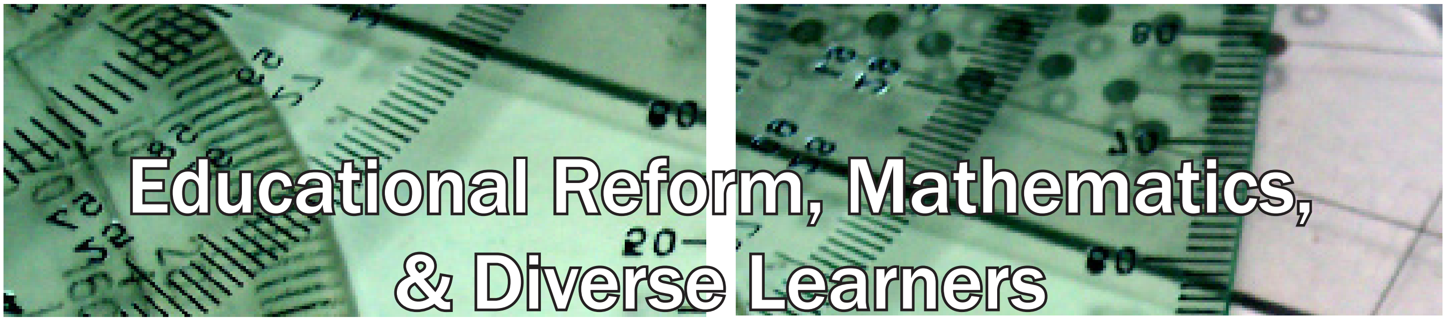
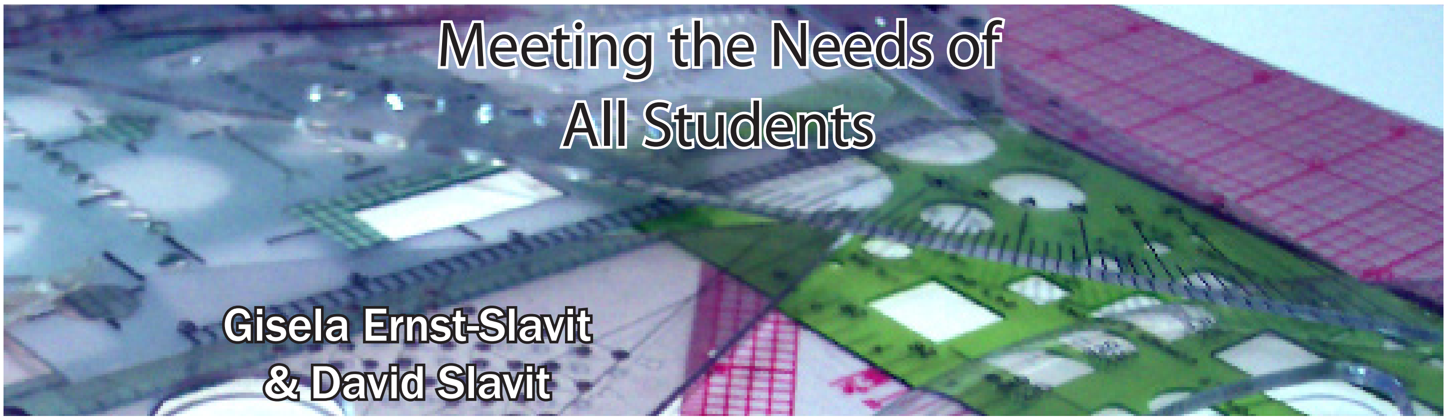


Educational Reform, Mathematics, & Diverse Learners



Meeting the Needs of All Students

Gisela Ernst-Slavit
& David Slavit

In a preservice mathematics methods class, David, a mathematics educator, shows the video "Good Morning Miss Toliver" (Toliver, 1993). This film features a public middle school teacher and her mathematics students from East Harlem in New York. The film depicts Miss Toliver using a variety of pedagogical strategies to arouse her students' interests in math.

Some of the strategies she uses include problem posing, integrating language arts into the teaching of mathematics, small group work, the application of mathematics to everyday real-world contexts, project-based learning, family involvement, and current event analysis and application to mathematics.

After viewing this short film, Becky raises her hand and states:

> That's all very nice but I can't be a Miss Toliver if I have a classroom like my practicum with eight ELLs who speak five different languages. How can I teach math if the students don't know their numbers in English? How can I plan lessons around the EALRs [state standards] and NCTM (National Council of Teachers of Mathematics) standards if many of my students can only speak in two-word phrases? They are supposed to communicate their understandings. How can I help my students learn to love math?

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Becky's words express her frustration and concern about reaching all her students. Becky, now in her early 30s, has come to school to pursue a masters degree with K-8 teacher certification after spending ten years as a successful professional in marketing. Becky is a very dedicated and thoughtful preservice student with strong math skills who has had prior success engaging small groups of English language learners (ELLs), low-income, and homeless students in mathematics during a summer school program. Although she has always appeared confident and resourceful, her comments today reflect the challenges she now faces in her practicum classroom.

Becky's worries are not unique. As teacher educators specializing in ESL/Bilingual Education (first author) and Mathematics Education (second author) we have heard many of these concerns from our preservice students and from teachers and administrators in K-12 schools. Throughout the years, as we taught in K-12 classrooms, carried out research in schools, taught or team taught methods and other teacher education courses, and worked with teachers and school administrators in professional development efforts, we have learned the importance of the interplay of language and culture in the teaching of mathematics.

This is particularly significant if we consider the increasing numbers of ELLs in today's classrooms. Current census data indicate that school districts throughout the United States are increasingly serving a student population whose home languages and cultures are diverse. For the 1993-94 school year, the National Clearinghouse for English Language Acquisition (NCELA) reported a national ELL student enrollment of 3,552,497. Ten years later, there were 4,999,481 school–aged ELLs in the U.S., reflecting approximately 10.3% of the student body.

States with historically large percentages of ELLs continue to show increases in this student population (e.g., Arizona, California, Florida, Illinois, New York, and Texas). However, current data also show large and unexpected growth of school-aged ELLs in states with historically low numbers (e.g., Tennessee, Indiana, Georgia, Nebraska, North Carolina, and South Carolina). While the number of linguistically and culturally diverse students is growing, the number of teachers with diverse backgrounds is not growing proportionally (Mercado, 2001; Nieto, 2004).

In part, because of this asynchronous aspect of public schooling, the challenges of understanding and incorporating students' linguistic and cultural capital into the instructional process intensifies for both individual teachers and school systems as a whole. But simultaneously, the demands of teaching mathematics to all children are also presenting their own specific challenges.

In 1989, the National Council of Teacher of Mathematics (NCTM) identified a clear set of standards for teaching and assessing mathematics, which were reinforced and elaborated upon in their more recent document (NCTM, 2000). These five standards apply to all grade levels and provide mathematics educators with a solid base on which to build instruction and curriculum:

1. To understand and value mathematics.

2. To reason mathematically.

3. To communicate mathematics.

4. To solve problems.

5. To make connections to contexts and other academic subject areas.

In 1991, NCTM produced an accompanying document that makes recommendations for teacher professional standards, stating that teachers should:

◆ Select mathematical tasks to engage students' intellect and interest.

◆ Provide opportunities to deepen students' understanding of mathematics and its applications.

◆ Orchestrate classroom discourse in ways that promote the investigation and growth of mathematical ideas.

◆ Help students use technology and other tools to pursue mathematical investigations.

◆ Help students seek connections to previous and developing knowledge.

◆ Guide individual, small-group, and whole class work. (NCTM 1991, p. 1)

According to the NCTM documents, teachers need to select activities that grow out of real-world problems relevant to the learner. That is very different from what most of us did as students in K-12 mathematics classrooms, where our lessons involved working on algorithms, manipulating mathematical expressions, and solving "recipe-type" problems. Skemp (1987) has referred to those past instructional norms as teaching "rules without reasons". Such lessons usually minimize the role of student experience and treat mathematics as a culture-free discipline.

In contrast, problem-based mathematics lessons engage learners in mathematical activity that involves overcoming an "intellectual impasse" (Schoenfeld, 1985) and elicits students' individual perspectives and problem-solving strategies. Teachers can utilize this mathematical activity in subsequent classroom discussions, furthering their students' mathematical development and ability to articulate their understandings. Problem-based lessons can also connect mathematical activity to relevant "real world" problems or other academic areas.

Tasks can be stated quite simply, such as "How many beach balls fit inside this classroom?" Perhaps a more complex task, taken from the Connected Mathematics Project (Lappan et al, 2004), asks students to redraw the map of the United States in a manner where area is proportional to population. Mathematical tasks can also be used to address cultural issues and teach social justice, such as a critical and historical analysis of a country's area and geographic boundaries or a statistical and economic analysis of the cost of war (Gutstein & Peterson, 2005).

In essence, (1) taking the meaning of "problem" seriously and (2) creating an environment where students are supported in their ability to solve, understand, and explain their mathematical activity lie at the heart of current mathematics education reform. Not only does this present significant content-related challenges for teachers such as Becky, who may not have confidence in their own mathematical ability, but it requires teachers to understand the mathematical perspectives of all their students and engage them actively in the mathematical learning environment. The latter assumes that the students' ability to communicate their understanding is not only possible, but an established environmental norm.

According to the National Research Council (2001, 2002), the ultimate goal of mathematics instruction for all K-12 students is mathematical proficiency. Steered by an appreciation of the value of mathematics, mathematically proficient learners are confident practitioners who are able to solve problems, compute and carry out procedures, understand important ideas, and communicate and reason mathematically.

Certain "verbs of doing mathematics" are present in most instructional recommendations, such as justifying, predicting, explaining, clarifying, elaborating, and describing (van de Walle, 2004). Implicit in many of these verbs are language demands that are often challenging to many students—including those speaking different English varieties and those learning English as a second language. Likewise, students' cultures and community histories, particularly if different from the mainstream, may play a role in how students access mathematical knowledge.

While equity and the phrase "mathematics for all" have been at the heart of the above recommendations, the disparities in mathematics achievement continue to be tightly coupled with social class and race (Ball, Goffney, & Bass, 2005). Thus, the potential of the Standards to respond to the economic, linguistic, and social disadvantages that provide context to classroom instruction has been challenged (e.g., Apple, 1992). As stated above, putting communication in the center of instructional reform places potential inequities on students whose languages and cultures are different from those of the school.

Further, while not part of the NCTM vision, the recent federal mandates to test children in mathematics in Grades 3-10 have increased the potential discrepancies between success among diverse groups. However, evidence is accumulating to suggest that problem-based, standards-based curriculum and instruction is effective in promoting mathematical proficiency (Senk and Thompson, 2003), including among traditionally marginalized groups (Silver and Stein, 1996; Gutstein, 2003).

Still, such success is usually accompanied by specific attention to a variety of instructional, cultural, and linguistic factors. Careful attention to these factors in the mathematics classroom can help educators fight issues of inequality and access for all their students.

Language and Cultural Factors To Consider when Teaching Mathematics to Diverse Students

> …students of all backgrounds deserve the very best our society can give them, and…their cultures, languages, and experiences need to be acknowledged, valued, and used as important sources of their education. (Nieto, 2004, p. xix)

Since the release of the 1989 NCTM Standards, several approaches for teaching mathematics that build on students' knowledge, cultural backgrounds, and experiences have been enacted (Ladson-Billings, 1995; Ortiz-Franco et al, 1999; Tate, 1997). A guiding principle behind much of this work is that the teaching and learning of mathematics can be extended and enhanced when participants' own styles and experiences (*vis-à-vis* their culture, language and identities) are mixed with those generalizations and conceptualizations offered in schools (Ernst-Slavit, Moore & Maloney, 2003).

Below we explore four selected cultural and linguistic factors to consider when teaching mathematics to diverse students.

1. The Role of Students' Languages, Cultures, and Communities in the Learning of Mathematics

> The connection of local knowledge to schooling is not an easy process…The challenge is to adapt local culture and knowledge to Western schooling without trivializing and stereotyping. (Lipka, 2002)

The Role of the First Language

As we contemplate ways of supporting teachers like Becky, who will have five, 10, and even 15 different language varieties in their classroom, it is imperative that we stress the need to capitalize on the inherent strengths of the students. For example, rather than viewing ELLs as "limited English proficient," we can consider these students already proficient or familiar with another language. Hence, their academic achievement can be significantly enhanced when they are able to use their native languages to learn in school (August & Hakuta, 1998; Baker, 2001).

Furthermore, research studies indicate that students' home languages can play a significant role in learning complex material such as that which is typically encountered in mathematics classrooms. This is especially true when students are afforded opportunities to use their home language (Cummins, 1981; Hornberger, 2003; Thomas & Collier, 2002).

The above research also suggests a correlation between literacy in the first and second language. Most ELLs have an intuitive understanding of the general structural and functional characteristics of a language as they approach the learning of a second and, in some cases, a third language.

In the case of mathematics, various symbols and representation systems can also transfer from culture to culture which are, in some ways, "language-free". More will be said about the role of language in mathematics classrooms in subsequent sections.

The Role of Culture

The role culture plays in mathematical teaching and learning is stressed by mathematics and ESL professional organizations (see, for example, NCTM Standards 2000 and TESOL PreK-12 English Language Proficiency Standards 2006). Both sets of standards support current research (e.g., Gay, 2000; Ladson-Billings, 1995; Lipka, 1998; Moll & Gonzalez, 1997; Nieto, 2003) claiming that teachers need to consider students' prior experiences, cultural backgrounds, community histories, and ethnic identities in the teaching of mathematics. This implies viewing "students' home cultures and languages as strengths upon which to build, rather than as deficits for which to compensate" (Gustein & Peterson, 2005: 3).

Clive, a mathematics teacher for over 30 years with a history of traditional instructional techniques, found himself in a small, rural school district with a student population of over 50% ELLs, most of whom were the children of migrant workers. Clive quickly found the need to abandon his life-long instructional tendencies and to focus on reaching out to his students' interests and take advantage of the cultural capital they possessed. He made selected home visits to better understand the backgrounds of his students, and modified his teaching techniques to allow students to communicate in their home languages, even though he may not speak their languages.

Clive also abandoned years of symbolically-based, "here-is-the-way-to-do-it" instruction and provided opportunities for students to conceive of and communicate their own ways of solving problems, even making attempts at incorporating students' interests into mathematical tasks. While Clive's approach was not perfect, nor did it significantly impact success on tests of student achievement, there was a noticeable impact on students' interests, motivations, and ability to discuss their mathematical understandings. In this case, utilizing the strengths of this diverse context not only supported students' learning of mathematics, but it promoted a renewed sense of purpose in this teacher's professional demeanor.

But cases such as Clive are not typical. The basic fact of utilizing students' home cultures and languages as strengths is often ignored when teaching Latino, Native, African, and Asian Americans, especially if they are poor. Instead they are often taught from traditional middle class, Eurocentric perspectives. A case in point is offered by Moody (2003), who studied African American students' experiences in mathematics classrooms. The students in her study found the traditional mathematics classroom culture to reflect analytical teaching and learning styles. That is, mathematics concepts and principles were presented in a manner that reflected the importance of order to the mathematical system itself (Stiff, 1990), similar to Clive's original approach.

In this type of classroom, competing for directness, precision, conciseness, the right answer, and a limited supply of appropriate knowledge made student survival very difficult. This type of mathematics environment contradicted students' learning styles and what they advocated as ideal mathematics settings. The latter reflect relational teaching and learning styles that value cooperative learning, have unlimited amounts of knowledge, skills and understanding to share, and where students' surviving is not an issue (Moody, 2003; Stiff, 1990). If diverse learners are to fully benefit from the schooling experience, the teaching of mathematics needs to be linked to their lives and circumstances and, in some respect, share their cultural norms.

The Role of Community Knowledge

Community knowledge is a valuable commodity in the classroom. Children come to school having acquired many conceptualizations and developed many skills that enable them to successfully engage in everyday life in their homes and communities. Gonzalez, Moll, and colleagues (Gonzalez, Moll, & Amanti, 2005; Moll, 1992; Moll & Gonzalez 1997) developed a "funds of knowledge" perspective as a means of enabling educators to capitalize on the tremendous amounts of resources communities can provide for learning.

In any community there are cultural practices and bodies of knowledge and information that households use to survive, to get ahead, or thrive (Gonzalez, Moll, & Amanti, 2005). A funds of knowledge perspective taps on those resources by recognizing and affirming the sociocultural dynamics and resources in the child's household.

Schools are often searching for parents and community members who have special talents, such as artists, photographers, storytellers, and writers. However, many other talents and skills are often overlooked. For example, in a study of 30 working-class Latino families, Moll and his colleagues found household funds of knowledge such as masonry, midwifery, farming, hunting, building codes, mining, appraising, ranching, roofing, carpentry, first aid, renting, herbal knowledge, selling, budgets, catechism, and Bible studies (Moll, 1992). Each of these knowledge funds have deeply quantitative or other mathematical dimensions. In the classroom, a funds of knowledge approach involves tapping on those resources and using those to scaffold new knowledge.

One example of the activation of students' and community knowledge into an integrated mathematics unit took place in one bilingual sixth-grade classroom in a working-class community in the Southwest (see Moll, Vélez-Ibañez, Greenberg, & Rivera, 1990). A young teacher invited students' parents and relatives involved in local construction to contribute their knowledge to an integrated unit. Students learned many things about measurement of height, perimeter, and area; the use of fractions for the mixture of mortar; the construction of arches; and calculation of money and time.

Literacy and oral reports were also part of the unit, as students explored topics such as cost versus quality. Through real-life, meaningful, community-based activities, this teacher was able to engage her students in problem solving while at the same time bridge the culture of school with the resources offered by the community.

2. The Specialized Language of Mathematics

Mathematical conversations at all grade levels incorporate the math register, or language specific to the discipline of mathematics. Embedded in the math register are various discourse and syntactical features that can make it difficult for speakers of different English varieties or ELLs to draw meaning from mathematical learning environments. These include the use of symbols, technical language, and the various ways that mathematical concepts are discussed and mathematical arguments are made.

The following discussion focuses on three important aspects of mathematics instruction: the difference between social and academic language, the distinct syntax of the math register, and the complexities involved in teaching and learning the math register.

Difference Between Social and Academic Language

Social language refers to the basic fluency used in everyday, face-to-face interaction. Many students come to school with this kind of fluency while others may need time to acquire it. ELLs are capable of learning everyday language within a relatively quick period of time—around two years. While this is the speech most used during recess, in the hallway, and outside the school, it is also much needed in the classroom.

Academic language, on the other hand, refers to the language needed to acquire new knowledge or skills, develop deeper understanding of a topic, and to communicate that understanding to others; it is the language students must use to effectively participate in the classroom environment. For ELLs, the acquisition of the academic language and literacy skills needed to succeed in the content areas may take anywhere from five to seven years (Cummins, 2005; Scarcella, 2003). However, and as indicated by Dale and Cuevas (1992), everyday vocabulary, such as "column," "table," and "pie" can take on alternative meanings in the mathematics classroom, suggesting a less than clear distinction between social and academic language in this setting.

Students must be prepared to deal with a range of language demands found in a variety of modes. These can include teachers' and fellow students' oral language, textbooks and other printed materials, and assessments. Students must also produce this language in appropriate ways through oral and written modes. These challenges are further compounded since, like other content areas, the math register includes unique vocabulary as well as syntactic and semantic features. The table below presents examples of different types of math vocabulary.

High Frequency Vocabulary: terms used regularly in everyday situations (e.g., small, orange, clock);

General Vocabulary: terms not directly associated with a specific content area (e.g., combine, describe, consequently);

Specialized Vocabulary: terms associated with a content area (e.g., divisor, least common denominator); and

Technical Vocabulary: terms associated with a specific content area topic (e.g., Pythagorean theorem, integrals, ratio)

Dale and Cuevas (1987) point to difficulties stemming from the many different ways in which mathematical processes and ideas are expressed. For example, multiplication can be represented by "x", "*", "( )", "·", and even nothing, as in the case of "xy" to represent the product of these two variables. Similarly, as an ELL realizes that "+" represents the action or concept of addition, she encounters words such as "add", "combine," "plus," "increased by," and "sum." Likewise, "–" can be expressed as "decreased by," "take away," "minus," "less than," and "subtract from." Consistent word choice and specific attention to connections between social and academic language that have similar semantic connotations can support diverse learners to follow and participate in mathematical conversations.

Distinct Syntax

At the sentence or syntactic level, there are language patterns and grammatical structures specific to mathematics. ELLs often encounter difficulties when they attempt to read and write mathematical sentences in the same way they read and write narrative text. That is, students may try to literally translate a mathematical concept expressed in words into a concept expressed in symbols. Dale and Cuevas (1992) demonstrate this lack of linear, one-to-one translation with the following example: The algebraic phrase "the number a is five less than the number b" is often translated into a=5-b, when it should be a=b-5. It should be noted that such translations are also difficult for students whose first language is English (Clement, 1982).

Other language features also present difficulties. These include the use of logical connectors (e.g., "consequently," "however') that in regular usage signal a logical relationship between parts of a text; in mathematics, they signal similarity or contradiction. Likewise, the use of comparative structures (e.g., "greater than," "less than," "n times as much as") and prepositions (e.g., "divided *by*," "divided *into*") pose serious difficulties for students who are trying to learn the content while, at the same time, learn the language of instruction.

Teaching and Learning the Math Register

Learning the math register can be a complex endeavor for diverse learners if we consider that many words cannot be translated from English to their native language. Lee and Fradd (1998) show that comparable terms and parallel ways of considering ideas may not exist across languages or, if they do exist, they may not be used with the same frequency or manner. But students can learn new word meanings and new terminology when it is presented through purposeful activities in meaningful contexts.

More specifically, vocabulary is best taught not as a separate activity, but as part of the lesson and with the support of realia (e.g., real objects and manipulatives) and visual supports (e.g., pictures, graphic organizers, charts, maps, bulletin boards). It has been recommended that the introduction of new vocabulary should be limited to no more than 12 words per lesson (Fathman, Quinn, & Kessler, 1992).

In addition, teachers can help students build their math vocabulary by offering diverse opportunities to use their newly acquired vocabulary in different contexts. For example, in mathematics, the word "table" can refer to a "times table" for multiplication facts or a "table of values" for graphing functions. "Table" may also have very different meanings and usages in non-mathematical contexts such as, "timetable" in social studies, "table of contents" in language arts, "water table" in physical science, and "periodic table" in chemistry.

Thus, pointing to the different meanings and usages across content areas can be very helpful for all students. Ultimately, the best way to learn a language is by using that language. This also applies to the learning of the mathematics register. All students, but particularly ELLs, need plenty of opportunities to hear mathematical language and to use their emerging math register by engaging in meaningful communicative activities.

3.
The Teaching of Mathematics to Diverse Populations

> It's true that people all over the world engage in mathematical activities to the extent of their needs and interests. They all count objects, they measure various quantities, they invent calendars and other ways to describe the passage of time, they design works of art, they plan buildings, and they play games that involve mathematical concepts. Furthermore, they invent terminology that enables them to discuss these activities. But each culture, each group, solves these problems in its own way. (Zaslavsky, 1996, p. 1)

Before children begin their formal schooling they have been developing mathematics concepts through attempts at quantitatively and spatially understanding their surrounding world (Nasir & Saxe, 2003). When children enter school, their experiences with formal mathematics instruction may be different from those initial, socially-based learning experiences. This is particularly true for immigrant children and for children from traditionally marginalized groups.

There is a growing literature that connects educational process to specific student ethnic populations (Froumin, 1996; Edwards, 1999; Moses, 1994; Ortiz-Franco et al., 1999). While care must be taken to avoid overgeneralization and to constantly consider context (as is the case when considering gender, language ability, learning style, or any other learner attribute), research suggests that specific attention can and should be given to students' linguistic, ethnic, and cultural background when constructing mathematical learning experiences (Secada et al., 1995).

Consider for example how children educated in selected Spanish-speaking nations have been taught to use problem-solving procedures (e.g., switching the position of the divisor and dividend in division problems) and mathematical symbols (e.g., a period instead of a comma in numerals that are multiples of one thousand and a comma instead of a period for decimals) different from what is taught in our schools.

These students not only need to learn new content in an unfamiliar language, but, in addition, they have to relearn some of their known procedures and symbol usages. Similarly, many students coming to the United States knowing the metric system encounter difficulties when they have to learn new units of measurement (e.g., inches, feet, yards, pints, quarts, ounces, pounds).

Important here is to recognize the need for a cultural frame of reference when teaching mathematics to diverse students. In other words, knowing the students, their cultures, and their communities will help frame the teaching of mathematics in the classroom**.** One well-known example of this kind of teaching is the Algebra Project founded by civil rights activist Bob Moses. Moses designed the Algebra Project to empower African American, Hispanic, and other minorities to master the basics of algebra, since algebra often serves as a gatekeeper to advanced mathematics and science courses (Moses & Cobb, 2001).

The Algebra Project draws on culturally specific norms by consciously encouraging students to express a descriptive representation of algebraic equations by using the native language of the students (Kammi, 1990; Silva et al, 1990). The project emphasizes a curricular process that draws upon students' existing funds of knowledge and experiences and links that knowledge to more fundamental and powerful ideas that are pivotal in the study of algebra.

An example on a much smaller scale involves a former graduate student, now teaching in a middle school in New Jersey. Max, a European American teacher, works in a school with a large number of diverse students. Most of his students are immigrants from Peru, Colombia, El Salvador, Mexico, and Nicaragua, among other nations. Max has decided to make use of the linguistic and cultural diversity in his mathematics classroom by examining various demographic, economic, and population data from his students' home countries.

Various skills and concepts involving numbers and operations, statistics, measurement, and probability have emerged in this discussion. But so have a host of debates about cultural norms, social equity, and economic power. The use of students' native languages and mathematical symbol systems in the presentation of findings has added another dimension to the learning experience.

What led to success for the students in the Algebra Project and the mostly Latino students in Max's classroom was that these educators purposefully linked students' construction of mathematical understanding with everyday, meaningful events drawn from their own linguistic, cultural funds of knowledge. Whether the themes dealt with history, music, or sports, they were connected to the students' interests, lives, and cultures.

4.
Language and Cultural Factors in the Development of Mathematics Itself

> "You have math in your blood"
> —Jaime Escalante

Mathematics has often been taught in schools as though it is an objective reality, independent of cultural influence in its origin (Lakoff & Nunez, 2000). Mathematics dates its beginnings to the time when humans began quantifying objects and phenomena in their lives. Although the process of counting (one, two, three…) was similar for different cultures, the symbols to represent quantities varied according to their own particular cultural conventions (Ortiz-Franco, 2005).

Thus, Incas, Babylonians, Mayans, Romans, Hindus, Egyptians, Aztecs, and other groups had different notations to represent numbers. However, as William Brownell said 70 years ago, numbers and operations are not handed down from on high, but are derived from human activity:

> If the number combinations were "number facts" as they are frequently said to be, children would encounter little difficulty when learning them. They can easily learn "two dogs and three dogs are five dogs," for this is a fact. But "2 and 3 are 5" is not a fact; it is a generalization . . . We learn number combinations as we learn other generalizations, not all at once by some sort of will or mind, but slowly, by abstracting likenesses and differences in many situations, by reacting to the number aspects of situations in steadily more mature ways. (Brownell, 1935, p. 22)

Both meaning, notation, and ways of counting are products of human invention. For example, in some cultures people use their toes and fingers to count while in the United States we mostly use our fingers (could this explain why our current number system is base-10?). An understanding of the historical development of mathematics, grounded in cultural heritage, can add to the quality and depth of any related discussion and exploration. In addition, as many teachers of mathematics have found, the use of students' cultural and linguistic backgrounds can be a motivation in the classroom.

Mathematical symbols, linguistic terms, and number systems vary, but historically there has been a consistent need in all cultures to develop and use mathematics to quantify objects and phenomena in their lives. Despite these different paths of mathematical development, the nature of mathematics has often led to a consistency in the ideas produced across cultures. Consider the following three examples:

EXAMPLE 1:
An Old English Children's Rhyme
(Approximately 1700 A.D.)

As I was going to St. Ives, I met a man with seven wives;
Every wife had seven sacks; Every sack had seven cats;
Every cat had seven kits.
Kits, cats, sacks, and wives, How many were going to St. Ives?

EXAMPLE 2:
Leonardo Fibonacci wrote *Liber abaci* in 1202 A.D. which helped spread the use of the Hindu-Arabic numeral system throughout Europe. In it he wrote the following problem:

There are seven old women on the road to Rome.
Each woman has seven mules; Each mule carries seven sacks;
Each sack contains seven loaves; with each loaf are seven knives; and
Each knife in seven sheaths. Women, mules, sacks, loaves, and sheaths, how many are there in all on the road to Rome?

EXAMPLE 3:
In the Rhind papyrus, 1650 B.C, the following is transcribed:

Estate	
Houses	7
Cats	49
Mice	343
Heads of wheat	2401
Hekat measures	16807
	19607

Could this same problem have been derived independently in each of these cultures? In any event, it is important to note the differences and similarities in the development and application of mathematics that have occurred throughout our history and across cultures. Doing so not only "humanizes" the discipline of mathematics, but it provides specific ways of connecting the learning environment to a variety of cultures and to reinforce the importance of the mathematical aspects of the students' home cultures.

Zaslavsky (1996) and numerous books on mathematical history provide a variety of additional examples. The work of Ortiz-Franco (2005) is particularly useful when bringing into the forefront discussions about cultural achievements of pre-Columbian cultures—a topic of great importance in classrooms with Latino students, particularly those of Mexican descent. Ortiz-Franco points to the earliest evidence of numerical inscriptions using positional systems of bars and dots which have been traced to the Olmecs (from Mesoamerica, the geographical region that encompasses the area from northern-central Mexico to northern Costa Rica) in approximately 1,200 B.C.

This date is significant, since some 800 years before Aristotle, Plato, and Euclid (whose society did not have a positional number system) began making contributions to Western culture, the Olmecs were already using a positional system (Ortiz-Franco, 2005, p. 73). Important to note is that the Hindu-Arabic number notation, which uses zero as a "placeholder", first occurred in 499 A.D. Other important accomplishments by Mesoamerican groups that can be easily brought into the forefront when teaching mathematics include the Mayan complex calendar and astronomical studies developed hundreds of years before the achievements of Galileo and Copernicus (Ortiz-Franco, 2005). Likewise, studying the quipu (system of strings and knots used to represent quantities) and the complex tax system can bring to life accomplishments by the Incas during the 15th and 16th centuries in South America.

The above examples, while grounded in the discipline of mathematics—in particular, its historical development—are frequently omitted in mathematics textbooks (Ball, Goffney, & Bass, 2005). Acknowledging the wide variety of cultural achievements in the development of mathematics can give recognition to cultures that achieved a high level of sophistication in mathematical thinking, and provide specific attention to the mathematical concepts and processes underpinning these contributions.

For example, a teacher might use a variety of "standard" algorithms from different cultures to discuss arithmetic procedures, giving credence to students' invented strategies as well as providing opportunity for discussion and justification of the embedded numeric and operational concepts and processes. Different base-number systems can also be used for several aspects of numeric development.

Likewise, a discussion of particular instances in the historical development of algebra might be used as a motivating tool in the teaching of specific ethnic populations, or the contributions of female mathematicians might be highlighted to motivate girls. As suggested by Ortiz-Franco (2005), this integrated approach can do much to instill pride in students' culture and also increase confidence in their ability to learn and succeed in mathematics classes and, perhaps, later in mathematics-based careers.

In the movie "Stand and Deliver," Jaime Escalante tells his Latino students, "Math is in your blood!." While those words served as a strong motivator to his students, they remind us of how little attention we pay to the important contributions that other cultures have made to the development of mathematics, and how narrow we can be regarding what constitutes appropriate mathematics and student mathematical activity.

Conclusions and Implications

The purpose of this article is two-fold: first, to explore the role played by linguistic and cultural factors in the mathematics classroom, particularly in relation to diverse learners; and, second, to provide insight into teaching, learning, and professional development that takes into account current mathematics education reform recommendations. The four major topics discussed in this paper, while broad and complex, do collectively create a perspective that values what students bring to the mathematics classroom.

More specifically, this instructional perspective considers the strengths that students with diverse language and cultural backgrounds bring to a mathematical environment. These include different ways of representing, speaking, and thinking about mathematical ideas and skills. Teachers not privy to the various cultural and linguistic norms which ground these mathematical dispositions may have difficulty engaging students in mathematical conversations—the centerpiece of current mathematics education reform. The above discussions provide suggestions and specific instructional approaches encouraging teachers to view cultural and linguistic diversity as a strength in the teaching and learning of mathematics.

But the inclusion of students' perspectives, understandings, and practices in mathematics implies considering major changes in the ways we think about teaching and learning. Building on the cultural and linguistic funds students bring to the classroom challenges the traditional power relationships in schools where the teacher is the authority and sole dispenser of knowledge.

In classrooms where students and teachers are learning from each other, traditional hierarchical lines are diffused as students have opportunities to shine while they share knowledge and practices in which the teacher may not be an authority. Additionally, when students bring their knowledge and perspectives to the mathematics classroom, teachers can build upon that knowledge to make connections with other content areas.

As Frankestein (1997) so aptly stated: "Knowledge is not created and recreated in the fragmented forms in which most school subjects are presented. Mathematics occurs in contexts, integrated with other knowledge of the world" (p. 13). When students' problems are presented within a context that reaches beyond the mathematical realm, then problems become more meaningful and mathematical knowledge can be better integrated with other knowledge.

By building on students' perspectives, by allowing a leveling of the power relations in the classroom, and by integrating mathematical knowledge with other types of knowledge, student learning can be enhanced (Nasir & Saxe, 2003). Furthermore, students will learn to use mathematical knowledge for meaningful purposes, or as Freire suggests, "to read the world" (Freire & Macedo, 1987). This type of classroom affords opportunities for students to explore and understand society, beyond the confines of the classroom.

It is important that this kind of teaching of mathematics be incorporated into the mathematical content and methods experiences of preservice teachers. As suggested by Garrett (2002):

> Teachers cannot take their students where they themselves cannot go….The exposure of individuals to diverse individuals and groups is insufficient to assure their ability to teach all of America's children and youth. Only through carefully directed activities, with ample opportunity for reflection, can [preservice teachers] grow to become the kind of educators who are capable of working with a diverse population. (p. 68)

Teacher education programs need to become the site at which college students' preconceived beliefs about linguistically and culturally diverse students and practices are interrogated. Unfortunately, most teacher preparation programs have yet to respond to this need.

The first step toward this goal is for teacher education faculty to recognize teaching students from different cultural and linguistic backgrounds as a salient and nuanced topic that needs to be included throughout the teacher education curriculum (Costa, McPhail, Smith, & Brisk, 2005). These authors provide some evidence that specific efforts to work with faculty in incorporating issues of language and culture into a teacher education program can change the landscape of the teacher preparation experience. The above instructional aspects are one map for such a reexamination in the context of mathematics.

Clearly, the incorporation of language and culture into the teaching of mathematics is a complex process, requiring, among other things, a self-examination of pedagogical and mathematical beliefs, a desire to utilize students' backgrounds in instructional planning and process, and insight into a variety of knowledge sets and dispositions related to specific aspects of language and culture.

Given the time demands teachers face, these challenges are significant. However, given the changing student demographics and the promise that such an instructional approach offers, it is critical that schools, districts, state and national agencies, and teacher preparation programs encourage the active attention to these dimensions in the teaching of mathematics.

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