Characteristics of Mathematics Teaching in Shanghai, China: Through the Lens of a Malaysian

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The mathematical performance of Chinese students, from mainland China, Hong Kong and Taiwan, is widely acclaimed in international comparisons of mathematics achievement. However, in the eyes of the Western educators, the environments established in Chinese schools are deemed relatively unfavourable for mathematics learning. This paper reports on a study that investigates the characteristics of effective mathematics teaching in five Shanghai schools. Findings reveal that those characteristics include (a) teaching with variation; (b) emphasis of precise and elegant mathematical language; (c) emphasis of logical reasoning, mathematical thinking and proofing during teaching; (d) order and serious classroom discipline; (e) strong and coherence teacher-student rapport, and (f) strong collaborative culture amongst mathematics teachers.

Over the past decades, Chinese students, particularly those from Mainland China, Hong Kong and Taiwan, have consistently excelled in a number of international comparisons of mathematics achievement (Mullis et al., 2000; OECD, 2001; Robitaille & Garden, 1989; Stevenson, Lummis, Lee, & Stigler, 1990). For example, in the Second International Mathematics Study (1976-1989), Hong Kong students ranked first in the 17-18 year old age group (see Fan & Zhu, 2004, p. 20). In the Second International Assessment of Educational Progress (1990-1991), students from Mainland China ranked first in the 13 year age group. Similarly, 15 year old Hong Kong students took first place in the Programme for International Student Assessment (PISA 2000). However, the mathematics learning conditions of Chinese classrooms were described as “crowded, large class size, passive learners, dominant teachers” while the teaching method was found to be “passive transmission” and “rote drilling” (Biggs, 1991, 1994; Morris, 1985). These features are, in the views of Western educators, not conducive to effective mathematics learning. This contradictory condition of mathematics learning of the Chinese learners is termed ‘the paradox of Chinese learner’ (Watkins & Biggs, 2001).

What are the significant characteristics of the Chinese learners that enable them to excel in many of the comparative mathematics achievement studies? What are the possible contributing factors? What can be learned from the Chinese way of mathematics teaching and learning so as to improve mathematics education? Indeed, these are questions that have attracted numerous mathematics educators and researchers, from both the West and the East. In fact, this has led to the publication of a book entitled ‘How Chinese learn mathematics: Perspective from insiders’ (Fan, Wong, Cai, & Li, 2004).
Taking the opportunity of a six months visit to Shanghai, China (from November 2004 to April 2005), sponsored by the Asian Scholarship Foundation, I set out to explore the culture of mathematics teaching in China, from my perspective as a Malaysian. In this paper my specific focus is centred on the characteristics of mathematics teaching in Shanghai. The discussion will be based on classroom observation, document study, and interviews with mathematics teachers and students, and will be supplemented by related research literature on mathematics education in China.

Conceptual Framework

Ball, Lubienski, and Mewborn (2001) noted that “what teachers and students are able [to] do together with mathematics in classrooms is at the heart of mathematics education” (p. 433). This highlighted the central importance of the role of both the mathematics teacher and the students, as key performers in mathematics classrooms. Prior to the commencement of a class, a mathematics teacher needs to plan the lesson, select and decide what content to teach now and what to teach later, as well as to choose the best teaching strategy that suits the students’ level of understanding. Moreover, the planned lesson needs to be able to motivate and to attract students to continue learning. Several studies have shown that all these decisions are very much influenced by a mathematics teacher’s level and depth of content knowledge (see Fennema & Franke, 1992; Ma, 1999) and teaching skills, as well as his or her philosophy, values and beliefs of mathematics and mathematics teaching (Bishop, 1991; Chin, 1995; Ernest, 1989; Thompson, 1992). In addition, students’ image of mathematics, and their values, beliefs and attitudes towards mathematics learning, need to be coherent with that of their mathematics teacher so as to guarantee a cohesive learning environment (Seah, 2005). Apart from the mathematics teachers and students themselves, the demand of the syllabus, public examinations, and school assessment could also influence the decision and way of teaching of the mathematics teacher. Last, but not least, the cultural context (Bishop, 1994), expectations of society and the parents’ demands (see Lim, Fatimah, & Tan, 2004) could also influence the way mathematics is taught in class. Thus, in search of the characteristics of mathematics teaching in Shanghai, all these factors will be explored.

The Study

Participants of the study

Five schools from Shanghai – one preschool, one primary, one middle school, and two high schools – participated in this study. These schools were located around the host university to which I was attached. These were not elite or selected schools but ordinary schools with students whose parents came from the middle to lower income groups. They were recommended to me through the host lecturer who happened to have some postgraduate students teaching in these schools.
For each school, two mathematics classes were selected for mathematics teaching observation and the mathematics teacher from each class was interviewed. As well, a number of students from the middle and high schools were interviewed. Not all students were interviewed because preschool and primary school students were deemed too young to articulate their thoughts fully. To gain a better overview of the organisational structure and administration of the school, the top administrators such as the principal and the head of the mathematics department of each school were also interviewed. Hence, a total of five principals, five mathematics department heads, 10 mathematics teachers and 12 mathematics students were interviewed. As some of the mathematics teachers allowed me to observe their teaching for more than one lesson, a total of 19 mathematics lessons were observed and video-taped for analysis.

Methods of Data Collection and Analysis

Qualitative data were collected using the following three methods:

1. A document study of the mathematics curriculum and textbook used in Shanghai schools: A detailed analysis was made of the major curriculum document of Shanghai, namely the Shanghai City primary and secondary mathematics curriculum standard (2004). The analysis aimed to identify the aims and objectives as well as the structure of Shanghai mathematics curriculum.

2. Video-taped classroom observations of mathematics teaching: Each of the 19 lessons was observed by at least the author and a postgraduate student from the host university. Some lessons were also observed together by two or three mathematics teachers of the same school. Each lesson lasted about 45 minutes. All the video-taped lessons were played back several times during analysis, in order to identify the special features or episodes of mathematics teaching in these lessons.

3. Interview transcripts with the administrators, mathematics teachers and students: Altogether 32 interviews were made [with five principals, five mathematics department heads, 10 mathematics teachers and 12 mathematics students]. All interviews were transcribed and analysed using the computer software, NUD*IST N5 (QSR, 2000).

All the data were collected from November 2004 to February 2005.

Findings and Discussion

Mathematics Curriculum and Textbook used in Shanghai Schools

All Shanghai schools subscribed to one major curriculum document, the Shanghai City Primary and Secondary Mathematics Curriculum Standard (2004). The document is divided into two parts. The first part outlines the theoretical framework, the aims and objectives, the structures and implementation of the general primary and secondary education in Shanghai. The second part is
subdivided into five sections: (a) introduction to the theoretical framework of the curriculum, (b) aims and objectives, (c) curriculum design, (d) content and demand, and (e) curriculum implementation.

In addition, the Shanghai curriculum standard divided the content into five key stages: (a) Primary Grades 1-2, (b) Primary Grades 3-5, (c) Middle school Grades 6-7, (d) Middle school Grades 8-9, and (e) High school Grades 10-11. For each key stage, the content was further divided into basic and extension. The basic content included numbers and operations, algebra and equations, graphs and geometry, statistics and probability, function and analysis. The extension section covered the same content areas, but each area was extended in breadth and depth. Each content area or topic was displayed in a table listing the learning content and displaying the detailed content demand and suggestions for teaching activities. Teachers and students were encouraged to teach and learn the extended content through various innovative and creative activities, such as investigation projects, solving daily life problems, mathematical puzzles and games as well as history of mathematics.

One significant feature of this curriculum document was that the document explains in clear detail how each of the objectives and content could be taught. These descriptions are supplemented with suggestions and examples. However, mathematics teachers are not required to follow the document strictly; instead they are very much encouraged to expand and explore further mathematical content as needed.

Analysis of Classroom Observations of Mathematics Teaching

A holistic approach was used to analyse the 19 video-taped mathematics lessons. I chose to use this approach because all the observed lessons were randomly selected. They varied in terms of topic and grade level. Hence, these lessons were analysed in accordance with a general lesson, which included the induction set, explanation or development of concept, skill acquired, summary, and homework given. Any noteworthy additional characteristics were also analysed. Table 1 summarises the general features and characteristics of primary and secondary school mathematics teaching in Shanghai.

A typical mathematics lesson in Shanghai primary or secondary schools, as illustrated in Table 1, appears orderly and routine. Nevertheless, a micro analysis of classroom practice, a document analysis of syllabus and interviews with mathematics teachers, students and school administrators, coupled with evidence from related research literature on mathematics classrooms in China, have illuminated otherwise. I will highlight and discuss some of these characteristics in the next section.
Table 1
General Features and Characteristics of Primary and Secondary School Mathematics Teaching in Shanghai

<table>
<thead>
<tr>
<th>Feature</th>
<th>Characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Induction set</td>
<td>Revise previous/related concept.</td>
</tr>
<tr>
<td>Concept explanation</td>
<td>Concept variation – different kinds of examples and difficulty level.</td>
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<tr>
<td>Skill acquisition</td>
<td>Demonstrate and practice examples and exercise questions with significant variation of difficulty level.</td>
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<tr>
<td>Summary</td>
<td>Teacher summarised.</td>
</tr>
<tr>
<td>Homework</td>
<td>Half an hour standard or can be completed in school.</td>
</tr>
<tr>
<td>Engagement of students</td>
<td>Calls individual student to demonstrate in front of class, answer or explain orally, seat work, group discussion.</td>
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<tr>
<td>Classroom atmosphere/discipline</td>
<td>Serious and orderly.</td>
</tr>
<tr>
<td>Emphasis</td>
<td>Primary school:</td>
</tr>
<tr>
<td></td>
<td>Precise language</td>
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<tr>
<td></td>
<td>Logical reasoning &amp; deductive thinking</td>
</tr>
<tr>
<td></td>
<td>Inspiring and encouraging voice</td>
</tr>
<tr>
<td></td>
<td>Using ICT such as Power point, multimedia presentation</td>
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<td></td>
<td>Using teaching aids</td>
</tr>
<tr>
<td></td>
<td>Secondary school:</td>
</tr>
<tr>
<td></td>
<td>Precise language</td>
</tr>
<tr>
<td></td>
<td>Strict format of writing</td>
</tr>
<tr>
<td></td>
<td>Logical reasoning &amp; deductive thinking</td>
</tr>
<tr>
<td></td>
<td>Using ICT such as Power point and Geometric Sketch Pad</td>
</tr>
<tr>
<td>Pace of teaching</td>
<td>Fast</td>
</tr>
<tr>
<td>Integration of daily life problems</td>
<td>More obvious in Primary school</td>
</tr>
</tbody>
</table>
Characteristics of Mathematics Teaching in Shanghai

Teaching with variation. During explanation of mathematical concepts, Shanghai mathematics teachers tended to use different kinds of examples. At least three examples, varying in connotation or difficulty level, were given in any one lesson. This was termed “concept variation” (Gu, 1994). For example, when teaching the concept of function, a secondary school mathematics teacher (SH4) developed the concept by giving the example: “Given \( f(x) = x^2 \), \( g(x) = 1 - 2x \), is \( f(x) + g(x) \) still a function?” This was then followed by a second example: “Given \( f(x) = \sqrt{x - 2} \) and \( g(x) = \sqrt{1 + x} \), is \( f(x) + g(x) \) still a function?” Later, another five different expressions were given to evaluate the students’ understanding:

(a) \( y = \sqrt{x - 2} - \sqrt{1 - x} \),
(b) \( y = \sqrt{x - 1} + \sqrt{1 - x} \),
(c) \( y = \begin{cases} x^2 + x + 2 & x > 0 \\ 2x - 3 & x < 1 \end{cases} \)
(d) \( |y| = x^2 + 2x \)
(e) \( y = x^n \)

Some of the given examples represent a function while some are examples of non-functions unless they fulfill certain criteria or conditions. By giving examples which vary in conceptual difficulty, teachers were challenging students to think and to develop a clearer understanding of function.

Similarly for skill requirement, the Shanghai mathematics teachers used procedural variation. This was evidenced in the use of multiple methods of solving a problem, and in the practice of giving classroom exercises as well as examination and test questions in a variety of formats and structures. Procedural variation provides students with ample opportunity for drill and practice. It is also used to diagnose students’ understanding of the concept at various levels and to test if they have mastered the requisite skills.

In fact, this way of teaching with variation has been applied either consciously or intuitively in China for a long time (Gu, Huang, & Marton, 2004). In particular, it was greatly promoted after Prof Gu Linyan’s (1994) fifteen years Qingpu experiment that aimed to study and implement the notion of “teaching with variation.” This feature was also described by Huang and Leung (2004) in their list of good teaching practices in Shanghai: “The teachers provided exercises with variation” (p. 376).

Emphasis on logical reasoning, mathematical thinking and proof during teaching. This was evident from classroom observations where high level thinking skill questions such as ‘why?’, ‘how?’, ‘what if?’ were asked during lessons. When teacher SH4, for example, was developing the concept of function, questions such as: “Is this a function? If yes, please give reasons, if not, you must also give reasons” were used. Likewise, when teacher SH2 was teaching the topic, shape, to the Primary One students, she asked questions such as: “Why couldn’t we build the truck’s wheels using rectangular shapes or cubes?” and “What are the differences between triangle and rectangles?” Similarly, when teacher SH8 was
teaching algorithmic index, she asked: “What rule have you found?” “This is the product of two numbers, what happen if there are 3 or 4 numbers?” It was common practice amongst the Shanghai mathematics teachers to ask questions such as “Why?”, “How?”, “What if?”, “How do you know that?” to stimulate their students to think and to make use of logical deduction. This kind of teaching emphasised mathematical reasoning and promoted much verbal discussion and interaction between the teacher and students as well as between the students themselves.

Perhaps this was not surprising because the above emphasis was found to have been incorporated in the _Shanghai City Primary and Secondary Mathematics Curriculum Standard_ (2004) as one of its objectives for the middle and high school level mathematics. The Standard states that students should be able: “to understand the function of using mathematical thinking method to practise mathematical thinking and to solve problems” (p. 35) and “to understand the importance of proof, master basic deductive reasoning principles and methods, and be able to explain logically and systematically the accuracy of deductive reasoning” (p. 38).

Analysis of the interviews with one of the mathematics teachers (SH5) also emphasised the fact that “mathematics should be helpful to develop a person’s logical thinking ... . I believe mathematics learning process needs to focus on thinking.” Consequently, teacher SH5 argued that he always made sure that his students were given ample time to think and to explore by themselves. When his students managed to solve a problem, they were asked to explain their solutions to the whole class. If they could not, the teacher would give a few hints or ask guiding questions to help them. Teacher SH5 further lamented that he strongly believed that only when the students were able to think for themselves, and organise their thoughts, would they be able to internalise the mathematical knowledge that they had learnt.

**Emphasis on the use of precise and elegant mathematical language.** This was evident from both classroom observations and the interviews with the Shanghai mathematics teachers. For example, when a student stated the definition of a perpendicular bisector to the whole class, teacher SH4 was not satisfied with the explanation although it was deemed reasonably correct. The student was asked to state it again using the least number of words. When another student was able to state the definition precisely and as close as possible to what was defined in the textbook, teacher SH4 then praised him for his aptitude in using elegant mathematical language.

In another case, a middle school mathematics teacher (SH3) emphasised the precise format of writing an algebraic expression. Likewise, another primary school mathematics teacher (SH1) stressed the importance of precision and the correct method of reading the unit of speed. In the context of students’ working on proof, a high school mathematics teacher (SH7) demanded that his students state a reason for every step, even if it was seen to follow through from the given criteria or previous condition. He said: “You must state that ‘based on previous step.’” He also tended to remind his students “to be careful” about possible mistakes or misconceptions.
During the interview, teacher SH7 explained that he emphasised strict format of a proof or mathematical algorithm because “in the high school mathematics examination paper, if you do not write according to the mathematical format required, marks will be deducted. Every mark in the high school examination counts and it can change a student’s future.” Hence, he always made sure his students wrote out their solution in a precise and logical format. Even though his remark reflected the influence of the public examination, it also reflected the curriculum requirement that mathematics be considered a strict and serious discipline.

*Order and serious classroom discipline.* In the 19 mathematics lessons that I observed, classroom discipline was always orderly and serious. Even in the primary school, when the students were given tasks to discuss with their peers, focused discussion, rather than off-task talk, predominated. The teacher was able to restore order quickly by clapping her hands. In the Primary One class that I observed, the students appeared noisy and busy discussing with their peers for several minutes. However, once they had completed their tasks, they immediately sat still and put both their hands behind their back, as if they were accustomed to do so.

Perhaps this phenomenon is not uncommon in China or Hong Kong, since it was described as one of the features of the Confucian Heritage Culture (CHC) learners’ phenomenon by Wong (2004). Wong (2004) argued that attention and discipline in class are first priority in the CHC learning environment. In such an environment, the teacher appears to be central, yet it represents, in my view, a student-centered approach to teaching. There is a cultural assumption that students know what should be done at every moment of the class, such as when to talk, when to do seat work, when to open one’s book and when to put up one’s hand. These “trainings are developed through reinforcement, social contracts, conformities, and social negotiations which are common in the CHC classroom and CHC teacher programs” (Wong, 2004, p. 526).

*Teacher-student rapport.* Although the classroom atmosphere was observed to be serious and orderly most of the time, the rapport between teacher and students appeared to be close and affirming. I found most of the Shanghai mathematics teachers often used inspiring and encouraging words during their teaching such as “Do continue with what you want to say”; “You must believe in yourself”; “Be brave and say what you think. If it is wrong, we can change”.

This was apparent in the interview with one student. When he was asked to describe his learning experience in a mathematics class, he explained it in this way: “During mathematics class, you are asked to answer questions. If you can do them, then you put up your hand, if you can’t, the teacher will explain. Nobody is going to force you to do it, but you must do it correctly. If you are serious in thinking, you are sure to be able to do it.” Another student had the same view:

Interviewer: If you know how to answer a question, would you volunteer to answer?
Student: Yes.

Interviewer: What would happen if you answered it incorrectly?

Student: Incorrectly? Just sit down and the teacher would ask another student to answer.

Interviewer: How do you feel then?

Student: Nothing, wrong means wrong. This time wrong, next time it will not be wrong anymore.

There seemed to be a strong coherence between the mathematics teachers’ teaching philosophy and the students’ beliefs about mathematics learning. Both Shanghai mathematics teachers and students view mathematics as an important tool for developing mathematical reasoning and mathematical thinking. They both strongly believe that it is not enough to simply practise answering questions. They both tended to believe that a variety of mathematical questions, at different levels and of different natures, were important to enhance mathematical understanding. These views were apparent in both the mathematics teachers and students’ interviews.

Strong collaborative culture among mathematics teachers. A significant characteristic of Shanghai mathematics teaching is that “teachers engage in continuous school-based collegial professional development through lesson study, and teaching research group” (Cai, Lin, & Fan, 2004, p. 544). Every school in the study had assigned an afternoon per week for the mathematics teachers to meet and discuss the lessons planned to be taught the following week. In addition, it was common practice for the mathematics teachers to observe their colleagues teach and exchange comments during ‘open class’ teaching. The mathematics teachers’ staff room was organised in such a way that all mathematics teachers teaching the same grade level usually sat together in the same room. This practice promoted interaction and collaboration among the mathematics teachers.

Limitations of the Study

Due to time and financial constraints, my research was restricted to five schools in Shanghai. Hence the characteristics discussed are limited to these five schools. I acknowledge that my findings should not be generalised to represent all Shanghai schools. Generalisation was not intended in this paper. In addition, as a visiting scholar, I acknowledge a possible researcher bias in that I might have been assigned the best classes within the schools to participate in my research. In view of this possible limitation, I have validated the interpretation of the data with my host and colleagues in China.
Conclusion

This paper explored the characteristics of mathematics teaching in five Shanghai schools through classroom observations and interviews with teachers and students. I highlighted only the major characteristics that I observed to be possible contributing factors to high mathematics achievement amongst Shanghai students. However, I acknowledge that we have to be cautious and smart in incorporating any practices from one culture to another. We need to take note of cultural differences, so that we know what to adopt, how to adapt and what we need to modify. Merely adopting foreign practices into our own culture may not necessarily work as well as we might hope.

Every culture is unique. Wholesale adoption of good teaching practice from one culture to another is not necessarily advisable (Watkins & Biggs, 2001; Wong, Han, & Lee, 2004). Hence, it is important to reflect on and evaluate different cultural contexts and values, keeping in mind one’s own culture’s strengths and incorporating alternative ideas in the best way possible.

Acknowledgements

This study was made possible by the support of the following institutions and people: The Asia Scholarship Foundation (especially Dr Salvador, Miss Somkamol & Miss Sasithara), University of Science Malaysia (my dean, Professor Aminah Ayob), East China Normal University (my host institution, especially Prof Li Shiqi, Associate Prof Zhou Xiaoping and Associate Prof Li Jun) and the many mathematics teachers, school administrators, students and parents of both Malaysian and Shanghai schools. To all these agencies and people, I express my deepest appreciation and thank you for giving me such fruitful experiences of research, academic and cultural exchange.

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