The Queensland project of the Numeracy Research and Development Initiative, on which this paper is based, focussed on identifying elements of learning environments that promote students’ numeracy learning outcomes. The project was undertaken in eight schools across a range of settings within the three schooling systems in Queensland and was coordinated by Education Queensland. A research team of ten primary mathematics educators formed from four Queensland universities collaborated with teachers and administrators in the project schools. Teacher–academic collaborations were found to have a positive impact on students’ numeracy outcomes.

The project included both quantitative and qualitative components. The qualitative component of the design was a combination of case study and collaborative action research. The case studies indicated there were six core elements that were essential to enhancing numeracy outcomes. These core elements were:

1. teacher mathematics knowledge;
2. teacher pedagogy knowledge;
3. planning (for units and lessons);
4. active learning (tools and talk);
5. classroom enquiry (empathy, tasks, mathematics terminology and dialogue that lead to the development of a community of enquiry and validation); and
6. classroom engagement (motivation, perseverance, and confidence in students and staff).

The six elements were identified from the case studies as ‘core’ because there was evidence of their impact (either positively or negatively) in at least six of the eight project schools.

It is the active learning and classroom enquiry elements which are the focus of this report.
Active learning

The need for learners to be cognitively and physically active in learning mathematics was identified in all project schools as an element crucial to enhancing students’ numeracy outcomes. Thus creating active/constructivist classroom learning environments became a major goal of all researchers. The case study results suggested that, along with a focus on student-centred-learning, engaging students in active learning entailed:

(a) the use of interesting and appropriate materials or tools to stimulate interest, represent abstract ideas and build effective mental models; and
(b) the use of teacher and student discussion or talk as a way of developing shared understanding and making links between the representations and connections within and across the strands.

The case studies strongly indicated that students’ numeracy outcomes were enhanced by active learning environments.

Tools

The hands-on use of tools was an important element of active learning environments that enhanced student learning outcomes in most project classrooms. These tools should range across real world (e.g., chairs), concrete (e.g., bundling sticks), virtual (e.g., computer copies of bundling sticks), and pictorial (e.g., place value charts) representations. However, tools do not enhance outcomes by their presence; they have to be appropriate to the mathematics and appropriately used. In one school, for example, a teacher provided a variety of space materials for the students to explore but failed to assist them elicit the mathematics from the materials. Thus, while students were manipulating materials, there was limited learning resulting from these manipulations.

A crucial aspect in using materials is to make connections between different representations. Student learning improved markedly when students were required to connect different representations of processes and concepts, for example, language, symbols, drawings, models and real-world enactments. It is also crucial that rich understandings are abstracted from the use of tools so it is important to use materials that will not limit the mathematics ideas generated. Thus, the range of tools provided should include non-prototypic (unusual) examples as well as the commonplace or prototypic; for instance, in developing hundredths, it is important to have students shade diagrams other than 10 × 10 grids.

The interrelationship between representations is enhanced if connections go both ways (e.g., from story to symbols and symbols to story) and if there is a multiplicity of representations. Generalisation and non-prototypic examples were found to reinforce a focus on the general mathematical ideas, rather than the particular use of the idea in specific contexts.

Talk

At the beginning of the project, cognitive scaffolding, and discussion of solutions and solution paths was evident in very few classrooms. Classroom dialogue was often restricted to instructions, and question/answer responses. In one class, for example, ‘answering’ was equated with giving the answer to a textbook exercise. If the student’s answer was incorrect, the teachers’ typical response was to immediately ask another student for her/his answer. In one observed lesson, the teacher asked six students in a row before getting the ‘correct answer’. However, instead of discussing with the students which of the answers was appropriate and giving reasons for their choice, the teacher moved on to asking the answer for the next textbook task.

Where genuine classroom discussion did occur, it was mostly with experienced teachers, a finding which suggests that substantive conversation, like cognitive scaffolding, places high demands on teachers’ mathematic and pedagogy knowledge. One teacher, for example, initiated discussions by disagreeing with a student’s response, thus ‘forcing’ the student to defend her/his position, a strategy that usually led to other students engaging in dialogue to support their peer. Another teacher regularly gave students an open task which was usually based on a student’s response to a particular question (e.g., find a way to show whether three tenths equals one third). This teacher was unconcerned with the amount of time she spent on any one concept that had elicited a misconception; her overarching concern was that the students all came to an agreement about the appropriateness of a given response.

Classroom enquiry

Closely aligned to establishing an active learning environment is establishing a community of enquiry which values representing, justifying and generalising mathematical ideas. Four key issues emerged as the project teachers endeavoured to establish such communities of enquiry in their classrooms, namely, empathy between teachers and students, the nature of the mathematical tasks chosen for student exploration, student’s mathematics terminology needed for engaging in meaningful dialogue, and the degree and depth of dialogue.

Empathy

The personal relationship between teachers and students and the extent that this encouraged students to feel good about themselves and their mathematics learning appeared to be quite crucial in many schools. Where it broke down, it prevented good mathematics knowledge and teaching skills from being effective with all students; where it was strongly in evidence, it appeared to overcome weaknesses in knowledge and planning.
Tasks
Initially, many of the project teachers relied heavily on textbooks for their instruction. Some teachers were unaware that they could select or develop activities to suit the needs of their students; others were too ‘maths-anxious’ to do so. Tasks that promoted classroom enquiry gave students opportunities to engage in justification and debate. Such tasks were open (more than one solution strategy and/or solution), cognitively challenging, motivating, and relevant, and they engendered discussions on a variety of levels. These tasks not only provided a forum for classroom discussion, but also allowed all participants to engage in the conversations that assisted students to understand the key ideas in a supportive environment.

Mathematics terminology
Observations of teacher–student and student–student talk indicated that mathematics terminology was important for communication that facilitates mathematical understanding. One teacher’s indiscriminate use of ‘number’ to refer to both a whole number (e.g., 46) and a digit within the number led to some misunderstanding, while another teacher’s incorrect and/or inconsistent use of spatial terms (e.g., oblong/rectangular prism) caused difficulties in communication. Improving terminology became an important component of teacher–researcher collaborations, particularly for mathematics topics with which teachers were less familiar (e.g., space and chance).

Dialogue
Positive empathy, well-chosen tasks and good mathematics terminology provide the basis for classroom dialogue that can drive enquiry. Classrooms that exhibited this enquiry appeared to have enhanced learning outcomes. In particular, teachers who allowed or encouraged students to disagree with them appeared to provide real opportunities for learning. Classrooms where enquiry was not evident appeared to repress aspects of learning that would enhance numeracy outcomes.

Dialogue appeared to be important within and outside the classroom. In some instances, the community of enquiry and validation occurred beyond the classroom walls for both teachers and students. In some schools, an added outcome of the project was an increase in the professional dialogue and debate about mathematics and mathematics teaching. As one school began to move towards becoming a ‘community of learners’, professional dialogue occurred not only within the project teachers and with the researcher, but also between the project teachers and the broader school community. The focus of these discussions was on delineating their own understandings of mathematics, sharing ideas about teaching mathematics, and getting to know about the mathematics taught in other year levels. Active learning and classroom enquiry impacted on both student and teacher motivation.

Summary
The teacher–academic collaborations impacted positively on almost all the project teachers in terms of teacher mathematics knowledge, mathematics teaching knowledge, and classroom engagement (especially confidence). These factors, in turn, enhanced teachers’ classroom practices and student numeracy outcomes. As two principals stated:

[The professional development] resulted in more knowledgeable teachers, enhanced numeracy outcomes of the students, and increased teamwork. (Principal 1)

Have a look [at the results of the Primary Maths Competition in 2002] and try and tell me that your work hasn’t had some impact on student learning outcomes at our school. The results are too good to pass off just as a coincidence. Thanks for your work with teachers, parents and students. (Principal 2)

The collaborations also assisted teachers in planning connected units of work based on students’ prior knowledge, in selecting and using appropriate resources, in organising and managing activity and discussion, and in reviewing their ideas about appropriate classroom enquiry. Teachers’ beliefs about mathematics and its teaching, learning and assessment were challenged and changed and this, in turn, was reflected in improved classroom practices that enhanced active learning, classroom enquiry and students’ numeracy outcomes.