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Are Giftedness and Creativity Synonyms in Mathematics?

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At the K–12 level one assumes that mathematically gifted students identified by out-of-level testing are also creative in their work. In professional mathematics, "creative" mathematicians constitute a very small subset within the field. At this level, mathematical giftedness does not necessarily imply mathematical creativity but the converse is certainly true. In the domain of mathematics, are the words creativity and giftedness synonyms? In this article, the constructs of mathematical creativity and mathematical giftedness are developed via a synthesis and analysis of the general literature on creativity and giftedness. The notions of creativity and giftedness at the K–12 and professional levels are compared and contrasted to develop principles and models that theoretically "maximize" the compatibility of these constructs. The relevance of these models is discussed with practical considerations for the classroom. The paper also significantly extends ideas presented by Usiskin (2000).

Creativity in mathematics is often looked at as the exclusive domain of professional mathematicians. The word *creativity* is "fuzzy" and lends itself to a variety of interpretations. What does creativity mean in mathematics? Is it purely the discovery of an original result? If so, then creativity is indeed the exclusive domain of professional mathematicians. Does student discovery of a hitherto known result or an innovative mathematical strategy also constitute creativity? Eminent mathematicians like Jacques Hadamard (1945) and George Polya (1954) have said that the only difference between the work of a mathematician and a student is that of degree. In other words, each operates at their respective levels, and we should recognize that students are also capable of being creative. Such a view is especially relevant to teachers of mathematically gifted students, who would expect gifted students to display creative traits. Does being mathematically gifted predispose a student to being creative? In other words, if a student has been identified as being mathematically gifted, then is he or she also creative in his or her approach to mathematics? Does mathematical giftedness imply mathematical creativity?

Kajander (1990) has stated that even among the mathematically gifted who displayed creative traits such as divergent thinking, mathematical creativity was a "special kind of creativity not necessarily related to divergent thinking" (p. 254). This statement begs the question as to whether or not mathematical creativity implies giftedness. It does at the level of professional research in mathematics. One could easily argue that professional mathematicians are gifted based on the fact that they have obtained a doctorate in the field and are active in research. However, even at this level, only a handful of professional mathematicians are classified as being truly "creative" (Usiskin, 2000).

An examination of Usiskin's (2000) eight-tiered hierarchy may help clarify the degrees of giftedness and creativity with regard to mathematics. Usiskin devised this hierarchy, which ranges from Level 0 to Level 7, to classify mathematical talent. In this hierarchy, Level 0 (No Talent) represents adults who know very little mathematics, and Level 1 (Culture level) represents adults who have rudimentary number sense as a function of cultural usage and their mathematical knowledge is comparable to those of students in grades 6–9. Clearly, a very large proportion of the general population would fall into the first two levels.

Thus, the remaining population is thinly spread throughout Levels 2 through 7 on the basis of mathematical talent. Level 2 represents the honors high school student who is capable of majoring in mathematics, as well as those who eventually become secondary math teachers. Level 3 (the "terrific" student) represents students that score[1] in the 750–800 range on the SATs or a 4 or 5 in the Calculus AP exams. These students have the potential to do beginning graduate level work in mathematics. Level 4 (the "exceptional" student) represents students who excel in math competitions and receive admission into math/science summer camps and/or academies because of their talent. These students are capable of constructing mathematical proofs and are able to converse with mathematicians about mathematics. Level 5 represents the productive mathematician. Although Usiskin's (2000) description of this level is vague, one can infer that it represents students who have successfully completed a Ph.D. in mathematics or related mathematical sciences and are capable of publishing in the field. Level 6 is the rarefied territory of the exceptional mathematician; it represents "mathematicians . . . [who] have moved their domains forward with notable conquests; they will be found in any history of the domains in which they work. These mathematicians are at the level of the Alfred P. Sloan fellows, the best in their age group in the country" (Usiskin, 2000, p. 156). Finally, at Level 7 are the all-time greats, the Fields Medal winners in mathematics.[2] This level is the exclusive territory of giants or exemplary geniuses like Leonard Euler, Karl Friedrich Gauss, Bernhard Riemmann, Srinivasa Ramanujan, David Hilbert, Henri Poincaré, and others.

In Usiskin's (2000) eight-tiered hierarchy of mathematical talent, the professional (gifted) mathematician is at Level 5, whereas the creative mathematician is found at Levels 6 and 7. Therefore, in the professional realm, mathematical creativity implies mathematical giftedness, but the reverse is not necessarily true. In this hierarchical classification of mathematical talent, students who are gifted and/or creative in mathematics are found at levels 3 and 4. The point that Usiskin emphasized is that these students have the potential of moving up into the professional realm (Level 5) with appropriate affective and instructional scaffolding as they progress beyond the K–12 realm into the university setting.

Motivation for Research

Numerous studies (i.e., Cramond, 1994; Davis, 1997; Smith, 1966; Torrance, 1981) indicate that very often the behavioral traits of creative individuals go against the grain of acceptable behavior in the institutionalized school setting. For instance, negative behavioral traits, such as indifference to class rules, display of boredom, cynicism, or hyperactivity, usually result in disciplinary measures as opposed to appropriate affective interventions. In the case of gifted students who "conform" to the norm, these students are often prone to hide their intellectual capacity for social reasons, and identify their academic talent as being a source of envy (Massé & Gagné, 2002). History is peppered with numerous examples of creative individuals described as "deviants" by the status quo. Brower (1999) has presented more than 50 examples of eminent writers, moral innovators, scientists, artists, and stage performers who were jailed because of society's fear of ideas that were "out there," yet powerful enough to create paradigmatic shifts in the public's mindset.

The stifling of creativity at the K–12th-grade levels is often collectively rationalized under the guise of doing what is supposedly good for the majority of the students, invoking the often misused term *equity*, appealing to curricular plans and school achievement goals, and so forth. The recent interpretation of the No Child Left Behind Act (NCLB; 2001) in the United States has brought to the forefront the debate about what to do with creative and gifted students in the classroom. Recently, Marshak (2003) wrote that the NCLB Act's call for accountability based on standardized testing for the traditional skills of reading, writing, and arithmetic valued in society's industrial setting is a giant step backwards to the 1940s. Based on recent reports released by the U.S. Department of Labor, Marshak has further stated that in addition to the three "traditional" R's (reading, writing, and 'rithmatic), additional skills such as problem solving and creative thinking are necessary for success in the global setting of 21st century. Even at the tertiary levels, there have been criticisms about the excessive amount of structure imposed on disciplines by academics and the limitations of "narrow, profoundly Western centric attitudes" (Creme, 2003, p. 273). Such criticism resonates particularly in the world of mathematics, especially at the K–12 level, where gifted/creative students with a non-Western ethnic background are rarely encouraged to express or use mathematical techniques they may be familiar with from their own cultures. Instead, they are encouraged to adopt a Western attitude. In summary, the literature indicates that giftedness is often associated with conformity, whereas creativity is viewed as a fringe commodity, tolerated and nurtured by some teachers, but typically not encouraged. There is clearly a schism between the value of creativity in the K–12 setting and in professional realms, which leads one to ponder how this schism can be bridged. This question is further explored in the present paper.

What About Problem Solving?

Professional mathematicians engage in problems that are full of uncertainty. However, most curricular and pedagogical approaches rarely offer students this open-ended view of mathematics. In fact, classroom practices and math curricula rarely use ill-posed or open-ended problems, or allow students a prolonged period of engagement and independence to work on these types of problems. Although problem solving in the mathematics classroom has received increased emphasis since the release of the original National Council of Teachers of Mathematics Standards (1989), nearly two decades later it has essentially become a dogmatic term invoked to act as panacea to remedy curricular ills. This rather strong statement receives considerable support from the extant surveys of the research literature on problem solving. For example, in the *Handbook for Research on Mathematics Teaching and Learning*, Schoenfeld (1992) described how the field of mathematics education in the United States has been subject to approximately 10-year cycles of pendulum swings between basic skills and problem solving. He concluded his chapter with optimism about the continuation of a movement that many at that time referred to as "the decade of problem solving" in mathematics education. However, since the 1992 handbook was published "the worldwide emphasis on high-stakes testing has ushered in an especially virulent decade-long return to basic skills" (Lesh & Sriraman, 2005b, p. 501).

Additionally, consider the following facts: Polya-style problem-solving heuristics—such as *draw a picture*, *work backwards*, *look for a similar problem*, or *identify the givens and goals*—have long histories of being advocated as important abilities for students to develop (Polya, 1945). But, what does it mean to "understand" them? Such strategies clearly have descriptive power. That is, experts often use such terms when they give after-the-fact explanations of their own problem-solving behaviors—or those of other people whom they observe. But, there is little evidence that general processes that experts use to describe their past problem-solving behaviors should also serve well as prescriptions to guide novices' next steps during ongoing problem-solving sessions. Researchers gathering data on problem solving also have the natural tendency to examine the data in front of them through the lens of a priori problem-solving models. Although there is great value in doing so, does such an approach really advance problem-solving research?

If one examines the history of problem-solving research, there have been momentous occasions when researchers have realized the restricted heuristic view of problem solving offered by the existing problem-solving research "toolkits," and have succeeded in redesigning existing models with more descriptive processes. However, the problem remains that descriptive processes are really more like names for large categories of skills rather than being well-defined skills in themselves. Therefore, in attempts to go beyond "descriptive power" to make such processes more "prescriptive power," one tactic that researchers and teachers have attempted is to convert each descriptive process into longer lists of more restricted, but also more clearly specified processes. If this approach is adopted, most of what it means to understand such processes involves knowing when to use them. So, higher order managerial rules and beliefs that specify when and why to use lower order prescriptive processes need to be introduced.

The obvious dilemma that arises is that short lists of descriptive processes have appeared to be too general to be meaningful. On the other hand, lists of prescriptive processes tend to become so numerous that knowing when to use them becomes the heart of understanding them (Lesh & Sriraman, 2005b). Furthermore, adding more metacognitive rules and beliefs only compounds these two basic difficulties. A decade after Schoenfeld's chapter in Grouws' (1992) *Handbook for Research on Mathematics Teaching and Learning* was published, in another extensive review of the literature, Lester and Kehle (2003) again reported that little progress had been made in problem-solving research, and that problem solving still had little to offer to school practice. Their conclusions agreed with Silver (1985), who long ago determined what we consider to be the core of the problem in problem-solving research. That is, the field of mathematics education needs to go "beyond process-sequence strings and coded protocols" in our research methodologies and "simple procedure-based computer models of performance" to develop ways of describing problem solving in terms of conceptual systems that influence students' performance (Silver, p. 257). Thus, the use of problem solving in the mathematics classroom arguably lends itself to a host of questions about its purpose, as well as its effectiveness.

Mathematical Creativity: The Lack of Domain-Specific Definitions in Mathematics

Given the relevance of giftedness and creativity to our society, we will now focus our attention specifically on the domain of mathematics with the purpose of generating appropriate definitions for these terms. The existing literature provides an overview of the various meanings of the terms *mathematical creativity* and *mathematical giftedness*; in addition, the literature demonstrates their compatibility and relevance at the professional and K–12 levels. Most of the extant definitions of mathematical creativity found in the mathematics and mathematics education literature are vague or elusive, which may be attributed to the difficulty of describing this complex construct. For instance, mathematical creativity has been defined as the ability to discern, or choose (Hadamard, 1945; Poincaré, 1948), to distinguish between acceptable and unacceptable patterns (Birkhoff, 1969), and to engage in nonalgorithmic decision-making (Ervynck, 1991). The literature on students who are mathematically creative in the K–12 realm is also vague. Exceptional mathematical ability (Level 4 talent) in the K–12 realm has been associated with Einstein syndrome (Sowell, 2001) and Asperger's syndrome (Jackson, 2002). The Einstein syndrome is characterized by exceptional mathematical ability, but delayed speech development, whereas Asperger's syndrome is a spectrum disorder characterized by "severe impairment in reciprocal social interaction, all absorbing narrow interests or obsession with a particular subject . . . [a]nd sometimes motor clumsiness" (James, 2003, p. 62). The dearth of specific definitions of mathematical creativity in the mathematics and mathematics education literature necessitates that we move away from the specific domain of mathematics to the general literature on creativity in order to construct an appropriate definition.

Creativity: General Definitions in Psychology/Educational Psychology

In the literature on creativity, numerous definitions can be found. Craft (2002) used the term *life wide creativity* to describe the numerous contexts of day-to-day life in which the phenomenon of creativity manifests. Other researchers have described creativity as a natural *survival* or *adaptive* response of humans in an ever-changing environment (Gruber, 1989; Ripple, 1989). Craft (2003) has also pointed out that it is essential we distinguish *everyday creativity* such as improvising on a recipe from *extraordinary creativity*, which causes paradigm shifts in a specific body of knowledge. It is generally accepted that works of extraordinary creativity can be judged only by experts within a specific domain of knowledge (Csikszentmihalyi, 1988, 2000; Craft, 2003). For instance, Andrew Wiles' proof of Fermat's Last Theorem could only be judged by a handful of mathematicians within a very specific subdomain of number theory.

At the K–12 level, one normally does not expect works of extraordinary creativity; however, it is certainly feasible for students to offer new insights into a math problem or a new interpretation or commentary on a literary or historical work. Students at the K–12 level are certainly capable of originality. For example, Weisberg (1993) suggested that creativity entails the use of ordinary cognitive processes and results in original and extraordinary products. Further, Sternberg and Lubart (2000) defined creativity as the ability to produce unexpected original work that is useful and adaptive. Other definitions usually have imposed the requirement of novelty, innovation or unusualness of a response to a given problem (Torrance, 1974). Numerous confluence theories of creativity have defined creativity as a convergence of knowledge, ability, thinking style, and motivational and environmental variables (Sternberg & Lubart, 1996, 2000), as well as an evolution of domain-specific ideas resulting in a creative outcome (Gruber & Wallace, 2000). For example, Csikszentmihalyi (2000) has maintained that creativity is a mutation resulting from a favorable interaction between an individual, domain, and field. Most recently, Plucker and Beghetto (2004) offered an empirical definition of creativity based on a survey and synthesis of numerous empirical studies in the field. They defined creativity as "the interplay between ability and process by which an individual or group produces an outcome or product that is both novel and useful as defined within some social context" (p.156).

Applying the General Definitions of Creativity to Mathematics

A synthesis of the numerous definitions of creativity can lead to a working definition of mathematical creativity at both the professional and K–12 levels. At the professional level, mathematical creativity can be defined as (a) the ability to produce original work that significantly extends the body of knowledge, and/or (b) the ability to open avenues of new questions for other mathematicians.

For instance, Hewitt's (1948) paper on rings of continuous functions led to unexplored possibilities and questions in the fields of analysis and topology that sustained other mathematicians for decades. A modern-day illustration of the far-reaching effects of Hewitt's paper is to "Google" the title of the paper, which results in over 120,000 hits.

On the other hand, mathematical creativity in grades K–12 can be defined as (a) the process that results in unusual (novel) and/or insightful solution(s) to a given problem or analogous problems, and/or (b) the formulation of new questions and/or possibilities that allow an old problem to be regarded from a new angle requiring imagination (Einstein & Inheld, 1938; Kuhn, 1962). The second part of this definition is very similar to definitions of creativity in professional mathematics.

The research also indicates that, at both the K–12 and the professional levels, creative individuals are prone to reformulating a problem or finding analogous problems (Frensch & Sternberg, 1992; Polya, 1945, 1954). These individuals are also different from their peers in that they are fiercely independent thinkers (Chambers, 1964; Gruber, 1981; Ypma, 1968), tend to persevere (Chambers; Diezmann & Watters, 2003), and reflect a great deal (Policastro & Gardner, 2000; Sriraman, 2003, Wertheimer, 1945).

Conditions That Enhance Mathematical Creativity at the Professional Level

Now that we have a working definition of mathematical creativity, we can explore the conditions under which such creativity manifests. In order to illuminate the conditions that enhance the manifestation of creativity at the professional level, Sriraman (2004c) conducted a qualitative study with five accomplished and creative professional mathematicians. The five mathematicians verbally reflected on the thought processes involved in studying mathematics. The results indicated that in general, the mathematicians' creative process followed the four-stage Gestalt model (Wallas, 1926) of *preparation, incubation, illumination,* and *verification*. It was also found that social interaction, imagery, heuristics, and intuition were some of the characteristics of mathematical creativity. Other characteristics that contributed to their research productivity were the time available in an academic setting for the pursuit of research, freedom of movement, the aesthetic appeal of mathematics, and the urgency/drive to solve problems with tremendous real-world implications. All five mathematicians spoke at length about the "Aha" or "Eureka" moment (Burton, 1999a, 1999b; Wallas) at which they gained a new insight into the problem, and were able to successfully construct a proof.

Mathematical Giftedness

A synthesis of the research literature on mathematical giftedness and characteristics of mathematical thinking revealed that the construct of mathematical giftedness has been defined in terms of the individual's ability in mathematical processes such as: (a) the ability to abstract, generalize, and discern mathematical structures (Kanevsky, 1990; Kiesswetter, 1985, 1992; Krutetskii, 1976; Shapiro, 1965; Sriraman, 2002, 2003); (b) the ability to manage data (Greenes, 1981; Yakimanskaya, 1970); (c) the ability to master principles of logical thinking and inference (Goldberg & Suppes, 1972; Suppes & Binford, 1965); (d) the ability to think analogically and heuristically and to pose related problems (Polya, 1954; Kiesswetter, 1985); (e) flexibility and reversibility of mathematical operations and thought (Krutetskii); (f) an intuitive awareness of mathematical proof (Sriraman, 2004c); (g) independent discovery of mathematical principles (Sriraman, 2004a, 2004b); (h) the ability to make decisions in problem-solving situations (Frensch & Sternberg, 1992; Schoenfeld, 1985; Sriraman, 2003); (i) the ability to visualize problems and/or relations (Hershkowitz, 1989; Presmeg, 1986); (j) the ability to infer behaviors that test for truth or falsity of a construct (Wason & Johnson-Laird, 1972); (k) the ability to distinguish between empirical and theoretical principles (Davydov, 1988, 1990; Vygotsky, 1962, 1978); and (l) the ability to think recursively (Kieren & Pirie, 1991; Minsky, 1985).

In addition, mathematical giftedness has also been associated with the capacity for learning at a faster pace (Chang, 1985; Heid, 1983). Most of the mathematical processes listed above are primarily cognitive and learned during K–12 schooling experiences. It should be noted that many of these studies involved task-based instruments with specific mathematical concepts/ideas to which students had previous exposure. Another important observation is that although many of these traits do play a role and are necessary in the setting of professional mathematics, they are not sufficient for creativity to manifest. In other words, in order to work as a professional mathematician (Level 5) and to create new mathematics, some abilities are more crucial than others. In particular, decision making; the abilities of abstracting and generalizing, inferencing, and constructing theoretical principles; and recursive thinking play an important role in how mathematics are created at the professional level. The processes of inferencing (Wason & Johnson-Laird, 1972), constructing theoretical principles (Davydov, 1988, 1990), and recursive thinking (Kieren & Pirie, 1991; Vitale, 1989) play a vital role in how new mathematics are created. Simply put, this process can be viewed as follows: The applied mathematician is trying to create mathematical models that say something about the physical world. The pure mathematician is willing to take those models and see what the implications are. During the modeling process, there exists some physical situation and the applied mathematician tries to identify the underlying principles. The pure mathematician steps back and abstracts to a setting in which these basic principles hold, and to see what the implications are. The pure mathematician who is dealing with the implications is working rather formally to see what is implied logically by this particular set of assumptions, without worrying about whether this is an appropriate model or not.

Wason and Johnson-Laird (1972) investigated whether adults, when given a set of assertions, were able to appreciate the logical implications. They were particularly interested in determining contexts that led adults[3] into drawing fallacious conclusions. According to Wason and Johnson-Laird, "The rational individual, in our sense of the word, is merely one who has *the ability to make inferences* [italics added]; he may not be rational in any other sense of the word" (p. 2). The process of inferencing can lead to mathematical generalizations. Thus, Wason and Johnson-Laird investigated how adults discovered general rules by setting up structured experiments, in which "subjects were presented with a hypothesis and they had to decide the items of evidence relevant for testing its truth. The experiments were designed to investigate the propensity of individuals to offer premature solutions based on confirming evidence" (p. 202).

The researchers had several interesting findings. First of all, subjects tended to make fallacious inferences when presented with affirmative statements. Another finding was that an overwhelming majority of the subjects in the study were prone to try to verify generalizations rather than to try and falsify them. Moreover, the researchers noted that the content of the material about which inferences were made was significant. Subjects tended to make "illicit conversions" and were biased towards verification when faced with material of an abstract nature, such as mathematical problems in n-dimensional space that can only be symbolically represented. However when the material was concrete and subjects had experienced a variety of connections with it, they tended to generate and assess hypothetical connections between facts. In other words, mathematical inference behavior is different from everyday inference behavior and mathematically gifted individuals are adept at logically and correctly connecting abstract constructs different from everyday constructs. This distinction between everyday (or empirical) and abstract (or theoretical) concepts was studied by Vygotsky (1962, 1978) in his investigations on concept formation and later pursued by Davydov (1988, 1990).

Initially Vygotsky (1962, 1978) explored the notion of scientific generalizations during his investigations on concept formation and distinguished between two types of concepts, namely spontaneous, or everyday, concepts, and scientific, or theoretical, concepts. Davydov (1988) continued this line of investigation on conceptualization and stressed that the important difference between everyday (empirical) concepts and theoretical concepts lies in their mode of formation. According to Davydov (1988), the difference between everyday concepts and theoretical concepts also lies in the type of abstraction one engages in, namely empirical abstraction versus theoretical abstraction. The former involves superficial comparisons for discerning similarities and differences, whereas the latter involves structural comparisons. Thus, empirical generalization requires the abstraction of similarities from collections of entities, which may themselves represent disparate functions and structures. For instance, Davydov (1988) said that the notion of "roundness" can be empirically abstracted from a dish, a wheel, and so forth. However, this empirical notion of circularity does not reveal the real objective content, which is the locus of points at a constant distance from a fixed point. This content is not apparent from the mere appearance of roundness. Davydov (1988, 1990) claimed that cultivating empirical generalizations useful only for the formation of everyday concepts is inadequate for the formation of theoretical generalizations, which characterize mathematics. Aside from inferencing behavior in theoretical situations, mathematical thinking is also characterized by "recursive thinking" a term borrowed from information processing.

According to Vitale (1989), recursion is the mode in which human beings tackle and represent problems. Kieren and Pirie (1991) have claimed that recursion is an appropriate metaphor "in looking at the complex phenomenon of the whole of a person's mathematical knowledge and mathematical understanding" (p. 79). Kieren and Pirie substantiated this claim by arguing that children are self-referencing and in their realm of existence there are a number of "behavioral possibilities." Consequently, because children are self-referencing, one of the primary means of cognition is recursive in nature, and their knowledge "is formed through thought actions which entail the results of previous thought actions as inputs" (p. 79). This aspect of cognition is particularly relevant in mathematics because mathematical knowledge building and understanding is a dynamic process in which one's present knowledge and understanding builds from and is linked to previous knowledge. Kieren and Pirie analyzed recursion in students' thoughts and actions within a problem-solving experience. They posed the well-known "handshake problem": How many handshakes are needed in a class with 35 students so that each person in a room shakes hands with every person exactly once?

One of the solution strategies used by a group was to first specialize the problem involving 35 people to one involving the group. The students devised a strategy whereby the people were lined up, and the person furthest from the door shook hands with everybody else, reported the number of handshakes to the last person with whom the handshake occurred, and then left the room. The second person furthest from the door then repeated the above procedure, reported the number of handshakes to the last person, and left the room. After many iterations, the last person added up the number of handshakes, namely 34 + 33 + 32 + . . . + 1, and left the room. Note that the above solution is easily generalized to the case with *n* people. One of the findings of this study was that most of the students never bothered to compute a solution for 35 people, because they had come up with a strategy that would generally work.

> The fact that the problem was never answered at any level suggests that the students sense that the solution to the problem does not reduce to an answer or result of a special case, but "calls" or uses the structure of that special case. The "structure" of this special case has in it both a substantially correct mathematical idea (a sequence of non-repeating handshakes) and a form by which it can be procedurally described (Kieren & Pirie, 1991, pp. 83–84).

The researchers envisioned this recursive structure diagrammatically as a triangle of activities, which included specializing, creating results, and then generalizing through interpretation and validation. The process of specializing to particular cases, conjecturing, and then generalizing through interpretation and validation is a common trait among mathematically gifted students (Krutetskii, 1976; Sriraman, 2003, 2004d). This process also shows similarities to how professional mathematicians interpret and extend results in their field (Sriraman, 2004c).

Discussion, Implications, and Recommendations for the K–12 classroom

The preceding discussion reveals that although mathematically gifted students possess many of the cognitive qualities required for work at the professional level, some cognitive traits are more important than others. This hierarchy necessitates a use of problems that call for the use of high-level inferencing, generation/discovery of principles, and recursive thinking.

The discussion on mathematical creativity indicates that many of the characteristics of mathematical creativity described by mathematicians as invaluable aspects of their craft, such as the freedom to choose and pursue problems in an academic setting, the freedom of movement required during work, the awareness of the distinction between learning versus creating, the aesthetic appeal of mathematics, and the affective urgency/drive to solve problems with tremendous real world implications, might be extremely difficult to simulate in a traditional classroom setting. A model involving the use of five principles to maximize creativity among the mathematically gifted in the K–12 setting is shown in Figure 1. I have outlined five general principles extracted from the literature and studies on mathematical creativity that can be applied in the everyday classroom setting in order to maximize the potential for mathematical creativity to manifest in the K–12 classroom.

Five Overarching Principles to Maximize Creativity

As seen in Figure 1, the five overarching principles that emerged from a synthesis and analysis of the literature as significantly enhancing mathematical creativity are labeled as (a) the Gestalt principle, (b) the aesthetic principle, (c) the free market principle, (d) the scholarly principle, and (e) the uncertainty principle.

The Gestalt Principle. The eminent French mathematicians Hadamard (1945) and Poincaré (1948) viewed creativity as a process by which the mathematician makes choices between questions that lead to fruition as opposed to those that lead to nothing new. These mathematicians were influenced by the Gestalt psychology of their time and characterized mathematical creativity as a four-stage process consisting of preparation, incubation, illumination, and verification (Wallas, 1926). Although psychologists have criticized the Gestalt model of creativity because it attributes a large "unknown" part of creativity to unconscious drives during the incubation stage, numerous studies with scientists and mathematicians (i.e., Burton, 1999a, 1999b; Davis & Hersh, 1981; Shaw, 1994; Sriraman, 2004c) have consistently validated this model. All of these studies found that after one has worked on a problem for a considerable time (preparation) without making a breakthrough, the person puts the problem

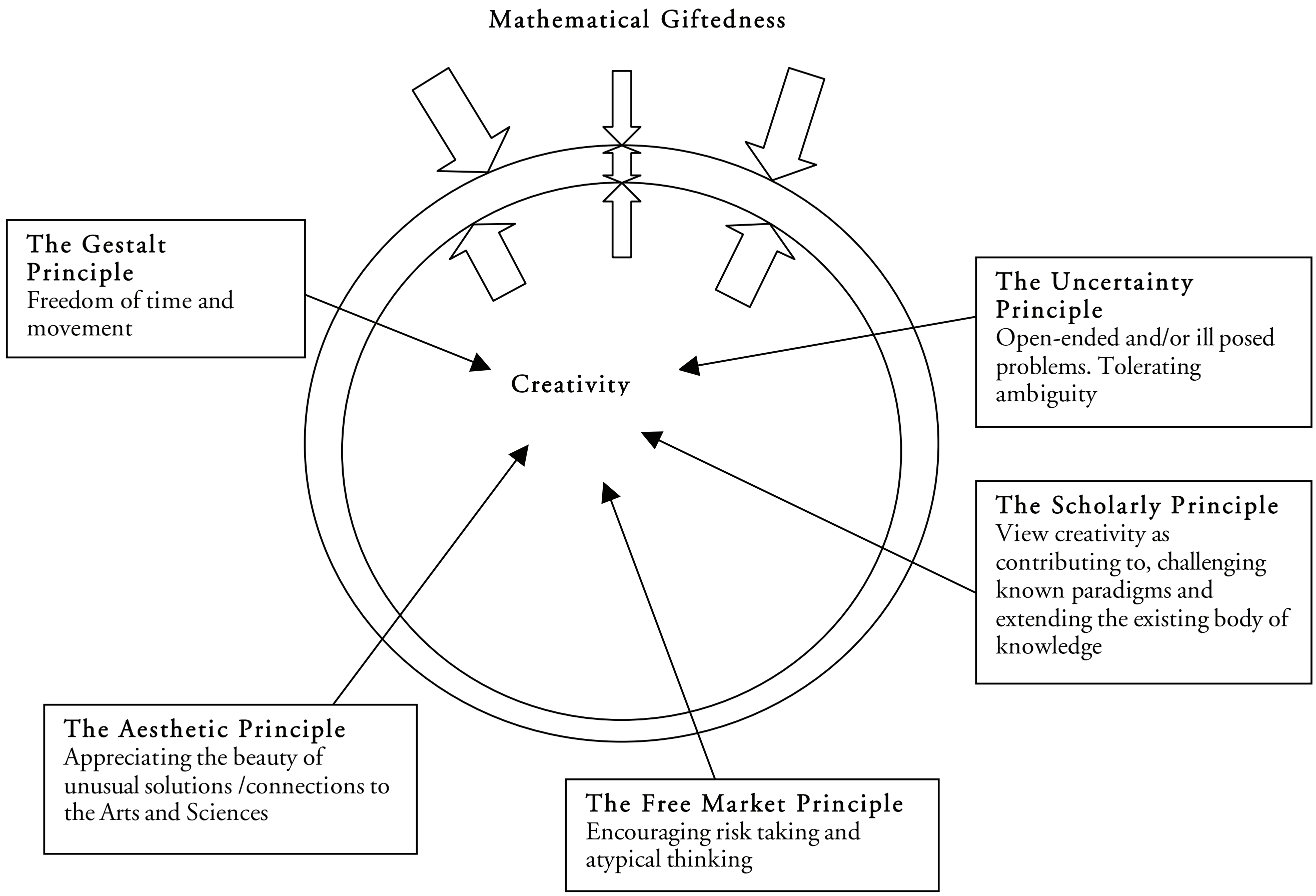

Figure 1. Harmonizing creativity and giftedness at the K–12 level

aside and other interests occupy the mind. This period of incubation eventually leads to an insight to the problem, to the "Eureka" or the "Aha!" moment of illumination. Most of us have experienced this magical moment. Yet, the value of this archaic Gestalt construct is ignored in the classroom. In fact, Krutetskii (1976) found that mathematically gifted children also experienced the sheer joy of creating that "included the feeling of satisfaction from the awareness of the difficulties that have been overcome, that one's own efforts have led to the goal" (p. 347). This implies that it is important for teachers to encourage the mathematically gifted to engage in suitably challenging problems over a protracted time period, thereby creating the opportunities for the discovery of an insight and to experience the euphoria of the "Aha!" moment.

The Aesthetic Principle. Mathematicians have often reported the aesthetic appeal of creating a "beautiful" theorem that ties together seemingly disparate ideas, combines ideas from different areas of mathematics, or utilizes an atypical proof technique (Birkhoff, 1956, 1969; Dreyfus & Eisenberg, 1986; Hardy, 1940). Wedderburn's theorem that a finite division ring is a field is one instance of a unification of apparently random ideas, because the proof involves algebra, complex analysis, and number theory. Cantor's argument about the uncountability of the set of real numbers is an often-quoted example of a brilliant and atypical mathematical proof technique (Nickerson, 2000). The eminent English mathematician G. H. Hardy (1940) compared the professional mathematician to an artist, because like an artist, a mathematician was a maker of patterns in the realm of abstract ideas. Hardy said,

> A mathematician, like a painter or a poet, is a maker of patterns. If his patterns are more permanent than theirs, it is because they are made with ideas. . . . The mathematician's patterns, like the painter's or the poet's, must be beautiful; the ideas, like the colors or the words, must fit together in a harmonious way. Beauty is the first test: there is no permanent place in the world for ugly mathematics (p. 13).

Recent studies in Australia (Barnes, 2000) and Germany (Brinkmann, 2004) with middle and high school students revealed that students were capable of appreciating the aesthetic beauty of a simple solution to a complex mathematical problem. Brinkmann found that even low achievers appreciated the struggle to get the insight that unlocked a seemingly unsolvable mathematical puzzle. Barnes noted that real-world problem selection and the careful "staging" of the discovery moment by the teacher were found to be the crucial elements for conveying an appreciation of mathematics to the classroom.

The Free Market Principle. Professional mathematicians in an academic setting take a huge risk when they announce a proof to a long-standing unsolved problem. Often times the mathematician puts his or her reputation at risk if a major flaw is discovered in the proof. For instance, in mathematical folklore, Louis de Branges' announcement of a proof to the Riemann hypothesis[4] fell through upon scrutiny by the experts. This led to subsequent disregard of his claim to a brilliant proof for the Bieberbach conjecture.[5] The Western mathematical community took notice of Louis de Brange's proof of the Bieberbach conjecture only after a prominent Soviet group of mathematicians supported his proof. On the other hand, Ramanujan's numerous intuitive claims, which lacked proof, were widely accepted by the community because of the backing of giants like G. H. Hardy and J. E. Littlewood. The implication of these anecdotes from professional mathematics for the classroom is that teachers should encourage students to take risks. In particular, they should encourage the gifted/creative students to pursue and present their solutions to contest or open problems at appropriate regional and state math student meetings, allowing them to gain experience at defending their ideas upon scrutiny from their peers.

The Scholarly Principle. K–12 teachers should embrace the idea of *creative deviance* as contributing to the body of mathematical knowledge, and they should be flexible and open to alternative student approaches to problems. In addition, they should nurture a classroom environment in which students are encouraged to debate and question the validity of both the teachers', as well as other students', approaches to problems. Gifted students should also be encouraged to generalize the problem and/or the solution, as well as pose a class of analogous problems in other contexts. Allowing students problem-posing opportunities and promoting the understanding of problem design helps them differentiate mathematical problems from nonmathematical problems, good problems from poor, and solvable from nonsolvable problems. In addition, independent thinking can be cultivated by offering students the opportunity to explore problem situations without any explicit instruction (English, in press; Sriraman & English, 2004). Teachers are also encouraged to engage in curriculum acceleration and compaction to lead mathematically gifted students into advanced concepts quickly and to promote independent scholarly activity. The longitudinal Study of Mathematically Precocious Youth (SMPY), started by Julian Stanley at Johns Hopkins University in 1971, generated a vast amount of empirical data gathered over the last 30 years, and has resulted in many findings about the types of curricular (e.g., acceleration, compacting) and affective interventions that foster the pursuit of advanced coursework in mathematics. More than 250 papers have been produced in its wake, and they provide excellent empirical support for the effectiveness of curriculum acceleration and compaction in mathematics (Benbow, Lubinski, & Sushy, 1996).

The Uncertainty Principle. Mathematics at the professional level is full of uncertainty and ambiguity, as indicated in some of the quotes presented earlier. Creating, as opposed to learning, requires that students be exposed to the uncertainty and the difficulty of creating mathematics. This ability requires the teacher to provide affective support to students who experience frustration over being unable to solve a difficult problem. Students should periodically be exposed to ideas from the history of mathematics and science that evolved over centuries and took the efforts of generations of mathematicians to finally solve. Cultivating this trait of perseverance will ultimately serve the mathematically gifted student in the professional realm. Keisswetter (1992) developed the so-called *Hamburg Model* in Germany, which is more focused on allowing gifted students to engage in problem-posing activities, followed by time for exploring viable and nonviable strategies to solve the posed problems. This approach captures an essence of the nature of professional mathematics, in which the most difficult task is to correctly formulate the problem (theorem). Conversely, some extant models within the U.S., such as those used in the Center for Talented Youth (CTY) at Johns Hopkins University, tend to focus instead on accelerating the learning of concepts and processes from the regular curriculum, thus preparing students for advanced coursework within mathematics (Barnett & Corazza, 1993).

Having presented five principles that can maximize mathematical creativity in the K–12 classroom, I present a model (see Figure 2) that captures the underlying essence of this paper and shows the relationship and compatibility of the constructs of mathematical creativity and giftedness between the K–12 and the professional realms of mathematics.

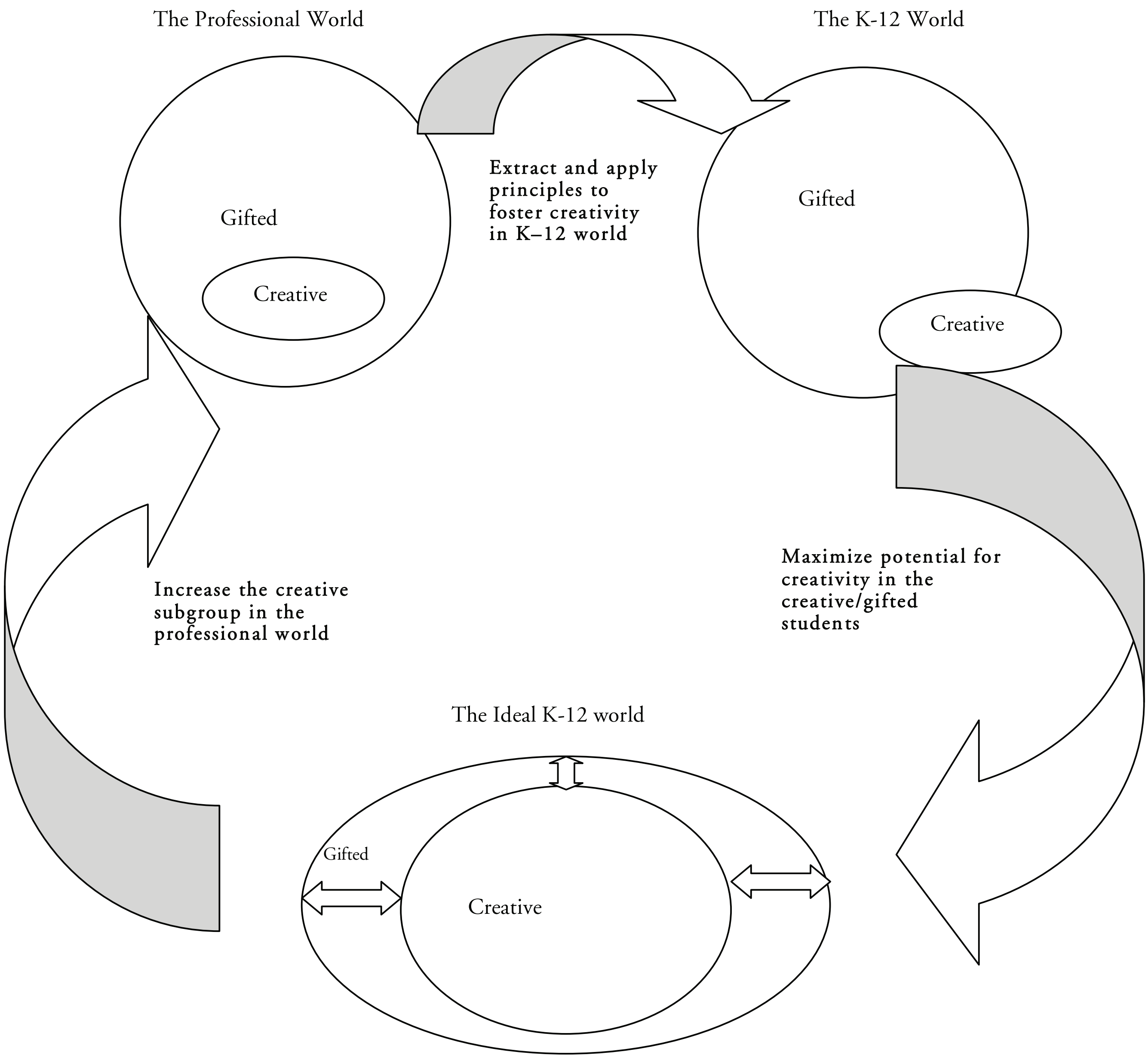

Figure 2. Maximizing the compatibility of creativity and giftedness

The Conceptual Model

The model presented in Figure 2 shows the dynamic nature of the relationship between mathematical creativity and mathematical giftedness and illustrates the possibilities for bridging the schism between the two. The K–12 world in the figure shows that mathematical creativity manifests in the "fringes" in the general pool of mathematically gifted students. On the other hand, the professional world of mathematics shows that creativity is a rare and sought after commodity. How do we bridge these two disjointed worlds? The model suggests that these realms of professional mathematics and the K–12 mathematics classrooms can be successfully bridged by paying increased attention to maximizing the creative potential of the mathematically gifted students in the "ideal" classroom. This can be accomplished by applying the five principles (see Figure 1) that work for creative mathematicians in the K–12 classroom. A classroom environment, especially classrooms with mathematically gifted students, in which the Gestalt, aesthetic, free market, scholarly, and uncertainty principles become a part of the classroom culture, maximizes the potential for creativity among the mathematically gifted. This increase in creativity serves these students as they progress into the world of postsecondary and research mathematics, or progress from Levels 4 to Levels 5 and 6. Progress into Level 6 increases the subgroup of creative mathematicians.

The conceptual model, although triadic in nature, is different from the general models proposed by Renzulli (1978, 1986) and Sternberg (1997) in that it shows the relationship between creativity and giftedness in the specific domain of mathematics, but it also has elements of Renzulli's (1978, 1986) three-ringed conception of giftedness, as well as Sternberg's triarchic view of giftedness. Renzulli's three-ringed conception suggests that giftedness is the interaction between above-average abilities, focused task-committed behavior, and creativity. The professional world of mathematics (Level 5) is characterized by above-average mathematical abilities among mathematicians and a commitment to research. However, mathematical creativity at this level remains an elusive commodity manifesting among a tiny subset of the general pool of mathematicians. In the ideal K–12 world, Renzulli's (1978) conception of task-commitment can be emphasized under the uncertainty principle, which suggests that difficult problems take extended time periods to solve and involve considerable struggle. Sternberg's triarchic view of giftedness suggests that gifted individuals possess a varying blend of analytic, synthetic (creative), and practical giftedness. This view particularly resonates in the world of mathematics.

Level 5 mathematicians that are productive in their areas of research have high levels of analytic and practical abilities. Practical abilities manifest in choosing problems that are accessible and publishable. However, creative mathematicians (Levels 6 and 7) have higher levels of synthetic abilities in comparison to the Level 5 mathematicians in that the papers they publish open up new research vistas for other mathematicians. An example of such creative work is the aforementioned paper by Hewitt (1948). These high levels of synthetic abilities are perhaps compromised by slightly lower levels of practical abilities. For example, mathematicians at Levels 6 and 7 often leave proofs half-finished or sometimes do not even bother to publish their work.

There are numerous examples in the history of mathematics that reveal such tendencies among highly creative mathematicians. For instance, Srinivasa Ramanujan (1887–1920), the Indian mathematician, had handwritten notebooks filled with numerous theorems without proof that still contribute to fertile directions in the growth of analytic number theory, elliptic functions, infinite series, and continued fractions. David Hilbert's (1900) list of 23 problems presented to mathematicians at the 1900 International Congress in Paris contributed both to the phenomenal growth of mathematics and the particular directions in which it has grown (Rowe & Gray, 2000). The Riemann hypothesis still remains an open problem with profound implications for numerous areas of mathematics. The most recent example is that of Paul Erdös, a contemporary and enigmatic mathematical genius, who was renowned for giving other mathematicians conjectures and/or problems with hints, partial solutions, or no solutions. The mathematicians that finished these problems and wrote up the results usually graciously listed Erdös as the coauthor of their papers. In fact, coauthors of Erdös gave themselves a number called "Erdös number 1," and mathematicians that coauthored a paper with "Erdös number 1" mathematicians gave themselves the number "Erdös number 2" and so on.

Elements of Sternberg's (1997) triarchic view of giftedness are also seen in the model in Figure 2. In order to maximize the potential for creativity to manifest in the mathematics classroom, teachers can encourage mathematically creative students to share their synthetic insights on connections between seemingly diverse problems with the other students in the class (Sriraman, 2004a). Historic examples of synthetic thinking in mathematics, which connect seemingly diverse ideas/concepts, can be used in the classroom to further illustrate the power and value of such insights. The scholarly, free-market, and aesthetic principles also contain aspects of Sternberg's triarchic view of giftedness.

Furthermore, the five principles encompass notions of polymathy, which can foster creativity in general by connecting notions from the arts and sciences to mathematics and vice versa. Common thinking traits of hundreds of polymaths (historical and contemporary) as analyzed by Root-Bernstein (1989, 1996, 2000, 2001, 2003) and many others are: (a) visual geometric thinking and/or thinking in terms of geometric principles; (b) frequent shifts in perspective; (c) thinking in analogies; (d) epistemological awareness, or an awareness of domain limitations; (e) interest in investigating paradoxes, which often reveals interplay between language, mathematics, and science; (f) belief in Occam's Razor, or belief that simple ideas are preferable to complicated ones; (g) acknowledgment of serendipity and the role of chance; and (h) the drive to influence the agenda of the times (Sriraman, 2005).

One recent example provided by Root-Bernstein (2003) is the effect of Escher's drawings on a young Roger Penrose, the mathematical physicist, who visited one of Escher's exhibitions in 1954. Stimulated by the seemingly impossible perspectives conveyed by Escher in two dimensions, Penrose began creating his own impossible objects such as the famous Penrose "impossible" tribar that shows a three-dimensional triangle that twists both forwards and backwards in two dimensions. Root-Bernstein wrote:

> Roger Penrose showed his tribar to his father L. S. Penrose, a biologist who dabbled in art . . . [who] invented the impossible staircase in which stairs appear to spiral both up and down simultaneously . . . [and] sent Escher a copy . . . [who] then developed artistic possibilities of the impossible staircase in ways that have since become famous. (p. 274)

Another well-known consequence of Escher's artistic influence on mathematicians is the investigation of tiling problems (both periodic and aperiodic) popularized by both Roger Penrose and Martin Gardner, which helped cystallographers understand the structure of many metal alloys that are aperiodic (Root-Bernstein, 2003).

The Changing Nature of Mathematics

Another important aspect of this discussion is the question of the balance between pure and applied mathematics. The literature suggests that the nature of mathematics relevant for today's world has also changed. In spite of the rich and antiquated roots of mathematics, Steen (2001) suggested that mathematicians should acknowledge the contributions of researchers in external disciplines like biology, physics, finance, information sciences, economics, education, medicine, and so on who have successfully used mathematics to create models with far-reaching and profound applications in today's world. These interdisciplinary and emergent applications have resulted in the field of mathematics thriving at the dawn of the 21st century. Yet, problem solving, as it is implemented in the classroom, does not contain this interdisciplinary approach and modeling of what is happening in the real world. In the U.S., the urgency of preparing today's students adequately for future-oriented fields is increasingly being emphasized at the university level. Steen (2005) writes that "as a science biology depends increasingly on data, algorithms and models; in virtually every respect it is becoming . . . more mathematical" (p. xi). Both the National Research Council (NRC) and the National Science Foundation (NSF) in the U.S. are increasingly funding universities that promote interdisciplinary doctoral programs between mathematics and the other sciences with the goal of producing scientists who are adept at "mathematizing" reality.

Mathematics at the secondary level exposes students to both breadth and depth of mathematical topics. However, most mathematics curricula are still anchored in the traditional treatment of mathematics, as opposed to an interdisciplinary- and modeling-based approach of mathematics used in the real world (Sriraman, in press). Sheffield, Bennett, Berriozabal, DeArmond, and Wertheimer (1995) lamented that not much had changed in terms of mathematics curricula at that point in time and remarked that gifted mathematics students were the ones who were most shortchanged and unable to utilize their talents, which could be viewed as a societal resource invaluable to maintaining leadership in a technologically changing world. Moreover, high school mathematics also serves as the gatekeeper for many areas of advanced study (Kerr, 1997). The traditional treatment of mathematics has little or no emphasis on modeling-based activities, which require teamwork and communication. Additionally, traditional mathematics has historically kept gifted girls from pursuing 4 years of high school mathematics. This deficit is difficult to remediate at the undergraduate level and results in the effect of low numbers of students capable of graduate-level work in interdisciplinary fields such as mathematical biology and bio-informatics (see Steen, 2005).

Any educator with a sense of history should foresee the snowball effect or the cycle of blaming inadequate preparation in high school on middle school and consequently elementary school mathematics, which suggests we work bottom up. That is, students should engage in the study and modeling of complex systems that occur in real-life situations from the very early grades on. Lesh, Kaput, and Hamilton (in press) reported that in projects such as Purdue University's Gender Equity in Engineering Project, when students' abilities and achievements were assessed using tasks that were designed to be simulations of real-life problem-solving situations, the understandings and abilities that emerged as being critical for success included many that are not emphasized in traditional textbooks or tests. Thus, the importance of a broader range of students and of deeper understandings and abilities naturally emerged as having extraordinary potential. Surprisingly enough, these students also came from populations, specifically female and minority, that are highly underrepresented in fields that emphasize mathematics, science, and technology, and they were underrepresented because their abilities had been previously unrecognized (Lesh & Sriraman, 2005a, 2005b; Sriraman, 2005). Thus, it may be more fruitful to engage students in model-eliciting activities, which expose them to complex real-life systems, as opposed to contrived problem solving. The mathematical conceptual systems arising from such investigations have great potential for being pursued by mathematically gifted students purely in terms of their implications, and because they create axiomatic structures through which theorems can be discovered that are analogous to what a pure mathematician does (Sriraman & Strzelecki, 2004).

Conclusion

In conclusion, the goal of the gifted education community is not simply to ensure that mathematically gifted students fulfill their potential by becoming productive pure and applied mathematicians, but also to ensure that the mathematically creative students among the mathematically gifted do not get overlooked. After all, these may indeed be the very students who have the potential to move the field forward through their atypical/unorthodox methods and insights. The butterfly effect of overlooking one of these potential Level 6 (creative mathematician) students in the classroom eventually affects the livelihood of a thousand potential Level 5 (productive mathematician) students. A case in point that illustrates the far-reaching ripples of such a butterfly effect is Hilbert's (1900) problems, which sustained both pure and applied mathematics, as well as Ramanujan's notebooks.

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End Notes

1 These scores place the students approximately in the 95–99 percentile band.

2 The Fields Medal was established by John Charles Fields (1863–1932) and is the equivalent of the Nobel Prize for the field of mathematics. These medals are

awarded every 4 years to mathematicians under 40 years of age, at the International Congress of Mathematics.

3 Behr and Khoury (1986) found that the inferencing behavior of younger school children were analogous to those found by Wason and Johnson-Laird (1972).

4 The Riemann hypothesis states that the zeros of Riemann's zeta function all have a real part of one half. Conjectured by Riemann in 1859, it has neither been proved nor disproved. This is currently the most outstanding unsolved problem in mathematics.

5 The Bieberbach Conjecture is easily understood by undergraduate students with some exposure to complex analysis because of the elementary nature of its statement. A univalent function f transforms a point in the unit disk into the point represented by the complex number f(z) given by an infinite series $f(z) = z + a_2z^2 + a_3z^3 + a_4z^4 + \ldots$ where the coefficients $a_2, a_3, a_4, \ldots$ are fixed complex numbers, which specify f. In 1916, Bieberbach conjectured that no matter which such f we consider, $|a_n| \leq n$. Louis de Branges proved this in 1985.