All at SIXES...

JOHN and ANN BAKER **highlight the pervasiveness and limitations of the Guess and Check strategy used by Year 4 and 5 students as they solve a problem with multiple solutions.**

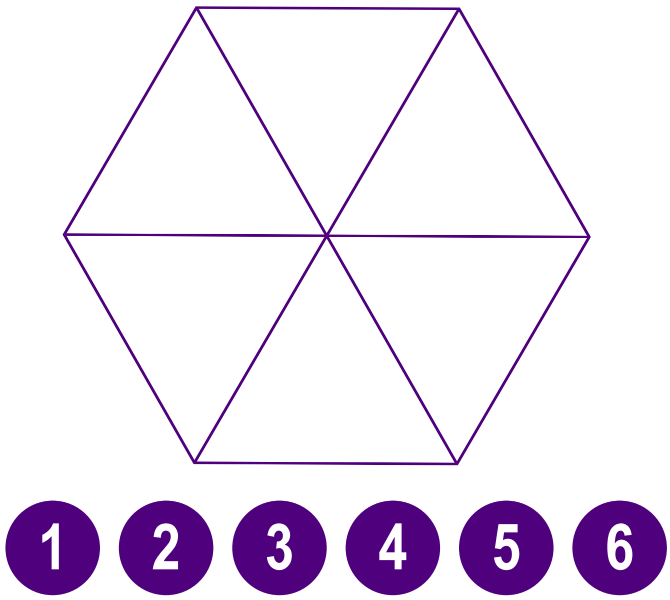
In this article we would like to look at a range of solutions submitted by Year 4–5 teams in the 2003 *Naturally Mathematical Challenge* (Baker, 2003a) to a question that involves no more than the numbers 1 to 6 and possible arrangements of them. In the context of other competition questions (Baker, 2003b; 2003c) and in our elaboration of mathematical process (Baker, 1990), we have described the way in which Polya's four step heuristic for problem solving (Polya, 1945), now popularly know as the *See, Plan, Do, Check model*, can be used by students not only to guide their initial thinking, but also as a framework for reporting on their method of solution. This article shows how the model can be used effectively by students in the middle primary years; but not all teams adhere to the model, and we also give examples where a *guess and check* approach is used, and we point out the severe limitations of this approach. In contrast, we also show how a systematic listing of possibilities can lead to a very complete solution.

Posing the problem

The challenges at each of the Year 4–5, Year 6–7 and Year 8–9 levels all consist of three problems. The first is usually a closed, arithmetic problem; the second has a spatial theme while the third is in the form of an investigation. The problems for the first challenge in the 2003 semester 2 competition took the hexagon as a theme. Thus we had questions on hexagonal transversals, on arranging numbers in a hexagonal pattern to make a magic hexagon-type of arrangement, as well as a question on locating prime numbers in a hexagonal grid. For the Year 4–5 group, the problem below

was posed for their investigation arose because we were looking at magic configurations, but soon found that the hexagon of side of length 1 has no magical properties. However, we also found that there is another possibility for arrangements of the numbers 1–6 in a simple hexagonal array, and this is the problem that resulted:

> Draw a hexagon divided into six triangles as shown. Now, write the numbers 1–6 on a set of counters and position one counter in each triangle of the hexagon.



> Make a list of the totals of the numbers in each row of triangles: your list should have six totals, because there are six different rows of triangles.
> Now for the challenge! By positioning the counters in a special way, make the list of totals into six consecutive numbers.
> In what different ways can this challenge be answered?

The way in which the problem was posed was such that we were trying to lead the Year 4–5 teams into the question step-by-step. Before thinking about the challenge of making the totals consecutive, we encouraged the teams to show that they understood just what was being asked by simply listing the possibilities and thus demonstrating that they knew how 6 rows (across, up and down) can be found in a hexagonal array.

We also made it explicit that the students should work in teams (see Bassarear, 1992) and that the teams should use manipulables (see Bright, 1999), in this case a set of numbered counters. Using numbered objects really does help in solving questions where a set of numbers have to be arranged in one way or another. In particular, the issues of inadvertently using a number twice in a solution and of knowing what numbers still need to be positioned are avoided if counters are used in this way. We do not encourage teams to write the numbers on scraps of paper, as a ceiling fan can cause havoc with an arrangement of paper numbers.

Jumping in

The following example shows that even when the question is clearly posed and aimed at taking things one at a time, some teams just could not resist jumping in.

Solution 1

> The problem is to find out the six consecutive numbers.
> We tried to solve the problem by finding the consecutive numbers on a large hexagon.
> We got our hexagon and wrote the numbers from 1-6 in the six spaces in order – biggest and then smallest. Then second biggest and then second smallest and so on. We succeeded on the first try with the hexagon.
> The numbers are consecutive.
> We are unsure if there is another solution. There is not one that we know of.

Using concrete materials

'One of the most obvious ways to teach mathematics differently is through the use of manipulables' (Bright, 1999). It is not always possible to persuade the teams that manipulables such as numbered counters really are a good thing to work with. Solution 2 ignores this advice, and the team was not quite as lucky as the Solution 1 team, and did not hit the jackpot on their first attempt.

Solution 2

Attempt 1

Drew a hexagon and wrote in the numbers 1–6 in any order matched each number to see if it would end up in the pattern, but failed.

Attempt 2

Rearranged the numbers, failed.

Attempt 3

Rearranged the numbers again, failed.

Attempt 4

Rearranged numbers again to 1 at the top 5, 6, 2, 3, 4 (clockwise). It was successful.

This solution concluded with:

> We do not believe that you could solve this question any other way apart from guessing and checking.

At times one has to wonder if students (at all level of schooling) know of any strategy other than guess and check! Certainly, without having recourse to some form of a system, it is very hard to know for certain whether a problem has solutions other than the one found.

Using counters

Even with the teams who persisted with a guess and check approach, the effect of using counters was immediately apparent:

Solution 3

See: We had to draw a hexagon with six equal triangles then write the numbers 1 to 6 on some counters so we can position one counter on each triangle. Then make a list of totals of the numbers in each row and end up with 6 totals because there are 6 rows of triangles. By positioning the counters in a special way the totals would be able to be rearranged into their consecutive order and we had to find as many answers as possible.

Plan: Our plan was to guess and check.

Do: These are our answers.

On closer inspection, these solutions are effectively the same as they are either rotations or reflections of each other. The phrase we prefer to use is 'the solutions are congruent', as congruent is a well-defined term and can be demonstrated by drawing of one of the solutions on a transparency and showing how it matches with each of the others.

A team from the same school had a very similar approach, including a reliance on guess and check as the method, but only two of their answers were congruent.

Some teams showed that they were thinking of something slightly beyond guess and check, even though it was their main strategy.

Solution 4

Plan: We planned to draw a picture and use numbered counters so that we could move them around the triangles. We plan to use guess and check and some logic to help us solve this problem.

Do: We followed our strategy and it took a while to get an answer. We saw that the totals we would need for our answers would be around 10. This is our solution.

The team saw that the totals would have to 'be around 10', and here is the germ of an idea that others picked up on.

Towards a systematic solution

The typical Year 4–5 student has mastered number facts to 20 and is confidently adding more than three on four single digit numbers together. This ability is all that is called for in finding a full solution to this question; the other requirement is the ability to take a systematic approach to solving a problem. Here is the first part of a complete solution that shows how a systematic approach can be used:

Solution 5

We placed the six counters in the triangles of the hexagon. We could get six totals of different rows, they are: 6, 9, 12, 15, 12 and 9.

We discovered that each triangle belongs to three rows. As we got the six totals, (the number in) each triangle was added three times, so the sum of the six totals was:

$(1 + 2 + 3 + 4 + 5 + 6) \times 3 = 63$

Having made this calculation, the team went on to find out what the consecutive totals should be, finding that the numbers 8, 9, 10, 11, 12 and 13 added to 63. They then took the next step towards systematically solving the problem. They produced the following list of ways in which the consecutive totals can be made.

Then we made the list:
8 = 5 + 2 + 1 = 4 + 3 + 1
9 = 6 + 2 + 1 = 5 + 3 + 1 = 4 + 3 + 2
10 = 6 + 3 + 1 = 5 + 4 + 1 = 5 + 3 + 2
11 = 6 + 4 + 1 = 6 + 3 + 2 = 5 + 4 + 2
12 = 6 + 5 + 1 = 6 + 4 + 2 = 5 + 4 + 3
13 = 6 + 5 + 2 = 6 + 4 + 3

From now on, they showed great skill in positioning the numbers for a row total of 8, and explored ways in which the diagram could be completed after that. Here is the way they used the 8 = 4 + 3 + 1 combination.

We chose three numbers which the total was 8:1,4 and 3.
We placed them into the triangles.
Then, we saw the totals which contain "1" among the rest of the numbers were:
12 = 6 + 5 + 1
9 = 6 + 2 + 1
We placed 6 and 5 into the triangles.
Then we put '2' into the last triangle.

By swapping the 5 and 6, the team showed another arrangement that worked, and then by rearranging the 1, 4, 3 combination, they discovered further possibilities. Rather than give the complete solution, we invite readers to visit the *Hexagonia* part of the Natural Maths website (www.naturalmaths.com.au/hexagonia) where the full set of twelve solutions are explained.

Conclusion

Our aim has been to give examples of the different approaches that primary school students gave to a numerical problem for which there is many more than one correct answer. We have pointed to the limitations of the guess and check strategy and shown that, where a systematic list was made of arrangements, the large number of possible solutions becomes accessible by Year 4–5 students. Also important in the problem solving process was the use of concrete materials to support the investigation. In most classrooms, the teacher will see the variety of approaches that we have shown here in evidence in the students as they work on a problem such as the one given. Through reflection, feedback and discussion, students can learn that different strategies can be successful and that a systematic approach often helps in ensuring that all possible solutions have been located.

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