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Improving Basic Multiplication Fact Recall for Primary School Students

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This study implemented a multiplication program based on systematic practice, aimed at improving children's recall of basic multiplication facts. Four Year 5 classes were recruited to participate in the study. Two classes practised multiplication facts using pencil and paper worksheets and another two classes practised on computers. Eleven practice sessions (each of 15 minutes duration) were conducted over a four week period. Both groups increased their recall of basic multiplication facts and maintained the increase for at least 4 weeks after the termination of the program. Implications for mathematics instruction, and for the overall development of mathematical proficiency, are discussed.

Students frequently find multiplication tasks to be a stumbling block in their mathematical progress. Many use inefficient and inaccurate counting methods and encounter difficulties in memorising tables (Geary, 2004; Kilpatrick, Swafford, & Findell, 2001; Koscinski & Gast, 1993) and later in dealing with larger single digit operands (Campbell & Graham, 1985; Steel & Funnell, 2001; Swan & Sparrow, 2000). Primary school children have stated they practise multiplication by writing down the series of numbers, by "looking at them," reciting them and listening to tapes (Steel & Funnell, 2001). Regrettably, if basic multiplication facts are not acquired during the primary school years, it is highly unlikely they will be practised in a structured manner in secondary school (Steel & Funnell, 2001).

Mulligan and Mitchelmore's (1997) two year longitudinal study of 60 Australian children in Year 2 and Year 3 identified a number of strategies employed by children to solve a range of multiplication problems. These strategies are described in Table 1. Improvements in speed and accuracy as students complete basic multiplication tasks in part reflect changes in the strategies children use. As they acquire new strategies they tend to abandon older, slower and less accurate ones (Lemaire & Siegler, 1995).

In mathematics education today, the emphasis is on developing children's understanding through exploration and discovery (Elkins, 2002; van Kraayenoord & Elkins, 2004; Westwood, 2003; Wu, 1999). Use of concrete materials, pictures, diagrams, and discussion increases students' familiarity with the process of multiplication and assists in their observation of regularities and patterns. For example, to learn the basic multiplication facts contained within the 0 to 9 times tables, over 100 multiplication combinations need to be mastered. Understanding *commutativity* – that the order of the two numbers does not affect their product (e.g., 5 x 8 = 8 x 5) – reduces the combinations by about half. Understanding the principles of multiplication by zero, by one and by two

Table 1
Calculation Strategies for Whole-number, Multiplicative Problems

Strategy	Definition
Direct counting	Physical materials are used to model the problem and the objects are simply counted without any obvious reference to the multiplicative structure.
Rhythmic counting	Counting follows the structure of the problem (e.g., "1, 2; 3, 4; 5, 6" or "6; 5, 4; 3, 2."). Simultaneously with counting, a second count is kept of the number of groups.
Skip counting	Counting is done in multiples (e.g., "2, 4, 6" or "6, 4, 2"), making it easier to keep count of the number of groups.
Additive calculation/ Repeated Addition	Counting is replaced by calculations (e.g., 2 + 2 = 4, 4 + 2 = 6).
Multiplicative calculation	Calculations take the form of known facts (e.g., "3 times 2 is 6" or derivatives from a known fact e.g., 3 x 2 = 2 x 2 + 2).
Commutative law	Changing the order of two numbers in a multiplication equation does not change the answer (e.g., 7 x 9 = 9 x 7).
Distributive law	a x (b + c) = a x b + a x c or vice versa (e.g., "9 times 14" is 9 x (7 + 7) = 9 x 7 + 9 x 7).

Note. Adapted from "Young Children's Intuitive Models of Multiplication and Division," by J. T. Mulligan and M. C. Mitchelmore, 1997, *Journal for Research in Mathematics Education, 28*(3), p. 311.

(which is the same as doubling the number), further reduces the number of combinations to learn to a manageable 28 (Hasselbring, Goin, & Bransford, 1988; Kilpatrick et al., 2001; Swan & Sparrow, 2000).

Whilst some children do reach a level of mastery in basic number skills through exploration and discovery, there are others who require classroom program adjustments to achieve similar levels of mastery (Elkins, 2002; Westwood, 2003). Many students invent their own calculation strategies and continue to use inefficient strategies when more efficient ones are available (Fuson, 2003; Kilpatrick et al., 2001). Systematic practice of key skills and knowledge to an automatic level of proficiency allows the recall of facts from memory without conscious effort (Hasselbring et al., 1988; Hasselbring, Lott, & Zydney, 2005).

The importance of automaticity becomes apparent when it is absent. Lessons may stall as students look up facts they should recall from memory. Thus conceptual understanding is necessary, but insufficient for mathematical proficiency (Bratina & Krudwig, 2003). Hasselbring et al. (1988) further advocate that to succeed in higher-order skills, these lower-order processes need to be executed efficiently. From an information-processing model perspective, commitment of basic mathematical facts to long-term memory frees up working memory (Ashcraft, 1994; Hunt & Ellis, 1999). Learning higher-order skills requires more working memory. When working memory resources are overburdened, performance deficits are likely to occur (Eysenck & Calvo, 1992; Faust, Ashcraft, & Fleck, 1996).

Basic multiplication facts are considered to be foundational for further advancement in mathematics. They form the basis for learning multi-digit multiplication, fractions, ratios, division, and decimals (Elkins, 2002; Howell & Nolet, 2000; Kilpatrick et al., 2001; Norbury, 2002). Many tasks across all domains of mathematics and across many subject areas call upon the recall of basic multiplication facts as a lower-order component of the overall task. To enable students to focus on more sophisticated tasks such as problem solving, proficiency in basic facts and skills is an advantage (Ashcraft, Kirk, & Hopko, 1998; Kilpatrick et al., 2001; Wu, 1999). Without procedural fluency and the ability to recall facts from memory, the student's focus during problem solving will be on basic skills rather than the task at hand, thus drawing attention away from the learning objectives of the task (Mercer & Miller, 1992). If the student cannot perform these basic calculations without the need to use calculators or other aids, higher-order processing in problem solving will be impeded (Westwood, 2003).

Systematic Practice

Researchers (e.g., Baroody, 1999; Steel & Funnell, 2001) believe that the development of multiplication recall is in part related to the frequency with which problems and opportunities for repeated practice are provided. However, it is not simply repetition that leads to improved performance. The structure of the practice needs to overcome plateaus in performance (Ericsson, Krampe, & Tesch-Romer, 1993). Studies (e.g., Harrison & Van Dervender, 1992; Koscinski & Gast, 1993; Williams, 2000; Williams & Collins, 1994; Wilson & Robinson, 1997; Wittman, Marcinkiewicz, & Harmodey-Douglas, 1998) have shown that multiplication programs aimed at improving the recall of basic multiplication facts have been successful with students of varying skill levels. A number of key factors were identified from these studies as essential for the success of any program aimed at the improvement of recall of multiplication facts.

Prior to engaging students in any program for improving the recall of basic multiplication facts, their current level of proficiency needs to be established. Levels of proficiency can be identified by giving students a pre-test of their mathematics facts (usually written), and asking students to complete as many questions as they can in a set amount of time (e.g., one or two minutes) (Howell & Nolet, 2000;

Stein, Silbert, & Carnine, 1997). Proficiency is then scored as the number of questions answered correctly on the pre-test. When answers are predominantly recalled from memory, the student should be able to answer approximately 40 basic mathematics questions correctly in one minute (Hasselbring et al., 1988; Howell & Nolet, 2000). To measure the effectiveness of the program, a test similar to the pre-test can be administered at the completion of the program.

The order in which facts are introduced and sequenced can assist students to become proficient in learning and recalling basic multiplication facts. Silbert, Carnine, and Stein (1990) suggest that facts that can be learned easily should be presented first during practice (e.g., 0, 1, 10, 2, 5, 9), then they should be followed by the more demanding multiplication sequences (e.g., 4, 7, 3, 8, and 6). Students should also be proficient at counting from 1 to 100 and be able to skip count.

Results from research (e.g., Chard & Kameenui, 1995; Cooke & Reichard, 1996; Koscinski & Gast, 1993; Williams & Collins, 1994) also show that the interspersion of known and unknown facts in each practice session increases the speed at which facts are committed to, maintained in, and retrieved from long-term memory. It also assists in the remediation of errors from previous sessions and improves the speed of retrieval of known facts from long-term memory.

When students initially learn the concepts of multiplication using concrete and semi-concrete materials, time restrictions are not appropriate. To improve speed of fact recall, students should be given a specific time to respond to a question or a constant time delay, typically starting at five seconds and gradually reducing to one and a half seconds. Reducing the response time forces the student to abandon inefficient counting strategies and attempt to retrieve the answer from memory (Hasselbring et al., 1988).

There are many ways of presenting multiplication facts for practice. Programs have used flashcards and pencil and paper methods, such as worksheets, recitation and computers. The comparative effectiveness of these modes of delivery has been the focus of a number of studies (e.g., Bahr & Reith, 1989; Christensen & Gerber, 1990; Harrison & Van Dervender, 1992; Williams, 2000). Practice on computers is said to afford some advantages over more traditional delivery modes. Students can progress at their own rate and practise using varying representations (horizontal or vertical). Feedback is immediate and scoring systems automatically monitor progress (Godfrey, 2001a). Students who used computers as part of their usual instruction generally learn more in less time and retain the information for longer (Godfrey, 2001b; Hasselbring et al., 1988).

Studies that focussed on the commitment and recall of multiplication facts from memory have been conducted with students with learning difficulties. Williams and Collins (1994) used flashcards with a five second constant time delay and material prompts to teach multiplication facts to four male students with learning disabilities, ranging in age from 9 years and 6 months to 13 years and 10 months. They were taught the 6, 7 and 8 multiplication tables, with one set of tables taught at a time. Each student practised 10 facts per session with three trials per session. The students' special education teacher conducted the sessions with two groups of two students. All four students learned their targeted multiplication facts.

Koscinski and Gast (1993) also used a constant time delay approach for teaching multiplication facts to six elementary school students between the ages of 9 and 10 years. All had learning disabilities. All students were identified as having difficulty memorising the multiplication tables above two. Multiplication fluency was assessed using timed pre-tests and post-tests. From the results of the pre-test, the gaps in each student's multiplication knowledge were determined. The students were then taught 15 unknown multiplication facts using computer-assisted instruction based on a five second constant time delay procedure. The results indicated that the constant time delay procedure was an effective method of teaching multiplication facts to those students.

In comparison to the previous two studies in which only previously unknown facts were presented, Cooke and Reichard (1996) compared the following interspersion drill ratios (known:unknown) 30:70, 50:50 and 70:30, in their study of six Year 5 students, five with learning difficulties and one with behavioural problems. The students themselves acted as peer tutors and presented 10 facts on flashcards to another student in the group. Each fact was held up for two seconds. If an incorrect answer or no answer was given in the time-frame, the tutor stated the answer. The same ten questions were randomly presented for seven minutes. A probe (test) containing 15 previously unknown facts was completed by the participants at the end of each session. The results showed that all students mastered the new facts as a result of the intervention, and that the rate of mastery depended on the interspersion drill ratio. Results from studies in a special education setting suggest that students can commit to memory and recall multiplication facts as a result of practice. However, these interventions were based on a teaching scheme in which the ratio of teacher/tutor:student was 1:1 or 1:2.

Research Questions

This study focuses on the development and implementation of a program of systematic practice for the improvement of basic multiplication fact recall. Very few studies examine the effect of such multiplication programs for normally achieving students at a classroom level. A number of questions were explored by this study. The first question asked whether or not a multiplication program, based on systematic practice, increases recall of basic multiplication facts. However, to be a truly useful program, the facts learned had to be retained after the completion of the program. Therefore, this study examined whether or not the change in multiplication recall endures over time. Finally, studies comparing computer-based instruction with pencil and paper instruction (e.g., Harrison & Van Dervender, 1992; Podell, Tournaki-Rein, & Lin, 1992; Williams, 2000) suggest that computer-based instruction is more effective than pencil and paper instruction. Owing to the age of the software used in those studies (the most recent software being written in 1994 and no longer commercially available), the advances in computer technology and increased access of students to computers, a more up-to-date comparison of computer-based instruction and pencil and paper instruction is needed. This study also addressed the question concerning computer-based instruction (CBI) effectiveness, in relation to pencil and paper instruction (PPI), for improving the recall of basic multiplication facts.

Methodology

This study uses a quasi-experimental, pre-test/post-test design as shown in Figure 1. The study was conducted during Term 3 of the school year. Classes were randomly assigned to an instructional approach – either pencil and paper instruction (PPI) or computer-based instruction (CBI).

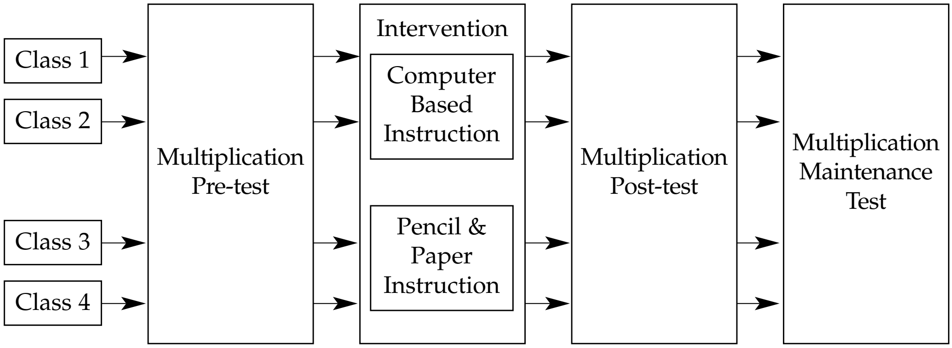

Figure 1. Research design incorporating pre-test, intervention and post-tests.

Participants

Participants were drawn from Year 5 classes at four inner-city Sydney, co-educational primary schools. To be included in the final data analysis, participants were required to attend at least seven of the eleven scheduled practice sessions, complete all the tests, and have written parental consent. The computer-based instruction group consisted of 37 students with an age range of 120 - 138 months (averaging 127.54 months). The pencil and paper instruction group consisted of 27 students with an age range of 116 - 140 months (averaging 127.15 months).

Test Instrument

Basic multiplication fact recall was measured by the number of multiplication facts answered correctly in a multiplication test completed in one minute. The multiplication pre-test contained 60 horizontally presented multiplication questions randomly chosen from the 0 to 10 times tables. The test was generated using Mathematics Worksheet Factory Lite (Worksheet Factory, 2001), a program available from the Internet. The post-test and maintenance test were generated by randomly re-ordering the questions used in the pre-test.

Program Instruments

A number of software requirements were identified from previous studies (Hasselbring et al., 1988; McDermott & Stegemann, 1987; Podell et al., 1992; Williams, 2000; Wittman et al., 1998). The following criteria were used to determine a suitable software package for computer-assisted instruction for this study: (a) availability of a Windows and Mac version of the software; (b) non-game format and no gimmicks or fancy graphics to avoid the introduction of possible novelty effects; (c) minimal keyboard skills necessary for the student to use the software, and (d) an unlimited amount of time available to answer each practice question. Although advancements in computer technology occur quite rapidly, many schools were using older computers in their computer labs. Thus, the software selected was required to run on Windows 98 and Windows ME, Apple PowerMac and Apple Macintosh computers. After reviewing a number of software packages, Back to Basics Maths Multiplication (GMA Software, 1999) was selected.

The software was provided to the schools for the purposes of the research. It allowed the participant to select the tables to be practised at each session. When using the software in practice mode, a question appeared on the screen in horizontal format. The question remained on the screen until the student typed in an answer or pressed a key. When the participant entered an incorrect answer, the correct answer was immediately displayed. If a correct answer was entered, a new question appeared.

Mathematics Worksheet Factory Lite (Worksheet Factory, 2001) was used to generate worksheets for the PPI group. Each worksheet contained 80 horizontally presented multiplication questions from the times tables sets to be practised for a particular session.

Procedure

The study was conducted during Term 3 of the school year. The pre-test was administered to all Year 5 students at their school one or two days prior to the commencement of the practice sessions. The post-test was administered within two days following the completion of the practice sessions. The maintenance test was administered approximately four weeks after the completion of the practice sessions. All tests were administered during each group's usual mathematics time with the classroom teacher in attendance. Participants were requested to complete questions in order and only skip a question if they did not know the answer. Participant confidentiality was maintained by assigning a code number to each participant's data.

Eleven practice sessions were conducted during a four-week period. Each practice session of 15 minutes duration was timed using a stop watch. The computer-based instruction participants received multiplication practice on the computer. The pencil and paper participants received their multiplication exercises in the form of worksheets in a standard classroom setting. In both cases, students were required to work independently throughout each session.

During each session, four sets of multiplication tables were practised. New and previously practised facts were interspersed during each session. The tables practised during each session are shown in Table 2. The order (0, 1, 10, 2, 5, 9, 4, 7, 3, 8, and 6) followed the pattern suggested by Silbert et al. (1990) with the exception that the tens multiplication tables were delegated to the latter practice sessions. For example, during session four, the 9, 4, 7 and 3 multiplication tables were practised. The 9, 4 and 7 multiplication tables had been practised in the previous session and the 3 multiplication tables were the new tables introduced.

Table 2
Multiplication Tables Practised by Session

	Tables Practiced		Use of
			Multiplication
Session	Revising	New	Table grid
1	1, 2, 5	1, 2, 5 and 9	✓
2	2, 5, 9	4	✓
3	5, 9, 4	7	✓
4	9, 4, 7	3	✓
5	4, 7, 3	8	✓
6	7, 3, 8	6	✓
7	3, 8, 6	10	✓
8	8, 6, 10	0	✓
9	6, 1, 2 and 5		
10	9, 4, 7 and 3		
11	8, 6, 10 and 0		

At the commencement of the first practice session, all participants were provided with a copy of a standard 1 to 10 multiplication table grid and shown how to use it. They were informed that it could be used to answer any questions they did not know. Use of the multiplication table grid was limited to the first eight sessions of the program so as to assist in the establishment of facts in long-term memory (Wittman et al., 1998). For sessions nine to eleven, the participants were informed that the multiplication tables grid could no longer be used and that they needed to try and recall the answer from memory.

During the first session, each participant in the CBI group was assigned a computer for use during the entire study. The groups were shown how to activate the Back to Basics software program and select a range of tables to practise. At the commencement of every session, the tables to be practised in the lesson were written on the board. Students were asked to select only those tables listed. At the completion of each session, a report was printed for each student, showing the results of their practice and which facts they had mastered (mastery was defined by correct answers on three consecutive attempts) and their data were cleared. Each student was reminded to try and complete three practice cycles.

PPI participants were provided with a worksheet at the commencement of each session. They were instructed to inform the researcher or classroom teacher on the completion of the worksheet, whereupon another worksheet was supplied. Three worksheets were available during each practice session with questions pertaining to the tables to be practised. The researcher retained the completed worksheets which were then marked (with the errors highlighted) and returned to the participant at their next practice session. If the participant completed all worksheets before the conclusion of the session, they were directed by their classroom teacher to continue with their usual class work. This procedure was repeated for all eleven sessions.

Teachers were not requested to alter their planned mathematics programs during the four weeks of multiplication practice. Other content taught was at the discretion of the classroom teacher.

Results

Data Analysis

All statistical analyses were completed using SPSS v11.0 for Windows (SPSS, 2002). Significance was accepted at $p < 0.05$ level. Between group (CBI and PPI) comparisons of pre-test scores were performed to determine if there was a significant difference between the CBI and PPI participants. An independent samples *t*-test was conducted to evaluate whether mean multiplication pre-test scores differed between the CBI group and PPI group. The pre-test mean multiplication score did not differ significantly ($t(62) = 0.111$, $p = 0.921$) between CBI group ($n = 37$, $M = 22.27$, $SD = 10.658$) and PPI group ($n = 27$, $M = 22.63$, $SD = 15.307$). An independent samples *t*-test was conducted to evaluate whether the mean age in months at time of pre-test differed between treatment groups. The mean age in months did not differ significantly ($t(62) = -0.299$; $p = 0.766$) for the CBI group ($M = 127.54$ months, $SD = 4.682$) and PPI group ($M = 127.15$ months, $SD = 5.803$). Gender balance across treatment groups was tested using a chi-square test. Gender distribution was consistent within each treatment group with approximately 60% males to 40% females therefore, adjustments for gender balance were unnecessary. Violation of normality was not detected for pre-test, post-test and maintenance multiplication test scores for the CBI group and PPI group using a normal probability plot. Thus, normality of the differences between scores was also assumed (Coakes & Steed, 2003).

Recall of Basic Multiplication Facts

Recall of basic multiplication facts was measured by the number of correct responses in a 60 question written multiplication test completed in one minute. Figure 2 shows the pre-test, post-test and maintenance test mean multiplication scores for the CBI group, PPI group and the entire sample. The mean

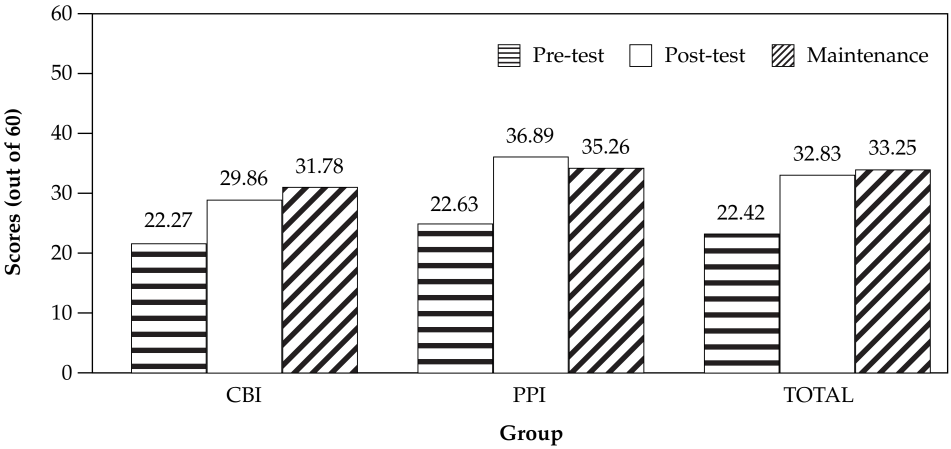

Figure 2. Mean multiplication test scores for pre-test, post-test 1 and maintenance test for CBI group (n = 37), PPI group (n = 27) and all participants (N = 64).

multiplication scores appear at the top of each column. Visual inspection shows that both post-test and maintenance test mean multiplication scores for the CBI group (n = 37), PPI group (n = 27) exceeded the pre-test scores. The improvement from pre-test to post-test for the pencil and paper instruction (PPI) group was greater than the CBI group.

Using a dependent samples (paired samples) t-test, multiplication pre-test scores were compared to post-test scores to examine whether test scores differed significantly as a result of the program. For the CBI group, mean multiplication scores differed significantly ($t(36) = -8.107$, $p = 0.001$), with post-test mean multiplication score increasing by 7.59 facts per minute (34%) from pre-test. All but six participants in the CBI group improved. Three students recorded no change in score from pre-test to post-test and the remaining three participants scored less than their pre-test. For the PPI group, mean multiplication scores differed significantly ($t(26) = -8.501$, $p = 0.001$), with post-test mean multiplication score increasing by 14.26 facts per minute (63%) from pre-test. All but three participants in the PPI group increased their scores. These three students recorded no change in score overtime. The lack of change in these results was in some cases due to a ceiling effect of the measure (for two students), while in other cases instruction appeared not to have an impact over the time the study was conducted. Follow-up investigation of the students' pre-skills, and strategies for answering questions would be needed to establish further reasons for the lack of change in their scores.

Retaining Recall of Basic Multiplication Facts

To determine whether changes in multiplication fact recall acquired during the program endured over time, a maintenance test was conducted approximately four weeks after the completion of the program. Using a dependent samples (paired samples) *t*-test, multiplication post-test scores were compared to maintenance test scores to examine whether there was significant change in scores during the four-week period following the completion of the program. For the CBI group, mean multiplication scores between post-test and maintenance test differed significantly ($t(36) = -2.115$, $p = 0.041$), with the maintenance mean multiplication score increasing by 1.92 facts per minute from post-testing. Although the difference in means was statistically significant, the post-test level of achievement of the CBI group was maintained overtime as maintenance scores increased. For the PPI group, mean multiplication scores between the post-test and maintenance test did not differ significantly ($t(26) = 1.065$, $p = 0.296$), with the maintenance test mean multiplication score decreasing slightly by 1.63 facts per minute from the post-test.

The maintenance of results past the end of the study was steady across both groups. The level of the students' conceptual knowledge was not measured in this study. Future studies may investigate this area more thoroughly, and establish what effect this knowledge may have had on maintenance of results beyond an intervention. Further, future studies could examine the level of interplay between conceptual knowledge and procedural proficiency. It is predicted that the link would be strong, with students with the strongest maintenance of results having proficient procedural knowledge and sound conceptual knowledge (Kilpatrick et al., 2001).

Treatment Effectiveness

To determine whether CBI or PPI was more effective, a multiple linear regression model was employed. Prior to performing the analysis, visual inspection of a scatterplot of pre-test and post-test multiplication scores indicated the existence of a strong positive linear relationship. A number of linear regression models were tested. Field (2005) recommends 10-15 cases for each predictor variable. If the initial model contains five predictor variables, at least 50 cases are preferred.

The initial regression model used multiplication pre-test scores, group, gender and age as predictor variables and post-test multiplication score as the dependent variable. Predictor variables were removed, based on the least significant coefficient first, followed by the next least significant and so on. The importance of each variable was assessed by the change in R^2 (Weinberg & Abramowitz, 2002). If the change in R^2 was significant, the variable was re-introduced into the model and the next significant predictor removed. The regression analysis appears in Table 3.

Three variables, type of school, age in months, and gender, were eliminated. A model with two predictors, pre-test and group, was selected as the most appropriate regression model and was statistically significant ($F(2, 61) = 89.555$, $p = 0.001$) with 74% ($R^2 = 0.746$) of multiplication post-test variance explained by the model. The regression model suggests that for the same pre-test score, at post-test, on average, a PPI participant answered an additional 6.703 facts per minute compared to a CBI participant. Thus for the participants of this study, PPI was a more effective method of improving recall of basic multiplication facts.

Table 3
Regression Models used to Predict Post-test Multiplication Scores

	B	SE	β	t	p
Model 1					
(Constant)	23.395	22.624		1.034	.305
Pre-test multiplication	.904	.083	.836	10.852	.001*
Group	-6.509	1.900	-.236	-3.426	.001*
Gender	1.630	1.976	.058	.825	.413
Age in months	-.059	.179	-.022	-.330	.743
Type of school[a]	-.441	2.139	-.016	-.206	.838
Model 2					
(Constant)	22.869	22.296		1.026	.309
Pre-test multiplication	.912	.073	.843	12.419	.001*
Group	-6.623	1.802	-.240	-3.675	.001*
Gender	1.740	1.887	.062	.922	.360
Age in months[a]	-.058	.177	-.022	-.325	.746
Model 3					
(Constant)	15.661*	2.375		6.594	.001*
Pre-test multiplication	.908	.072	.839	12.627	.001*
Group	-6.649	1.787	-.241	-3.721	.001*
Gender[a]	1.668	1.860	.060	.897	.373
Model 4					
(Constant)	16.680*	2.082		8.012	.001*
Pre-test multiplication	.893	.070	.826	12.790	.001*
Group	-6.703	1.783	-0.243	-3.759	.001*

Note. Model 1: R^2 = .750; Model 2: R^2 = .750, ΔR^2 < .001; Model 3: R^2 = .749, ΔR^2 = -.001; Model 4: R^2 = .746, ΔR^2 = -.003.

[a] This predictor variable was dropped in next regression model.

* $p < 0.05$.

This model was further reviewed to determine what type of practice was best suited to students with low pre-test scores. Participants were grouped into a low recall group (up to the 20th percentile of sample), average recall group (21st-79th percentile) and high recall group (80th percentile and above). Two dichotomous dummy variables were created to represent the three categories (Fields, 2005). The low recall group was designated as the baseline. Another regression analysis was completed using four predictor variables: pre-test score, group, pre-test D1 (average recall group) and pre-test D2 (high recall group), and post-test multiplication as the dependent variable. The regression model was statistically significant, $F(4,59) = 47.68$, $p = 0.001$, with 76% ($R^2 = 0.764$) of multiplication post-test variance explained by the model. The regression coefficient statistics appear in Table 4. Although pre-tests, D1 and D2, were not significant predictors, Table 4 does show that the differential effect of PPI and CBI instruction was reduced as students' pre-test scores increased.

Table 4
Multiple Regression Coefficients for Different Pre-test Levels

	B	SE	β	t	p
Model 1					
(Constant)	15.390	2.400		6.412	0.000*
Pre-test multiplication	0.765	0.150	0.707	5.096	0.000*
Group	-7.547	1.824	-0.273	-4.138	0.000*
Pre-test D1	5.868	3.054	0.212	1.921	0.060
Pre-test D2	6.219	5.842	0.183	1.065	0.291

Discussion

The results suggest that systematic practice of basic multiplication facts was an effective method of improving students' recall of these facts and that their improved recall remained for at least four weeks after the last practice session. While the results supported the position that practice, structured to overcome plateaus of achievement, leads to increased procedural proficiency in recalling multiplication facts, the results do not support the assertion that computer-based practice is more effective than traditional pencil and paper practice approaches (e.g., Harrison & Van Dervender, 1992; Koscinski & Gast, 1993; Williams, 2000).

A possible explanation for the difference in improvement between the two groups may be that the assessment of multiplication recall was based on a written test. The writing practice received by the PPI group may have given those participants an unfair advantage in the post and maintenance tests, as they may have been able to write faster and may have benefited from the more experience they had of being tested in this medium.

Another factor to consider is that a written test may fail to be a good indicator of multiplication recall as poor performance can be attributed to any one of four factors: (a) the student is slow at retrieving multiplication facts; (b) the student calculated the answer slowly; (c) the student is slow at writing digits, or (d) or a combination of all three factors depending on the question (Howell & Nolet, 2000). Poor reading speed may also adversely affect the proficient recall of multiplication facts (Wilson & Robinson, 1997).

Further studies may overcome the slow writing issue through the use of a verbal test, where questions are read aloud and verbal responses are given by the students. However, this may need to be completed on an individual basis and it was not possible to administer a verbal test in this study due to time constraints. Alternatively, reducing test bias could be minimised by having both groups respond using a pencil and paper test, and a computer-based test.

Adaptation to the Classroom

Marking the pre-test highlighted some frequent conceptual and procedural misunderstandings. For example, some students consistently calculated the A x 0 multiplication fact incorrectly by writing the value of the A, whereas its commuted counterpart, 0 x A was answered as 0. The revision of concepts and procedures needs to be included in the practice sessions to ensure students possess the necessary pre-skills to answer questions accurately (Howell & Nolet, 2000; Mercer & Miller, 1992). This can be achieved through building conceptual knowledge though the use of manipulatives (Anstrom, 2006), through building 'generalisable' rules (Silbert et al., 1990), and through providing practice in the use of virtual manipulatives. Virtual manipulatives allow the introduction of concepts, and provide practice and remediation (Zrofass, Follansbee, & Weagle, 2006).

The efficacy of implementing a computer-based, classroom program will be hampered by the practical limitation of the small number of computers found in most mathematics classrooms and by time constraints. The computer group participants in this study were required to move from their normal classroom to the computer laboratory within the school. A 15 minute practice session took the students out of the class for 20 to 25 minutes as they moved to and from their classroom, started the computer, started the computer program, waited for the other students (so as to commence the practice session at the same time), terminated the program and shutdown the computer. As there were not enough computers for the entire class, the CBI groups were divided into two, with each half undertaking their practice in separate sessions, which resulted in further disruption. Peer-tutors have been used effectively (Baker, Gersten, & Lee, 2002) for practising multiplication facts and could be a workable alternative to computer-based practice.

While proficiency in multiplication facts is important, there are also other basic facts that require practice to maintain ongoing development of mathematical proficiency, such as addition and subtraction facts (Hasselbring et al., 1988; Podell et al.,1992). Therefore practising other basic arithmetic skills could be included in practice sessions like those used in this study. Initially, multiplication facts may be practised separately to promote proficiency; later they could be mixed with other facts to allow students to become more proficient in selecting from and discriminating between operations. Although it is a more traditional program, Mathematics Worksheet Factory Lite (Worksheet Factory, 2001) allows the user to customise the basic fact questions on the worksheets by selecting the arithmetic operations required (+, -, x, ÷), number of questions (maximum 100) and the format (either vertical or horizontal). Finally, the effects of fine-tuning the program could be explored by varying session durations, the number of sessions and the total length of the systematic practice program.

The study provided tentative support for the use of technology (i.e., computers). Technology can be used to assist teachers to provide quality instruction for students with learning difficulties (Swanson, 2005). As outlined in this paper, factors to consider when using technology include: whether the level of difficulty can be adjusted to meet the needs of students; what type of feedback is provided for students; whether proficiency can be promoted by adjusting the time requirements for completing tasks; the frequency in which skills or problems are presented is likely to build proficiency; use of discrimination examples; type of response; and clarity of directions (Murray, Silver-Pacuilla & Helsel, 2007). Evaluating technology in the light of all these considerations requires time, and teachers should be prepared to spend some time working their way through the array of technology available to ensure they can maximise its potential to assist students in their classroom.

In summary, the belief that the development of basic multiplication fact recall is enhanced by practice has been supported. Results generalised to the study group have shown that a systematic practice of basic multiplication facts by interspersing known and unknown facts improved students' recall of these facts for all but a few.

For these few students, revisiting multiplication concepts may be necessary. Without this improved recall of basic multiplication facts, working memory is consumed by the most fundamental of problems. Releasing working memory capacity allows students to tackle more difficult tasks such as multi-step problems or questions demanding higher-order thinking.

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