Creating mathematics performance assessments that address multiple student levels

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In recent times there has been considerable commentary regarding the need to enhance mathematical assessment as evidenced by *Numeracy, A Priority for All: Challenges for Australian Schools* (2000). This emphasis on assessment is timely because although the mathematical reform movement has produced much-needed improvements in both curriculum and instruction, changes in assessment have not kept pace (Firestone & Schorr, 2004; Morgan, 1998). As Ridgway (1998, p. 2) states, "As an issue of policy, the implementation of standards-based curricula should always be accompanied by the implementation of standards-based assessment. In fact, incremental change in assessment systems will foster concurrent improvement in professional and curriculum development."

There is a need to assess a much wider range of mathematical abilities than has been the case heretofore, including problem solving and posing, representing, and understanding. This broadened view of mathematical assessment is supported by curriculum guides from numerous nations. For example, Australian frameworks suggest that numeracy not only includes the ability to perform basic calculations, but also a thorough, connected understanding of number and operation (Leonelli & Schmitt, 2001). Traditional mathematics assessments tend to communicate that mathematics is an endeavor that involves determining a quick answer using a pre-existing, memorized method (Bell, 1995; Clarke, Clarke & Lovitt, 1990), thus failing to represent the true complexity of mathematics (Galbraith, 1993). The measurement of decontextualized technical skills should be replaced with measures that reflect what is known about what it means to understand and do mathematics (AAMT, 2002; NCTM, 2000). Both the *Assessment Standards for School Mathematics* (NCTM, 1995) and the *Principles and Standards for School Mathematics* (NCTM, 2000) state that assessment tasks communicate what type of mathematical knowledge and performance are valued. Therefore, standards-based instruction is complemented by standards-based assessment (Dunbar & Witt, 1993).

Parallelling reform in mathematics curriculum and instruction have been calls to authenticate student assessment in all subject areas. Terms such as, "authentic assessment," "alternative assessment," and "performance assessment" have become banners to rally focussed efforts to change paradigms about the nature and purpose of assessment. Mathematics educators have particularly focussed upon the use of performance assessments as both a means to align assessment with new reform curricula (Firestone & Schorr, 2004) and to improve the links between teaching practice and assessment (Pelegrino, Chubowsky & Glaser, 2001).

One of the challenges associated with performance assessments is designing them in such a way so that they can be delivered on an individual student's level, i.e., developmental assessments (Pegg, 2003). In today's classrooms, teachers are usually faced with the responsibility of teaching children with a wide range of abilities. It is quite conceivable that a well-designed performance assessment could be administered and yet fail to provide useful data if the assessment is delivered at a level that is either too difficult or too easy for the student being assessed. The purpose of this article is to suggest ways to create such assessments while honoring what is known about the traits that characterize high quality performance assessments in mathematics.

Creating a performance assessment

The content of number and operations forms the cornerstone of the entire mathematics curriculum internationally (Hogan, Murcia, & van Wyke, 2004; NCTM, 2000; Reys & Nohda, 1994), therefore I have chosen to focus on this content in helping teachers begin to learn to design performance-based assessments for their classrooms. This section describes steps I use in teaching teachers to create performance assessments; it reflects input from teachers involved in professional development workshops over several years.

1. Choose a topic for your grade level, e.g., division of whole numbers, about which a problem can be developed that leads to the types of concepts you want to develop (e.g., Ma, 1999).

2. Create the performance task. Write an engaging, real-life word problem that incorporates the concepts you have chosen. Craft it so that any size numbers would make sense in the wording, which will allow for assessing multiple levels through one task. For example, here is a fifth grade division task:

   > I have ________ pieces of candy that I am going to put into bags and each bag will contain _______ candies. How many bags will I have?

   Although this task, or problem, is simple in its construction, we have found it to be quite engaging, and children have created a number of novel solution strategies in solving it. It also allows for a great deal of reasoning and representing, particularly when accompanied by the wise use of probing questions by the assessor as discussed subsequently in step 6.

Note that numbers of varying numerical complexity can be inserted in the blanks depending upon the child's estimated level. The term "level" possesses multiple definitions in mathematics education, but in a number and operations context it often relates to the complexity of the numbers involved in the problem a child is asked to solve. Therefore, one of the ways the level of a problem can be regulated is by regulating the complexity of the numbers in the problem. The teachers with whom I have worked have helped to develop a "hierarchy of numerical complexity" relative to number and the four operations to guide them in determining the level at which a child is capable of problem solving; they have found it to be quite accurate and useful (see Table 1).

3. Design a quickly administered inventory to estimate level. This inventory should call for the solving of simple exercises of varying complexity in the operation associated with the grade and can be administered to an entire class in written form prior to administering the performance assessment. Exercises in a possible fifth grade inventory might look like the ones that appear below.

    1. $8 \div 2 =$
    2. $12 \div 2 =$
    3. $7 \div 3 =$
    4. $60 \div 2 =$
    5. $120 \div 4 =$
    6. $65 \div 2 =$
    7. $150 \div 7 =$
    8. $78 \div 3 =$
    9. $60 \div 20 =$

    Note that this inventory is quite procedural in nature. The teachers with whom I have worked have found that procedural performance can be used to find a quick, rough estimate of the level at which the performance assessment should be administered. However, if a child's instructional experience has not included the development of solid connections between concepts and procedures, this estimate is likely to be too high. If the child's responses in the initial stages of the performance assessment indicate that the level estimate obtained from the inventory is inaccurate, the teacher makes an immediate adjustment by re-administering the task with numbers of differing complexity.

4. Select criteria to serve as standards for judging the performance. I have found seven criteria to be quite useful in gauging the quality of a student's mathematical performance: 5 analytical criteria based upon the NCTM Process Standards (2000) in agreement with the earlier suggestions of Dunbar and Witt (1993) and

Table 1

Level	Number sense	Addition	Subtraction	Multiplication	Division
A	Rote counting	Joining sets	Separating sets	1 digit × 1 digit = 1 digit 2 × 3 = 6	1 digit ÷ 1 digit = 1 digit 8 ÷ 2 = 4
B	One-to-one correspondence	Single digit addends & sum 3 + 2 = 5	1 digit – 1 digit = 1 digit 5 – 3 = 2	1 digit × 1 digit = 2 digits (composing) 2 × 6 = 12	2 digit ÷ 1 digit = 1 digit 12 ÷ 2 = 6
C	Single digit < 5	Single digit addends & double digit sum 3 + 9 = 12	2 digits – 1 digit = 1 digit (decomposing) 13 – 5 = 8	10 × single digit 10 × 3 = 30	1 or 2 digits ÷ 1 digit = 1 digit with remainder 7 ÷ 3 = 2 r 1
D	Single digit > 5	Multiple single digit addends 3 + 2 + 4 = 9	2 digits – 1 digit = 2 digits (no decomposing) 27 – 5 = 22	10 multiple × single digit = 2 digits 20 × 3 = 60	10 multiple ÷ 1 digit = 10 multiple (no decomposing) 60 ÷ 2 = 30
E	2 digit >15 < 20	2 digits + 2 digits no composing 32 + 24 = 56	2 digits – 1 digit = 2 digits (decomposing) 27 – 9 = 18	10 multiple × single digit = 3 digits (composing) 30 × 4 = 120	3 digit 10 multiple ÷ 1 digit (no decomposing) 120 ÷ 4 = 30
F	2 digit > 9 < 16	2 digits + 2 digits with composing 32 + 29 = 61	2 digits – 2 digits = 2 digits (no decomposing) 36 – 24 = 12	2 digits × 1 digit (no composing) 13 × 2 = 26	2 or 3 digits ÷ 1 digit = 2 digits (no decomposing) 65 ÷ 2 = 32 r1
G	2 digit > 20	3 2–digit addends with composing 32 + 25 + 46 = 103	2 digits – 2 digits = 1 or 2 digits (decomposing) 32 – 18 = 14	2 digits × 1 digit (composing) 14 × 3 = 42	2 or 3 digit 10 multiple ÷ 1 digit = 2 digits (decomposing) 150 ÷ 7 = 21 r 3
H	3 digit	3 digits + 3 digits varying composing 391 + 467 = 858	3 digits – 2 or 3 digits = 1, 2 or 3 digits (decomposing involving 1 zero) 406 – 178 = 228	10 multiple × 10 multiple = 3 digits (composing) 20 × 20 = 400	2 or 3 digits ÷ 1 digit = 2 digits (decomposing) 78 ÷ 3 = 26
I	3 digit, zeroes in ones or tens places	3 3-digit addends with composing 323 + 257 + 469	3 or 4 digits – 2 or 3 digits (1 decomposing) 1469 – 635	2 digits × 2 digits (no composing) 13 × 12	3 digits ÷ 1 digit = 3 digits 432 ÷ 2 = 216
J	4 digit	3 4–digit addends with composing 3235 + 2579 + 4696	4 or 5 digits – 4 or 5 digits (2 alternating decomposes) 4628 – 1809	2 digits × 2 digits (one composing on first row) 23 × 14	3 digits ÷ 1 digit = 3 digits (zero in quotient) 412 ÷ 2 = 206
K	5 digit	4 digits + 4 digits varying composing 4625 + 1856	Variable digit number (2 consecutive decomposes) 631 – 253	2 digits × 2 digits (one composing on second row) 43 × 24	2 or 3 digits ÷ 10 multiple 60 ÷ 20 = 3
L	6 digit		Variable digit number (3 consecutive decomposes) 54363 – 14581	2 digits × 2 digits (two composes) 23 × 65	
M	6 digit, zeroes		Variable digit number (decomposing involving 2 or more zeroes) 4001 – 1376	2 digits × 2 digits (larger digits) 67 × 98	
N	7 digit				

two holistic criteria as suggested by the Learning Principle, also part of the *Principles and Standards* document. The 5 analytical criteria are:

a. problem solving — accurately solving a worthwhile task using multiple strategies;
b. communicating — explaining problem solving strategies clearly;
c. reasoning — justifying those strategies in a mathematically sound manner;
d. representing — showing or modelling mathematical ideas in multiple ways;
e. connecting — explaining the connections between strategies and/or representations.

The two holistic criteria are:

f. conceptual — demonstrating an overall understanding of the mathematics involved with solving the task;
g. procedural — demonstrating knowledge of the rules or algorithms involved with solving the task.

5. Design a rubric using those criteria. One way to go about designing this rubric is to create a scoring hierarchy based upon the degree of assessor prompting required in order for a student to experience success in the assessment (see Table 2). In other words, the more assistance a child requires, the lower the rubric score. The incorporation of prompting as a factor in distinguishing rubric levels results in a blurring of the line between instruction and assessment in harmony with current assessment philosophy (McMillan, 2004).

6. Create questions or prompts to probe student thinking. Appropriate questions insure that opportunities are provided for students to express themselves verbally as well as in written form (Dunbar & Witt, 1993; Glaser, Raghavan & Baxter, 1992) and insure that students were invited to display behaviour that addresses all analytical criteria, i.e., the Process Standards (Mewborn & Huberty, 1999). In this way you can be confident that important mathematical knowledge is assessed (NCTM, 2000; Morgan, 1998; Dunbar & Witt, 1993) and that the interpretations associated with the assessment possess construct validity (Messick, 1989), particularly in terms of assessing deep, connected conceptual understanding. Some possible prompts and questions for a fifth grade division assessment appear below:

a. Which operation would you use to find the answer? (problem solving)
b. Explain how you solved the problem. (communicating)
c. Solve the problem in a different way. Explain or show me. (problem solving)
d. What would happen to the numbers in the question if you multiplied them? (If student multiplied, ask what would happen to the numbers if they divided them.) (reasoning)
e. Show this problem as a fraction. (representing)

Table 2

Rubric level	Problem Solving	Communicating	Reasoning	Representing	Connecting	Procedural	Conceptual
4 Independent Understanding	Can solve the problem in two ways independently	Can clearly explain the problem solving strategies	Can clearly justify the problem solving strategies	Can represent the problem in at least two ways independently	Can independently connect representations or strategies	Can solve the problem using a procedure independently	Can show thorough understanding of the problem and of the associated mathematics independently
3 Understanding with minimal help	Can solve the problem in two way with minimal help or one way independently	Can clearly explain all but one part of the problem solving strategies	Can justify all but one part of the problem solving strategies	Can represent the problem in two ways with minimal help or one way independently	Can connect representations or strategies with minimal help	Can solve the problem procedurally with minimal help	Can show some understanding with minimal help
2 Understanding with substantial help	Can solve the problem at least one way with help	Can explain portions of the problem solving strategies	Can justify portions of the problem solving strategies	Can represent some of the problem with help	Can connect representations or strategies only with substantial help	Can solve the problem procedurally with substantial help	Can show some understanding with substantial help
1 Little understanding	Cannot solve the problem even with help	Cannot explain the strategies even with help	Cannot justify the strategies even with help	Cannot represent the problem even with help	Cannot connect representations or strategies even with help	Cannot solve the problem procedurally even with help	Cannot show understanding even with help

   f. Solve this problem using pictures, manipulatives, etc. (connecting)

7. Create a form for students to record their work and a teacher recording form.

Performance Assessment
Teacher Recording Form

Name ____________________ Grade ____

March date _______________

Operation /Algorithmic Sense

____ Level (Number Complexity)

____ Problems Solving (multiple ways)
____ Communicating (explaining)
____ Reasoning (justifying)
____ Representing (show multiple ways)
____ Connecting (representations, explanations)
____ Procedural understanding
____ Conceptual understanding

Anecdotal notes:

Mathematics Performance Assessment
Student Recording Form

Name ____________________ Grade ____ Date ________

There are ____________ pieces of candy. We need to put them into ________________bags. How many pieces of candy will be in each bag?

Working:

Conclusion

By following a few simple guidelines, very useful performance assessments in mathematics can be designed and administered. These assessments will allow you to determine the level at which students comprehend and perform important mathematics. The data derived therefrom will inform your instruction and send a clear, consistent message to your students about what it means to know and to do mathematics.

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From Helen Prochazka's

Scrapbook

On two occasions I have been asked, by members of Parliament, "Pray, Mr. Babbage, if you put into the machine wrong figures, will the right answers come out?" I am not able rightly to apprehend the kind of confusion of ideas that could provoke such a question.
Charles Babbage,
19th century mathematician and inventor

Medicine makes people ill, mathematics make them sad and theology makes them sinful.
Martin Luther,
16th century religious reformer

I can calculate the motion of the heavenly bodies but not the madness of crowds.
Isaac Newton,
19th century mathematician and astronomer