INDIVIDUAL INTERVIEWS *as insight into* children's computational thinking

HEIDI J. HIGGINS and **LYNDA R. WIEST** illustrate the meaning of computational understanding.

Learning mathematics may seem to students a tedious process of memorising algorithms that have little meaning or value. It may involve countless hours practising computations with little focus on why the procedures work and when to use them. Students may, therefore, forget these procedures over time. Accessing and executing procedural knowledge is easy with today's technologies. Understanding it well enough to use it in varied contexts is another matter.

Teachers who believe "practice makes perfect" may engage students in repetitive, perhaps timed, computational exercises. If we teach students to understand the procedures they practice, however, they will not need as much drill and they will have more flexible use of the computations they perform. Three-quarters of a century ago, William Brownell began arguing for meaningful arithmetic. In this article we describe some of Brownell's ideas about children's understanding of school arithmetic. We include transcripts of Brownell-inspired individual interviews with two second-grade students that illustrate one method for assessing computational understanding.

Brownell's meaning theory

Brownell's work was first published in 1928 and continued for more than four decades. His theories relate directly to the issues we face today. Most of his work (e.g., Brownell, 1945; 1956) pertained to the teaching and learning of arithmetic but can easily apply to other mathematics content areas. Brownell's theory involved arithmetic having both a mathematical aim and a social aim. He believed that both aims are essential to a functional curriculum. Instruction should be organised around the ideas and relations inherent in arithmetic within a context that is both mathematical and practical. Students should both make sense of the mathematics itself and know how it applies to the real world. Mathematics experiences, Brownell said, must be meaningful to students at the time of learning. Accordingly, he named his theory the *meaning theory*.

Students have different levels of mathematics understanding. Rather than being either present or absent, understanding falls along a continuum. A student may give a low-level response to 18 + 8 by counting out 18 cubes and then 8 more before re-counting all of them starting with one. Another student may put 18 in her head and count on 8 more. Another strategy is to break the problem into two steps using a "make-ten" strategy: 18 + 2 = 20 and add the remaining 6 (from the 8) to get 26. Still another student may add the ones (8 + 8 = 16) and then the tens (16 + 10 = 26). A student with much computation experience may at once know that the sum of 18 and 8 is 26.

Clearly, these sample responses show varied levels of understanding. Instruction should be organised so that the students will ascend to the level of "meaningful habituation," in which students' responses in mathematical situations are automatic but have a firm basis in understanding (Brownell 1935). Students' understanding of mathematics is based on the amount and kinds of experiences they have. Understanding a mathematics concept fully means knowing its function, structure, and relationships. We might infer understanding by what students say and do, but we should also look at what students do not say or do.

Assessing student thinking

It is easy to assume that students have a higher level of understanding than they actually do when — for instance — a child immediately answers 28 + 13 correctly. Having students explain and support their solution methods for problems gives a window into their thinking. One student may rapidly count up 13 from 28 to give the answer of 44, whereas another may decompose the numbers into 28 +10 = 38 and 38 + 3 = 41. The second student's strategy may take longer, but it demonstrates better number sense.

Another assessment technique is to see what students do in the presence of error (Brownell, 1956). Looking at students' work is not very useful here. Instead, teachers can note what students do when confronted with perceived error by posing questions while students are discussing their work with peers. However, individual conferences with students can be more useful and informative in gauging understanding.

Interviews with second graders

Heidi Higgins noticed during classroom discussions that one of her second graders usually carried out computation correctly but offered justifications that did not support her solutions. She knew she was not getting enough information through classroom observations and questions. So, Heidi decided to meet with her student one-on-one.

Heidi began the interview by asking the student to

Interview 1

Assessment questions	***Student's responses***
What is the sum of 14 and 3?	[Student writes 14 + 3 on paper.] 14 plus 3 equals 17.
How did you come up with that answer?	Because 4 and 3 is 7, and there's a one on the front, so it makes it 17.
What is the sum of 17 and 9?	[Student writes 17 + 9 on paper. She puts up 7 fingers and counts on her fingers.] 15. [She looks down at the paper and says that 15 does not make sense.] 17 [puts up 9 fingers and counts them], 18, 19, 20… 26.
What is the sum of 24 and 17?	[Student repeats the same procedure of putting the number in her head and counting 17 more fingers.] The answer is 41.
Could you solve this same problem using a different strategy?	I usually do it this way. 24 +17 52 [She starts with 7 + 4, putting 7 in her head and adding 4 more to get 12, and placing the 2 in the ones column and the 1 in the tens column. She then adds the tens and got 5.]
Do you notice that you have two different answers?	Oh, I added 7 plus 5 instead of 4, and I counted wrong over here [pointing to the tens column].
What is the sum of 56 and 38?	[Student writes 56 + 38 on her paper, uses the counting on procedure for the ones column, gets 14, places the 1 above the tens column, adds the tens column and gets 94.]
No, I think the answer is 84.	[Student rewrites the problem again and goes through the same procedure. She comes up with 94 again. She looks at Heidi and smiles and says that she was only 10 off and she was really close.]
Do you have any other way to check and see if the answer is 84 or 94?	No, I did it this way but I don't know what I did wrong. [The student tries the procedure again but cannot tell Heidi if she has the right answer.]

Interview 2

Assessment questions	*Student's responses*
What is the sum of 26 and 8?	It is 34.
How did you come up with that?	Well, you add 4 to 26, which would give me 30, and add the last 4 from the 8, and it would give me 34.
What is the sum of 54 and 38?	If you take the 4 from the 54 and give it to 38, you would have 42, and then you'd add 50. You would get 92.
What is the sum of 67 and 84?	[The student forgets the problem so Heidi tells him again and asks him to write it on the paper.] 71. I took the 4 from the 84 and gave it to 67. And then add the 80. You would get 151.
What is the sum of 77 and 28?	[Student writes 78 + 27.] Oops, I wrote the wrong problem. It doesn't matter. I'll still get the same answer.
How do you know?	Because all I did is switch the number in the ones column. 8 and 7 is 15 no matter which way you write it. Therefore, the answer to this problem and your problem is 105.
The sum of 39 and 46 is 84.	I don't see how. You're wrong, because if you take the 6 from 46 and give it to 39, you'd have 45, and 40 more is 85. You added your ones wrong. I'll even show you on paper. [The student writes the problem on paper and tells Heidi she can also check this using her fingers if needed.]
What is the sum of 554 and 128?	I know that it is in the 600s. Because if you take 554 and add 100 to it, you would have 654. With 8 more you would get 672, and 20 more would be 682.
No, the answer is 672.	I know that you're wrong. When you were adding your tens together, you missed a group.
How so?	For one thing, 8 plus 4 is 12, not 2. Somewhere in your addition you forgot to put that 10 back in.

add two numbers (see Interview 1). She noticed that the girl was most comfortable with the method of counting on. Asked if she knew how to use a different strategy, the student opted for the traditional algorithm of adding the ones column, regrouping, and then adding the tens. However, when Heidi intentionally disagreed with her answer, the girl repeated the algorithm but seemed convinced that she had done something wrong that she could not identify. (Shortly thereafter, Heidi told her that her answer was correct.) This child could perform the standard addition algorithm but had not developed meaning for it. Here, counting on characterised the student's level of computational understanding.

During another individual interview (see Interview 2), Heidi found that a male student displayed no confusion when faced with error. The boy did not hesitate to tell his teacher (Heidi) that she was wrong and even went so far as to say she could use her fingers to check her answer if needed. (Heidi's students work in a respectful classroom climate where they feel free to question and disagree with ideas, as long as they explain why.) The boy was able to break the numbers apart and recombine them accurately. He did not need to count his fingers or rely on a memorised method. This student appeared to have a higher level of understanding — and accompanying confidence — than the first student interviewed.

Implications for instruction

William Brownell's meaning theory promotes understanding of mathematical procedures. As teachers plan instruction, they should consider how to help students make meaning of computation. Instruction should start slowly using varied concrete materials and move increasingly toward symbols or other abstractions. Number relationships can be explored and discussed. Teachers should also structure opportunities for students to apply mathematics concepts in real-world contexts (Brownell, 1935).

The meaning theory does not suggest that students should never engage in repetitive practice. However, drill should only be introduced after students have achieved understanding of a concept or skill. It should be used to increase proficiency and make the learning permanent, leading to "meaningful habituation" (Brownell, 1956).

Conducting one-on-one interviews is one way to assess student understanding of and confidence with computational procedures. The two individual interviews discussed in this article illustrate sample questions teachers might use and provide possible student responses. The insights into students' thinking that this technique affords are a great way to inform both whole-class and individualised instruction.

References

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