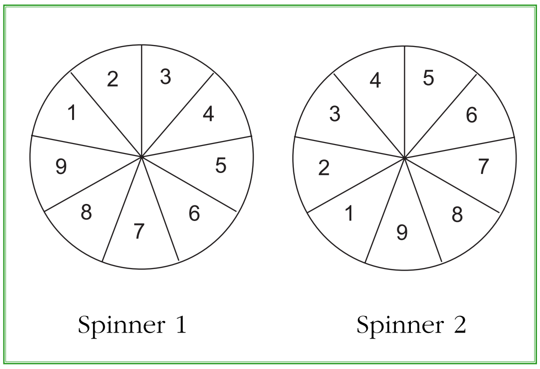

Figure 1. The spinners used in the "Two Spinners Game"

Students are instructed to "play the game a number of times and see what happens." They are then asked:

- Is this a fair game?
- Who, if anyone, has a better chance of winning?
- Why? Justify your answer.

The teacher's resource book makes a brief suggestion about focusing on how many combinations of numbers add to make even and odd numbers (Feely, 2003, p. 116), but does not provide any additional comments.

Before discussing the game in more detail, stop and see if you think the game is fair and decide how you might answer that question with a Grade 5 class. Also, what Chance ideas can be developed, what aspects are challenging, and what are students likely to think of if they play the game?

What the activity offers

The activity suits group work, and fosters problem-solving skills. In terms of Chance, this activity allows exploration of "sample space," "probability," and "fairness" (and some number reasoning as well). The activity links these three important Chance concepts, highlighting the need to know the sample space in order to calculate probability, and the need to calculate probability in order to decide if the game is fair. Although the activity is not as straightforward as it may seem, if presented in the right way it can provide experiences that help develop good understanding of these concepts.

Our teachers were successful in exploring some of these ideas, but not all of them. So, what is really going on, and how can we use this activity to maximum effect? It is worth beginning a class exploration by actually playing the game, as our teachers did.

Sample pace

After students have played the game a few times, it is important to look at the sample space (i.e., the complete set of outcomes). There are many approaches for enumerating the sample space, and students should be encouraged to propose their own methods. These can be discussed so that the class can decide if they are appropriate. Some possible approaches are listed below.

Method 1

The outcomes can simply be listed, and counted at the end.

1+1	1+2	1+3	1+4	1+5	1+6	1+7	1+8	1+9
2+1	2+2	2+3	2+4	etc.				

Figure 2. Systematic listing of possible outcomes

Students may need to be encouraged to develop a systematic way of listing the possibilities. Note that 1+2 and 2+1 (and similar) are different situations that must be counted separately because the numbers come from two different spinners.

Method 2

Alternatively, students can draw up a grid as shown in Figure 3.

		Spinner 1								
		1	2	3	4	5	6	7	8	9
Spinner 2	1	2	3	4	5	6	7	8	9	10
	2	3	4	5	6	7	8	9	10	11
	3	4	5	6	7	8	9	10	11	12
	4	5	6	7	8	9	10	11	12	13
	5	6	7	8	9	10	11	12	13	14
	6	7	8	9	10	11	12	13	14	15
	7	8	9	10	11	12	13	14	15	16
	8	9	10	11	12	13	14	15	16	17
	9	10	11	12	13	14	15	16	17	18

Figure 3. Table showing all possible outcomes.

The grid is a nice way to enumerate the outcomes, because addition patterns emerge, and students can feel confident that they have all the outcomes recorded. In Methods 1 and 2, listing all the outcomes reveals 81 possible combinations: 41 even and 40 odd.

Method 3

Rather than list all possibilities separately, students can use properties of odd and even sums to calculate the total number of odd and even outcomes.

- Spinner 1 has 5 odd numbers and spinner 2 has 5 odds, which leads to 25 sum combinations. Since Odd+Odd=Even, this gives 25 even outcomes.
- Spinner 1 has 5 odds and spinner 2 has 4 evens, which leads to 20 combinations. Since Odd+Even=Odd, this gives 20 odd outcomes.
- Similarly, spinner 1 has 4 evens, spinner 2 has 5 odds, which gives 20 odd outcomes.
- Finally, spinner 1 has 4 evens, spinner 2 has 4 evens, so this gives 16 even outcomes.
- Students can then calculate the total number of even and odd outcomes.

Method 4

A tree diagram can be used to find the number of outcomes in similar fashion to Method 3 as shown in Figure 4.

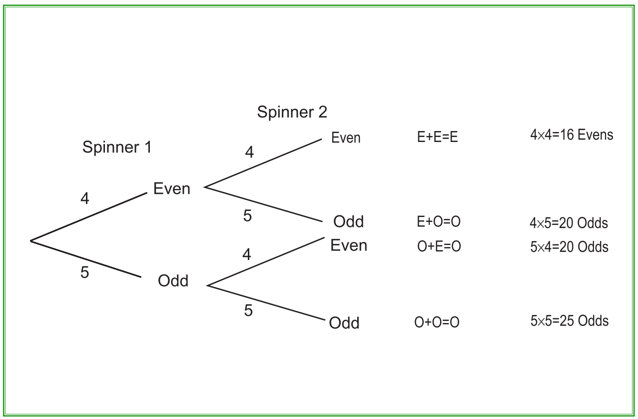

Figure 4. Tree diagram for all possible outcomes

Not-quite-right methods

As suggested earlier, students will have their own suggestions for thinking about the sample space. In the classrooms we observed, an initial reaction was that the spinners both have more odds on them, although at first no conclusion was drawn from this. Later, several students suggested that because Odd + Odd = Even, Even + Even = Even and Even + Odd = Odd, this must mean that there is a 2/3 chance of getting an even number and a 1/3 chance of getting odd. Unfortunately this argument correctly concludes that even is more likely, but the reason is incomplete and the true difference is actually much smaller.

Probability

Now that we have the full sample space or the total numbers of evens and odds, the probability of an odd or even outcome can be calculated. [We emphasise that before doing this activity students should have encountered numerical representations of probabilities using examples involving a smaller sample space, such as throwing a die.] For the "Two Spinner Game" we have:

$$\text{Probability of even} = \Pr(\text{Even}) = \frac{\text{number of even outcomes}}{\text{total number of outcomes}} = \frac{41}{81}$$

$$\text{Probability of odd} = \Pr(\text{Odd}) = \frac{\text{number of odd outcomes}}{\text{total number of outcomes}} = \frac{40}{81}$$

If the sample space has been listed using Methods 1 or 2 above, it is also possible to calculate the probabilities of individual outcomes. For example, the probability of the sum being exactly 10 is:

$$\text{Probability of a 10} = \Pr(10) = \frac{\text{number of 10s}}{\text{total number of outcomes}} = \frac{9}{81} = \frac{1}{9}$$

Fairness

Having calculated the probabilities of odd and even outcomes, we are ready to answer the question "Is this a fair game?" Since there are more even outcomes than odd outcomes, "even" is more likely to win. This makes the game unfair. [Students need to understand that fairness is not about playing by the rules. A game is unfair if, even when both players are playing by the rules, one player is more likely to win no matter what the other tries to do.] However, note that there is only a miniscule difference between the probability of odd or even outcomes—just $\frac{1}{81}$. Students should discuss the meaning of this difference: will it be noticeable after playing the game just a few times? (It won't.) This is one of the complexities of the "Two Spinner Game": it is unfair, but you won't notice while playing it. This makes it difficult to explore what "unfair" means, even if you believe the game is biased.

The problems with the activity … and with probability

As you can see, there is much that can be covered with this activity. There are also some problems. First, the size of the sample space makes the task of listing each outcome and then calculating probabilities quite complex. Given that the activity is directed at Grade 5, this may be too advanced for students just beginning to learn about numerical probabilities. It also makes it harder for students to comprehend what the probabilities mean.

A further problem is that the difference in probabilities between getting an even or an odd number is only $\frac{1}{81}$ –small enough that it would not be noticeable during a single game or even several. If the effect of the probability cannot be observed, then Grade 5 students may have difficulty making sense of it.

Finally, there are some tricky things about probability itself. Students need to learn that numerical probability describes a long-term phenomenon (the proportion of times we expect to see a particular outcome), but then there is also the idea of variation. If we toss a coin the probability of getting heads is 1/2, because in the long-term we get heads half the time. Variation is seen when we actually toss a coin 100 times: we may get 54 heads, or 47 heads, or possibly—but by no means definitely—50 heads. Such variation is not a consequence of the coin or the person tossing it, it is how random processes work

Some variations on the Two Spinners Game

It is possible to adapt the Two Spinners Game to reduce the above problems. Different adaptations will have different effects on the activity. Pairs of spinners for these games are included at the end of this article.

Variation 1: Two spinners each with just 1, 2 and 3

With fewer numbers there is a reduction in the complexity of enumerating the sample space and determining numerical probabilities, and an increase in the bias of the game towards one player (1/9). However, with the reduction in the numbers, the opportunity to make generalisations about even and odd sums may be lost.

Variation 2: Two spinners, one with 1, 2, 3 and the other with 4, 5, 6

This is like Variation 1, but this time there is a bias towards odd numbers. This makes it much easier to be convinced of the incorrectness of the incomplete reasoning that leads to probabilities of 2/3 even, 1/3 odd (as discussed in the Sample Space section).

Variation 3: Spinners with 1, 3, 5, 7, 10 and 2, 4, 6, 8, 9

For students who can cope with the complexity of having more numbers, this combination has 25 outcomes, only 8 of which are even. This bias in favour of odd outcomes should become evident very quickly when playing the game.

Variation 4: The original spinners with multiplication

Use the original 1 to 9 spinners, but instead of adding the two numbers, multiply them instead. This will provide practice in multiplication, will allow you to make generalisations about products of even and odd numbers, and the sample space in this case is significantly biased towards even (56/81 to 25/81) so it will be evident while playing.

Variation 5: Design your own

Many other variations are possible, which adjust the outcomes and allow you to focus on different aspects of the game. When making modifications, you should consider all aspects of the game that will be changed (sample space, probability, and fairness). We suggest students could design their own variations, too.

A suggested teaching sequence

Here we suggest a plan for using the Two Spinners Game over a two or three lesson period.

- Start with Variation 2 (rather than the original version). Allow students to play it and record how many odds and evens they get, and think about what is happening. After playing a few times, tally the class results, see if there is a difference between the number of odd and even outcomes, ask students if they think the game is fair, and get students to suggest any ideas they have for working out what is happening.
- Now have students develop the sample space for Variation 2, then determine the probabilities of even and odd outcomes. See if the class's tallied proportions match the theoretical proportions. There should be roughly 5/9 odds and 4/9 evens, but to see this you need to ensure you have recorded many trials (e.g., at least 200, so combine the class results). Discuss how knowing the probabilities helps us understand why the game is unfair.
- Try Variation 3 next. Have students make a quick prediction for what they think will happen, then play the game, and explore the sample space. Some students may want to omit the playing part, and do the sample space immediately. This is fine, but the connection between the sample space calculations and the real data should be explored because you can discuss the variation between theory and experiment. Again, consider fairness, and whether this game is fairer than the first (it is worse, and this will be much more noticeable in the real data).
- Now try the original Two Spinners Game. Let students choose if they want to play or work out the sample space directly. Encourage alternative approaches to exploring the sample space (e.g., "Does anyone have a quick way of working it out?"). Determine the probabilities of even and odd outcomes, and then consider the 1/81 difference. Ask if this means that the game is unfair? (Yes.) Ask if students think it is a noticeable difference? Students may be uncertain about this, which could lead to some good discussion. Collect some experimental data to see what happens; if you collect 50 sets of spins from each pair of students you should find that about half of the pairs have odd ahead and half will have even winning, showing that the difference isn't noticeable. Even if you combine all the results, you still may not see even coming out ahead. You could also try Variation 4 after this.
- Finally, have students design their own pairs of spinners and discuss the properties with the class. Students should talk about what they designed the spinners to do, how unfair their game is, and what they think would happen if they played the game many times.

Conclusions

The Two Spinners Game seems simple at first glance, but proves to be a source of rich and complex mathematics. Teachers can guide students in their exploration and help students develop and understand new techniques and key probability concepts. Above all, we cannot leave the teaching of Chance to chance!

References

Feely, J. (2003). Nelson Maths for Victoria: Teacher's Resource Year 5. Melbourne: Thomson Nelson.

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Spinner Games