Paper folding FRACTIONS

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This classroom activity can be used to describe areas of folded polygons in terms of a standard unit of measure (inches or centimetres). We begin with a square sheet of paper, say 5 units by 5 units and fold it along the two diagonals (see Figure 1).

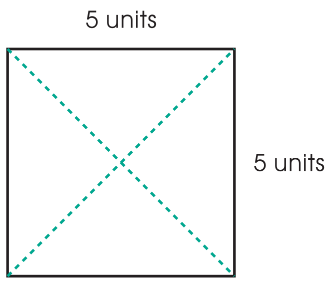


Figure 1. Fold along diagonals.

How many different triangles are formed? Actually, there are four small triangles and four large triangles the small triangles are all congruent and the large triangles are all congruent. What is the area of each of the small triangles and what is the area of each of the large triangles? Since the area of the square is $5 \times 5 = 25$ square units, your students should be able to reason that the area of each small triangle is $\frac{25}{4} = 6\frac{1}{4}$ square units since the original square is partitioned into 4 congruent parts. The area of each large triangle is $\frac{25}{2} = 12\frac{1}{2}$ square units since either diagonal partitions the square into two congruent triangles.

Next, we fold each corner of the square to the centre of the square making a fold line that is perpendicular to the diagonal to that corner, at the midpoint of the segment whose endpoints are the centre of the 5×5 square and the corner (see Figure 2). This forms a small triangle called a "corner triangle" because one of its vertices is the corner of the original square.

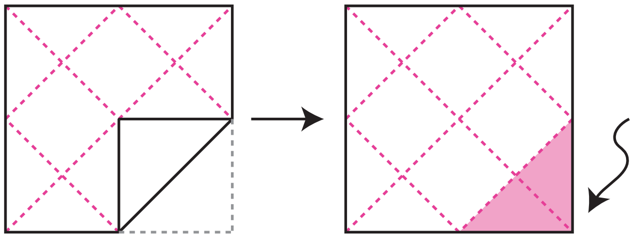


Figure 2. Folding the corners to the centre.

Note that the base of each "corner triangle" is the segment formed by the midpoints of the sides of the square that form the corner. This creates other shapes whose areas we can discuss by reasoning (or computation). For example, by folding in all corners your students will discover that the four "corner triangles" have the same area as the inner square because the triangles "cover" the inner square. Since together the four folded triangles and inner square make up the original square, the area of each (inner square or folded triangles) must be $\frac{25}{2} = 12\frac{1}{2}$ square units. Thus the area of one of the folded triangles is $\frac{1}{4} \times \frac{25}{2} = \frac{25}{8} = 3\frac{1}{8}$ square units, and half of one of these folded triangles is $\frac{1}{2} \times \frac{25}{8} = \frac{25}{16} = 1\frac{9}{16}$ square units. Also, since the inner square is divided into four smaller squares, each smaller square has area $\frac{1}{4} \times \frac{25}{2} = \frac{25}{8} = 3\frac{1}{8}$ square units, the same as a folded triangle. This makes sense because two of the halves of the folded "corner" triangle will fit to make a small square.

Using this information and folding or not folding on the various fold lines students can create different polygons and use reasoning to find their areas. If, as a teacher, you want to have your students verify that the formulas for finding areas of squares, rectangles, triangles, and trapezoids are consistent with their reasoning this can be assigned too (see *Paper folding fractions worksheet*). Note: this can be done without formulas and without resorting to the Pythagorean Theorem. If formulas are used it will be necessary in some cases to rely on using the Pythagorean Theorem. Students may also discover that all shapes can be partitioned into the smallest triangle (with area $\frac{25}{16}$ square units), thereby allowing one to find the area of all shapes by counting the number of the smallest triangles and multiplying by $\frac{25}{16}$ to find the area of the folded shape.

From Helen Prochazka's

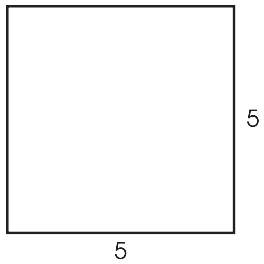
Scrapbook

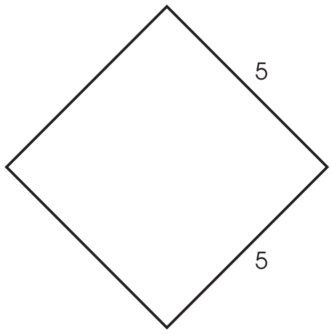
Every science that has thriven has thriven upon its own symbols: logic, the only science which is admitted to have made no improvements in century after century, is the only one which has grown no symbols.

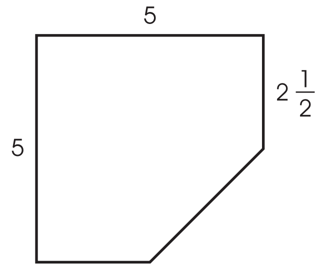
Augustus De Morgan (1806–1871)
Transactions vol. X, 1864, p. 184,
Cambridge Philosophical Society.

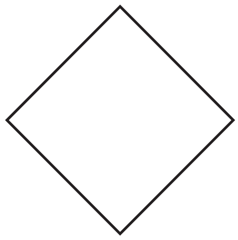
Paper folding fractions worksheet

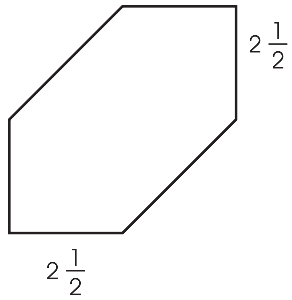
Directions: Fold your paper to obtain each polygon below and find its area. (Original shape is to scale.)

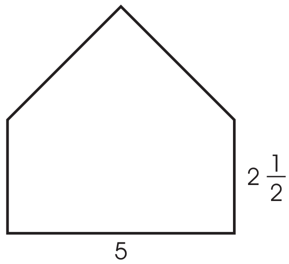


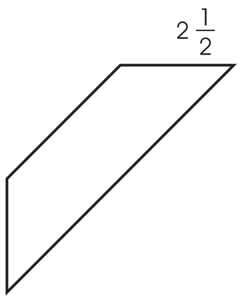


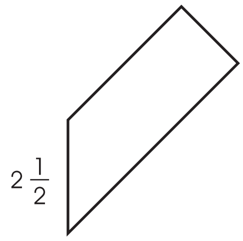


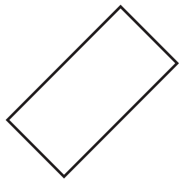


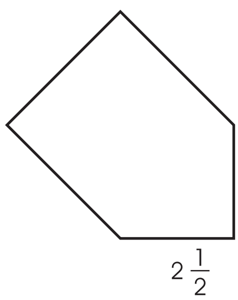


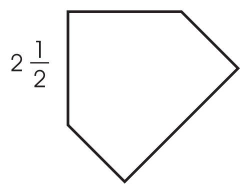


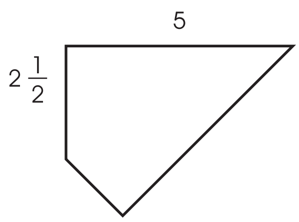






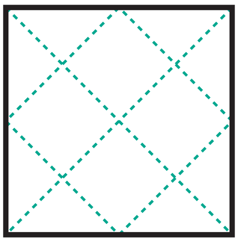




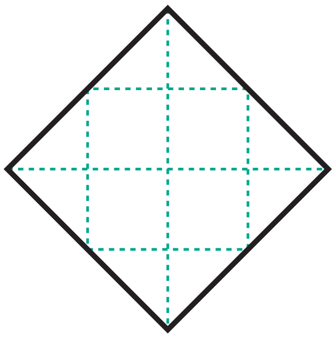


Paper folding fractions worksheet — Solutions

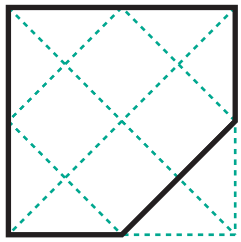
All answers are in square units.



$A = 25$

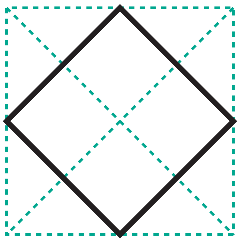


$A = 25$

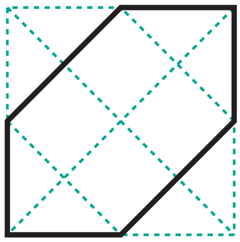


$A = 25 - \frac{25}{8} = 21\frac{7}{8}$

or $A = \frac{25}{2} + 3\left(\frac{25}{8}\right) = \frac{175}{8}$

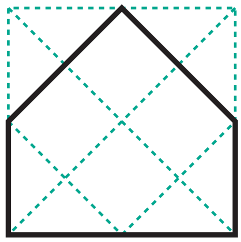


$A = \frac{25}{2}$

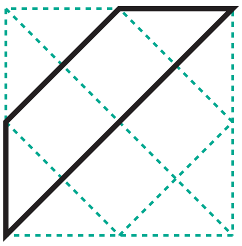


$A = \frac{25}{2} + 2\left(\frac{25}{8}\right)$

$= \frac{50}{4} + \frac{25}{4} = \frac{75}{4} = 18\frac{3}{4}$

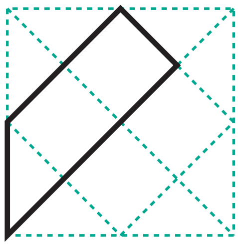


$A = \frac{25}{2} + 2\left(\frac{25}{8}\right)$



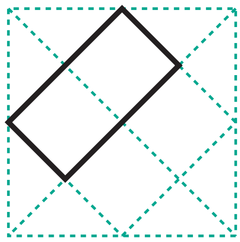
$A = \frac{1}{2} \times \frac{25}{2} + 2\left(\frac{25}{16}\right)$

$= \frac{50}{8} + \frac{25}{8} = \frac{75}{8}$

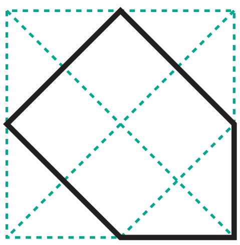


$A = \left(\frac{1}{2} \times \frac{25}{2}\right) + \frac{25}{16}$

$= \frac{100}{16} + \frac{25}{16} = \frac{125}{16}$

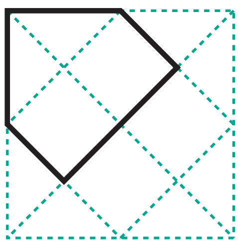


$A = \left(\frac{1}{2} \times \frac{25}{2}\right) = \frac{25}{4}$



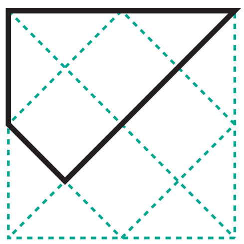
$A = \frac{25}{2} + \frac{25}{8}$

$= \frac{100}{8} + \frac{25}{8} = \frac{125}{8}$



$A = \left(\frac{1}{2} \times \frac{25}{2}\right) + \frac{25}{8}$

$= \frac{50}{8} + \frac{25}{8} = \frac{75}{8}$



$A = \left(\frac{1}{2} \times \frac{25}{2}\right) + \frac{25}{8} + \frac{25}{16}$

$= \frac{100}{16} + \frac{50}{16} + \frac{25}{16} = \frac{175}{16}$