SITTING AT THE BACK

Noticing 'Math Noticing'

LEN SPARROW

highlights the significance of helping children make connections in mathematics via problem solving. He describes how teachers encourage children to think mathematically by 'taking notice' of the mathematics they encounter.

Introduction

Recently, I have been fortunate to be a visitor in a number of classrooms in the United States, England and Europe. On one such visit, I was placed at very short notice in an Elementary School in Columbia, Missouri. The teachers accepted me into their normal classrooms for their normal lessons (they had no time to prepare any special 'visitor lessons'). I moved between three classes: Grade 1, Grade 2 and Grade 4. The following is a snapshot account of what I saw and my reactions to it.

Reform mathematics

The latest *Principles and Standards for School Mathematics* document from the National Council of Teachers of Mathematics (NCTM, 2000) as well as the earlier *Curriculum and Evaluation Standards for School Mathematics* document (NCTM, 1989) suggested that children of all ages should be engaged in working in a mathematical way via problem solving. Problem solving is regarded not only as a goal of mathematics but also as a means of learning mathematics. They suggested that:

> Students should have frequent opportunities to formulate, grapple with, and solve complex problems that require a significant amount of effort and should then be encouraged to reflect on their thinking (p. 52).

Part of what has become known as 'reform mathematics' is appearing more often in classrooms. Reform mathematics

involves children interacting with mathematics beyond the traditional style of listening to the teacher and reproducing the methods demonstrated. More responsibility is placed on learners to solve problems, make sense of situations, and apply and use mathematics in contexts.

Curriculum documents from an Australian context, for example *A National Statement on Mathematics for Australian Schools* (Australian Education Council, 1991), the *Curriculum Framework for Kindergarten to Year 12 Education in Western Australia* (Curriculum Council, 1998) and the latest syllabus document from New South Wales (Board of Studies, 2002) all highlight and include children working mathematically. For example,

> Students should develop their capacity to use mathematics in solving problems individually and collaboratively. (AEC, 1991, p. 12)

Booker, Bond, Sparrow and Swan (2004) summarised the change in emphasis in mathematics teaching and learning encapsulated in the reform movement in mathematics education:

> An ability to think with and about mathematics has come to be the dominant feature of what has to be learned rather than the set procedures and directed solving of straightforward problems that occurred in the past. (p. 37)

In a study of effective teachers of numeracy in the primary school, Askew (1999) noted that they seemed to pay attention to:

- connections between different aspects of mathematics, for example, addition and subtraction or fractions, decimals and percentages;
- connections between different representations of mathematics: moving between symbols, words, diagrams and objects;
- connections with children's methods — valuing these and being interested in children's thinking but also sharing their methods.

I used these features to guide my observations in the classrooms I visited.

First Grade (5/6 years old)

'How many pets do you have?' asked the teacher. She received various responses from her enthusiastic children. At one point, it appeared to become a competition to see who had the most pets with truth and accuracy replaced by fantasy and exaggeration, for example, one child admitted to three pets at the start of the discussion but later offered that he had ten pets. The teacher then produced a *Dr Seuss* book that the children knew and reminded them about the dream where there had been twelve pets. The pets (cats and dogs) were moving around so quickly that it had been impossible to count them accurately.

Teacher (T): How many cats and how many dogs might have been in my dream?
Sam: 10 dogs and 2 cats.
T: How did you know?
Sam: I know 10 add 2 is 12.
Alison: 11 dogs and 11 cats.
T: Does that make 12 altogether?
[Some children murmur, 'No'.]
T: How might Alison show this?
Chris: She could use cubes — yellow for cats and blue for dogs.
[Alison puts 11 yellow and 11 blue cubes together.]
T: Let's count them.
[All count to 22.]
T: So Alison has 22 pets. How can she make 12 pets?
Chris: She needs to take off 10 dogs.
T: How will that make 12?
Chris: 22 take 10 is 12.
T: Can you explain this please?
Chris: I have 2 tens and 2. If I take off 1 ten I will have 1 ten and two left, which is 12.

The children were set the problem, 'Find how many combinations of dogs and cats might have been in my dream'. They were given a largish sheet of paper and told to record their 'math thinking'. They were told they can use cubes, words, numbers, and pictures and yes they could use coloured pens but only for 'math recording and not art'. Chris noted it was like the 'Peas and carrots' activity they had explored earlier. Children returned to their seats and I followed Alison to see what she did.

As I reached the table, I noticed that Sam had already started and had:

11 dogs and 1 cat
11 cats and 1 dog

He said something about changing them round. I commented that it was a great idea and a good way to work. I noted that he continued:

8 cats and 4 dogs
5 cats and 7 dogs

No sign of the switching strategy. Meanwhile Alison had:

11 cats and 1 dog
10 cats and 2 dogs

I asked what might come next. She offered confidently:

9 cats and 3 dogs

Alison told me about the 'going down' pattern of 11 to 10 to 9. She continued to follow the pattern. Her only difficulty in the task appeared to be the horizontal alignment of cats and dogs in her personal recording system. In drawing the lines in a freehand style between the left and right hand side of the paper she sometimes connected the wrong pets. She had almost completed her combinations when the teacher called all children back to the mat area for a discussion of their findings. I noted that Sam had only completed a couple more random combinations of pets. Other children had a recording style of dog and cat pictures and had only a few combinations completed. There were a variety of recording styles throughout the room.

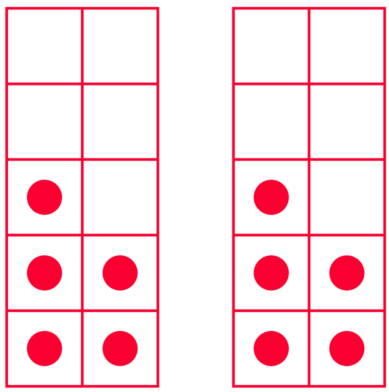
The teacher selected children to offer their way of recording and modelled an initial letter and number version for the children to see.

Dogs	Cats
7 d d d d d d d 4 d d d d	5 c c c c c 8 c c c c c c c c

The teacher reminded the children to talk only about their 'math noticing' when someone in typical Grade One style offered, 'I like the green pen'. Throughout the discussion she emphasised 'math noticing'. In particular, she wanted them to notice the 'flip flop' of 11 dogs and 1 cat with 1 dog and 11 cats. Some children managed to offer examples but, as she reported later, the majority of children found this idea difficult and beyond their understanding at the present time. She ended the session with pairs of children showing each other their work and talking to each other about the 'math noticing' they had done in the activity.

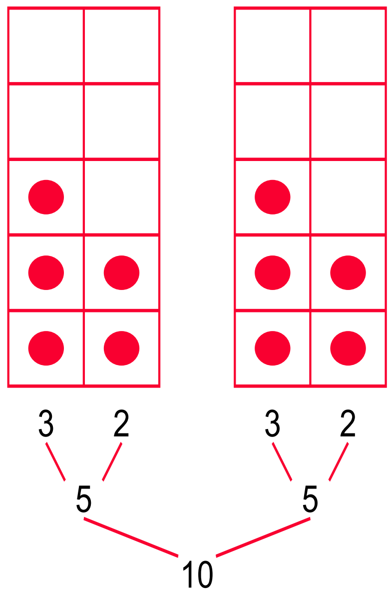
Grade 2 (6/7 years old)

The teacher flashed onto the screen via the overhead projector the following pair of ten frames. The children saw them for approximately five seconds before the OHP was turned off. They were set the task to write an incomplete number sentence to 'figure out the number of dots shown'.

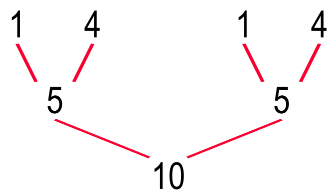


For those who needed it, there was a second flash, again only for a very short time, far too short a time for children to count single dots. Children were invited to share their strategy for 'figuring out the number of dots'.

Jack spoke about seeing groups of dots. The teacher asked him to describe this and the number sentence he wrote.



Maree saw a different pattern



Each time, the teacher asked for clarification and confirmation from the child that she was recording exactly what the child wanted. She also connected the described strategies to other general computational strategies the children knew, for example 'doubling' or 'quick ten' and 'doubles plus one'. Another pair of ten frames and dots was then flashed onto the screen and the process repeated.

Beat the calculator

The teacher employed a second task for the children that also embedded mental strategies. One child was given a calculator and was told that s/he had to use it for the task. The other child could only do the calculation 'in the head'. The pair was given a ziplock pack of cards onto which were written incomplete number sentences that embedded mental computational strategies, for example make 10 or double, of the form:

$\boxed{5 + 2 + 5 + 1 =}$ $\boxed{7 + 2 + 3 + 2 =}$

The children drew a simple recording chart for the results of their activity:

Brain	Calculator

The cards were placed face down in front of the pair of children. The child with the calculator was instructed to turn over the card and key into the calculator all parts of the exposed number sentence. The other child had to calculate the answer using only the brain. The pair recorded on the chart which category was first to offer the correct answer — brain or calculator. Roles were reversed and the process continued until all cards were used.

On most occasions on most charts the 'brain' easily outpointed 'the calculator'. This surprised some children. The teacher highlighted the fact that it is often quicker to use the brain than the calculator. She also noted that finger counting was a very slow strategy and probably accounted for the calculator being quicker when children used that strategy. Some children were asked to share their strategies for beating the calculator. The teacher again connected children's strategies to those familiar to the class.

Fourth Grade (9/10 years old)

As the children arrived in their classroom, the problem for the morning was already available, much as it was on most days. Trafton's (1999) idea of 'routine' was established and the children quietly and confidently began work without

teacher intervention, as they were familiar with the expectations of the task. Children had to calculate in any way they wished the following:

53×4
74×8

The calculations were a problem for the children, as they had not been taught a standard method to find the answer. This was a generic task similar to those presented by McIntosh, De Nardi and Swan (1994) called, 'How did you do it?'. The children knew they would be expected to explain to the rest of the class their method of solution, that they could expect support from the teacher and their peers, and that they would hear other people's solutions but they would not be told how to do the calculation.

After quite a short time for the children to complete the task the teacher asked for children to volunteer with their solutions. She reminded the class that they had to listen, to think and to check the solution to see if it worked. The children who spoke did so with confidence. This was a change from the usual reluctance of adults and children to explain mathematics in a public forum.

Bri-Ellen offered:

53×4 as

$50 \times 4 = 200$
$3 \times 4 = 12$
212

Here 53 is seen as 50 + 3. The number is understood as a quantity (50 and 3) rather than a pair of digits (5 and 3).

Terry offered:

53×4 as

$50 \times 2 = 106$

$106 \times 2 = 212$

The child here used the powerful doubling strategy having seen 4 as 2×2 or double, double.

The teacher took only two examples but noted she had seen others. She commented how the methods, while being different, were similar and used ideas the class had talked about in previous lessons.

For the second calculation Kane offered:

74×8 as

$70 \times 8 = 560$
$4 \times 8 = 32$
592

This is the typical way the calculation is completed with the standard vertical method. These children, however, had not been shown this method.

Sam offered:

74×8 as

$74 \times 2 = 148$
$74 \times 2 = 148$
$74 \times 2 = 148$
$74 \times 2 = 148$

These were added 592

Another child commented that it was a good way to complete the calculation but it could be done in a slightly different way by using eight as $2 \times 2 \times 2$ or double, double, double.

$74 \times 2 = 148$
$148 \times 2 = 296$
$296 \times 2 = 592$

The first child had used 8 in an additive sense of 2 + 2 + 2 + 2 while the second child had used a multiplicative strategy having seen 8 as $2 \times 2 \times 2$.

The teacher noted the methods suggested by the children and later wrote them onto large sheets of paper that were displayed in the classroom for the children to see and maybe use at a later date. She also had an expectation that children would use these displays to make personal records of available methods in their mathematics notebooks. They already had examples for addition and subtraction to which they could refer. The teacher was aware of the different strategies that could be used for calculations. She did not, however, teach these strategies directly but made sure that in the reporting sessions children were exposed to many possible methods. She selected the calculations she gave to the children to embed specific methods and so exposed the children to the variety available. The teacher also set the

requirement that individual children were not allowed to ask her for help unless all their team was stuck or unsure, thus forcing the children to talk to each other and cooperate in understanding the task and solution.

Commentary

It was most pleasing to note, after earlier observing in another state low-level textbook work, the mathematical thinking and noticing by children as I sat at the back of the room. These were not specially prepared lessons; they were the ones that normally happened in the classroom. In all cases, the responsibility for thinking and ‘math noticing’ was firmly with the children. Mathematics started rather than finished with a problem. The teachers rarely gave explanations or methods for working. Children did not have teacher or textbook models to remember and replicate. Textbooks, the Investigations series, were part of the school but did not dominate and direct the mathematics that happened in classrooms. This was the responsibility of the teachers. The books followed a ‘reform curriculum’ based on the *Principles and Standards* outlined by the National Council of Teachers of Mathematics (NCTM). Textbooks and blackline master sheets were not overtly present in the classes I observed. The tasks for the children formed part of the overall philosophy of the NCTM curriculum and had possibly been gleaned from the teacher’s book of the *Investigations* textbook series.

The teachers were aware of the potential for mathematics learning of the tasks and were quite directed and strategic in their work with children, while at the same time appearing not to lead the children’s thinking. Teachers were selective about who showed their strategy to the class. This avoided repetition of familiar methods that were already comprehended by the majority of the class but at the same time exposed new ideas, thinking and methods. Teachers also used this time to be explicit about the connection of the mathematics suggested by a child to other things in mathematics that the children had experienced. Reporting to the class was not therefore an ad hoc, haphazard, and low-level ‘show and tell’ exercise but something a whole lot richer.

The teachers constantly asked questions of children rather than supplying answers. They asked for clarification, explanation and development of ideas and methods. They helped children connect ideas in mathematics. The purpose of the lesson or task was made obvious to the children. They required children to think and notice mathematics.

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