Mathematical fiction for senior students and undergraduates: Novels, plays, and film

Janice Padula
<janicepadula@yahoo.com>

Introduction

Mathematical fiction has probably existed since ideas have been written down and certainly as early as 414 BC (Kasman, 2000). Mathematical fiction is a recently rediscovered and growing literature, as sales of the novels: *The Curious Incident of the Dog in the Night-time* (Haddon, 2003) and *The Da Vinci Code* (Brown, 2004) attest. Science fiction has been called the literature of ideas. These days fiction, not just science fiction, is likely to contain mathematical ideas.

This article describes a novel and two plays with mathematical themes and then investigates the mathematics in the film, *A Beautiful Mind* (Howard & Grazer, 2001). The topics discussed include: modular arithmetic, cryptography, vector calculus, probability, financial mathematics, and game, chaos, and number theory, including Germain primes.

Traditionally, primary teachers (Padula, 2004) have used children's literature to encourage mathematical thinking by providing an engaging and often humorous context for the introduction of mathematical-scientific ideas. It is generally acknowledged, for example, in a National Council of Teachers of Mathematics (NCTM) curriculum yearbook (Balas, 1997), that (children's) literature can help students meaningfully connect their world to the world of mathematical ideas.

For senior students, research into the effects of literature on learning mathematics is not extensive, although Bickmore-Brand (1993) has made a contribution to "the effects of reading and writing on mathematics-learning" debate. Bickmore-Brand (1993) identified seven language-learning strategies that can be applied to enhance the learning of mathematics, one of which is creating a meaningful and relevant context for the knowledge, skills and values of mathematics. Smith (2003) successfully uses science-fiction stories to teach mathematics to students in Years 9–12 and declares, after teaching many years, that he has come to the conclusion that you cannot teach mathematics without (teaching) communications. Padula (2005) attempts to classify the different kinds of literature suitable for teaching mathematics to secondary-school students, and lists some examples of each class, including classic mathematical-fiction novels such as *Flatland* (Abbott, 1884, 1932) and one of its many sequels, *Flatterland* (Stewart, 2001).

Sriraman (2003) writes that mathematical fiction successfully "marries" literature and mathematics creating the ideal forum to nurture critical thinking and introduce sophisticated mathematical ideas. Sriraman, (2003, 2004) has shown how ninth graders as young as 13 or 14 years of age can engage with *Flatland* (Abbott, 1884, 1932) and the first five chapters of its modern update *Flatterland* (Stewart, 2001) to: develop problem-solving and critical-thinking skills; better understand different geometries, such as fractal geometry and different dimensions; develop the skill to make unbiased valid inferences; and, understand and discuss the philosophy of mathematics — albeit at an elementary level.

If, as Sriraman (2004) continues, Stewart's, *Flatterland* (2001) in its entirety poses a challenge even to university mathematics and physics majors, it does so within a context that introduces complicated mathematical ideas (such as dimensions of space and non-Euclidean geometries) and takes students on an adventurous journey with the "Space Hopper" and his "Virtual Unreality Engine", lashing them with his ready wit[1].

A small selection of mathematical fiction follows in the succeeding paragraphs.

Mathematical fiction: A novel for the mature teenager, or young adult

Cryptonomicon (Stephenson, 1999) is a family saga describing the way the flow of information affects history through cryptology, both encoding and decoding. The World War II "Enigma", "Ultra", and other wartime codes are described and the book relates fictionalised attempts by American and English servicemen to prevent the Germans and Japanese from discovering their codes have been cracked.

Security of information is an essential concern in the younger generation's attempt to establish a world bank with a safe IT code. Cryptonomicon has mathematician Alan Turing and Americans, Lawrence Waterhouse and his grandson, Randy, chasing, decoding and devising codes in countries from Australia, England, Finland and America, to a fictional South-East-Asian sultanate, "Kinakuta" (Brunei). There are amusing diagrams and graphs, and mathematical references throughout the book.

This novel may seem overly long (918 pages), and full of digressions to non-technologically-minded readers, but the mathematical digressions are fun. It includes a good introduction to modular arithmetic and its role in cryptography in an early chapter (Stephenson, pp. 68–71), and there are references to Riemann's zeta function and Gödel's theorem. It is written in an ironic style that is similar to Heller's *Catch 22* (1994) and is sure to be popular

1. Abbott & Stewart's [2002] annotated book on Flatland provides teachers with further insights into Abbott's satire and is a valuable resource for teachers of senior students.

with mature, male, mathematics and IT students, and "techies" of both genders. It does, however, contain some glaring mathematical errors according to Kasman (2000). Stephenson makes several references to factoring large primes and writes about Alan Turing creating information theory, and that a decade too soon.

Teachers could use just the Appendix and other mathematical sections of the book, e.g., the description of how to share out the mathematical family's matriarchal household goods in a fair way (Stephenson, pp. 625–628). As "Uncle Red" describes it:

> The question reduces to a mathematical one: how do you divide up an inhomogeneous set of *n* objects among *m* people… i.e., how do you partition the set into *m* subsets (S_1, S_2, …, S_m) such that the value of each subset is as close as possible to being equal? (p. 625)

The question is followed by an answer. Using vectors to form a value matrix the answer is:

$$\sum_{i=1}^{n} V_i = \tau$$

where τ is a constant, according to Stephenson.

Where emotional attachment to a particular piece of furniture is considered along with financial value the mathematics becomes:

$$\sum_{i=1}^{n} V_i^{e} = \tau_e \quad \text{and} \quad \sum_{i=1}^{n} V_i^{\$} = \tau_{\$}$$

where τ_e is emotional and $\tau_{\$}$ is financial worth to family members.

In the Appendix, Bruce Schneier, (Stephenson, 1999) author of a book on cryptography, describes a cryptosystem called Solitaire, the Pontifex code in the book. Students can try this system with a pack of cards. A 54-card deck is used as a 54-element permutation (counting the jokers in the pack). According to Schneier there are 54! or approximately 2.31×10^{71} possible different orderings of a pack. There are 52 cards in a deck (without the jokers) and 26 letters in the alphabet (or 403 291 461 126 605 584 000 000 combinatorial possibilities). What could possibly be better for devising codes, Schneier asks?

Plays

As Devaney (2003), of the Department of Mathematics at Boston University, remarks: Tom Stoppard's play *Arcadia* (1993) has a mathematical theme and offers teachers of both mathematics and the humanities an opportunity to join forces in "a unique and rewarding way". Fast-paced and witty, the play relates the story of a sixteen-year-old English lady, Thomasina, living in Byron's time, who discovers the mathematics for the growth of animals on her estate. Her workbook is discovered generations later by one of her descendants, a young mathematician, Valentine.

The mathematical ideas involved form one of the main sub themes of the play. Chaos theory and fractals form an integral part of the plot and Fermat's last theorem and the Second Law of Thermodynamics play important roles. There are graphs illustrating an algorithm used by population biologists: $F_k(x) = kx(1 - x)$ at the *Chaos, Fractals, and Arcadia* (Devaney, 2003) site at http://math.bu.edu/DYSYS/arcadia/sect6.html.

> Here x represents the population in question normalised so that x lies between 0 and 1. The constant k is a parameter; k is for grouse, and so on. Given an initial population x_0 and a particular value of k, you can then iterate F_k to find the populations in successive years. For example, if $k = 1.5$ and $x_0 = 0.2$, then:
>
> $$\begin{aligned} x_0 &= 0.2 \\ x_1 &= 0.24 \\ x_2 &= 0.2736 \\ x_3 &= 0.2981... \\ x_4 &= 0.3138... \\ &\ldots \\ x_{20} &= 0.3333\ldots \\ x_{21} &= 0.3333\ldots \end{aligned}$$
>
> (Devaney, 2003, p. 1)

> If you select $k = 3.2$ and $x_0 = 0.2$, then after several iterations, you find that the population begins to cycle back and forth. One year the population is high, the next, low, thus:
>
> $$\begin{aligned} x_0 &= 0.2 \\ &\ldots \\ x_{20} &= 0.7994 \\ x_{21} &= 0.5130 \\ x_{22} &= 0.7994 \\ x_{23} &= 0.5130 \end{aligned}$$
>
> (Devaney, 2003, p. 1)

There is also an interesting appendix by Devaney detailing why Thomasina's algorithm works. Instructions on how to play the Chaos Game are included at the site; the game results in the Sierpinski triangle.

David Auburn's (2001) play *Proof*, the winner of the 2001 Pulitzer Prize for drama and a "Tony" on Broadway, was staged in Melbourne in 2003, starring Rachel Griffiths. It also has a mathematical theme. The audience discovers that mathematics is not simply about figures but about pure research and the future; it is something that goes far beyond just symbols on a page. Within the play the audience is made to realise that pure mathematics is something that a mathematician will continue living with, just as a composer does with music (Rickard, 2003). As Dave Bayer (2000), mathematics professor and the play's reviewer, states: "This play is ultimately a love letter to mathematics, and one can only respond to its generosity in kind" (Bayer, 2000, p. 1084).

In Scene 2, Catherine, who has cared for her mentally ill mathematician father until his death, quotes from a letter Gauss wrote to Sophie Germain. Germain was a French mathematician born in the eighteenth century who sent the mathematician, Gauss, some proofs involving a certain class of prime numbers (Germain primes) using a man's name. When Gauss discovered she was a woman he wrote:

> A taste for the mysteries of numbers is excessively rare, but when a person of the sex which according to our customs and prejudices, must encounter infinitely more difficulties than men to familiarize herself with these thorny researches, succeeds nevertheless in penetrating the most obscure parts of them, then without a doubt she must have the noblest courage, quite extraordinary talents, and superior genius. (Auburn, 2001, p. 36)

The first few Germain primes are: 2, 3, 5, 11, 23, 29, 41, 53, 83, 89, 113, 131… (double a prime and add one, and you get another prime). A somewhat larger $92\,305 \times 2^{16\,998}$ is Catherine's second example in the play. Gauss's warm endorsement of females in mathematics is beautifully expressed and heartening.

Screenplays and films

Films can be either a starting point for teaching mathematics, or a reward for work completed. In recent years movies such as: *A Beautiful Mind* (Howard & Grazer, 2001), π, or, *Pi — The Movie* (Watson, 1997), *Good Will Hunting* (Damon & Affleck, 1998), *Stand and Deliver* (Law, Evans & Gideon, 1987), *Enigma* (Jagger & Michaels, 1995), and *The Bank* (Maynard, 2002) have all broached the subject of mathematics. Screenplays of films such as these are available from the libraries of technical and university colleges teaching media courses. In collaboration with mathematics teachers, English or Drama teachers may use selected scenes from these screenplays in their classes.

There are two scenes in *A Beautiful Mind* (Howard & Grazer, 2001), essentially a love story, where the mathematician John Forbes Nash Jr and his friends are in a bar and a beautiful blonde walks in. The screenwriter, Akiva Goldsman, uses these scenes to describe Nash's Nobel-Prize-winning contribution to mathematics and economics: Game Theory. (Game Theory is a branch of mathematics that uses models to study interactions with formalised incentive structures, or games. It has applications in many fields including: economics, evolutionary biology, political science and military strategy.) It is explained in the later scene as John tells his friends the best strategy for winning the attentions of the second (very attractive) girl. There is a simple, longer explanation of Game Theory for students, and more about Nash and *A Beautiful Mind*, at www.simonsingh.net/A_Beautiful_Mind.html. Producers Ron Howard and Brian Grazer employed (Professor) Dave Bayer on the set as a hands double for Russell Crowe; it is actually his hands you see writing equations and diagrams on the window glass of Nash's room at Princeton.

Although some of Kasman's (2000) contributors have criticised the film for leaving out important parts of Nash's life, Lynne Butler, a mathematics professor, thinks *A Beautiful Mind* is as concise and unexpected as an elegant proof. She and her husband share a home with her brother, who lives with schizophrenia, and she finds the film compelling, truthful and important. She includes "The Noncooperative Game" in *A Beautiful Mind* at www.haverford.edu/math/lbutler/review.html as part of her review of the film (Butler, 2002).

> The movie suggests that a motivating example for the discovery of Nash equilibria might have been the strategies of five suitors most attracted to the same woman in a group of five. As suggested by the movie's visuals, positive outcomes occur only when each woman is approached by one suitor. In the two-person version of this game, each of two suitors, say John and Martin, decides with what probability, say x and y respectively, he will approach the more attractive of two women. The expected payoff to John is $xa(1-y)+(1-x)by$, where $a > b > 0$ since John prefers the more attractive woman. Likewise the expected payoff to Martin is $(1-x)cy+xd(1-y)$, where $c > d > 0$. Two Nash equilibria for this game, as suggested in the movie, occur when $x = 1$ and $y = 0$ (with payoffs a and d) or when $x = 0$ and $y = 1$ (with payoffs b and c). The only other Nash equilibrium is when $x = c/(c+d)$ and $y = a/(a+b)$ (with payoffs $ba(a+b) < b$ and $dc/(c+d) < d$). At a Nash equilibrium, neither player can improve his expected payoff by unilaterally changing his strategy. (p. 456)

Singh (2002) describes the Nash equilibrium as a situation in which both players have a perfect strategy that results in stability. Players maintain this strategy because anything else will only worsen their own position. Most young men (and women, for quite a different reason) would find the strategy, as described by Butler above, a great incentive for studying probability and game theory. Singh also relates how the British Government made £23 billion in an auction of third-generation mobile-phone licences by using the same strategy.

The vector-calculus problem John writes on the blackboard when he first meets his future wife, Alicia, is a real one, in de Rham cohomology. (De Rham cohomology is a tool belonging to algebraic and differential topology.) The answer is one, according to Kasman (2000). The problem was chosen because the film's mathematics adviser, Dave Bayer, the reviewer of *Proof* and an algebraic geometer, thought that it was the kind of problem Nash himself would have chosen, in that it is exceptional and interesting. As Mackenzie (2002) describes, it is really a complicated version of a physics problem:

> It is a more complicated version of a classical physics problem: determining whether a static electric field (the F in lines one and two) necessarily has a potential function (indicated by g). If the "electric field" is allowed to be infinite or simply nonexistent at certain points (collectively indicated by X), the question becomes physically unrealistic but mathematically very rich. The answer depends not only on the geometry of the set X, but also on one's assumptions about the field F, as the fictional Nash explains to Alicia rather brusquely when she offers her stab at a solution. (pp. 789–791)

Actor Russell Crowe's comment to Jennifer Connelly (Alicia) when she hands in her attempt at solving the problem is: "I never said (that) the vector fields were rational functions". Do you know a curious physics teacher or lecturer who would enjoy the challenge of describing in equations the problem in de Rham cohomology?[2]

However, if you are looking for simpler examples of vector calculus there are notes of two lectures by Subrata Mukherejee (1995), lecturer in applied mechanics at Cornell University, at:
http://omega.albany.edu:8008/calc3/vectors-dir/cornell-lecture.html and http://omega.albany.edu:8008/calc3/3d-geom-dir/cornell-lecture.html: "An introduction to vectors", and "The geometry of vectors". The latter concludes with two exercises that can easily be executed by students using *Mathematica* or similar software (Mukherejee, 1995, January 25, p. 3). They are:

Identify the following sets of points (in 3D)

1. $x^2 + y^2 + z^2 = 1$

   $z = \frac{1}{2}$

The first equation is that of a sphere, while the second is a horizontal plane. The plane cuts the sphere parallel to the *xy* plane creating an intersection set of a circle (Mukherjee, 1995, January 25; see Figure 1 below).

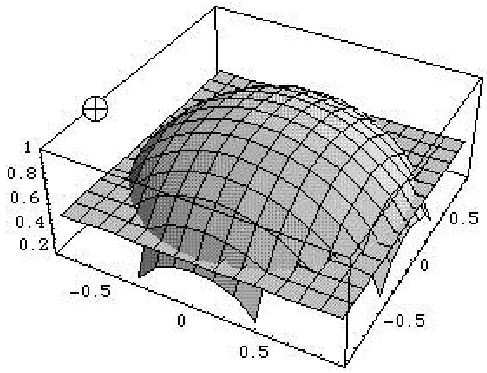

Figure 1. A plane cuts the sphere parallel to the xy plane creating an intersection set of a circle, generated by Mathematica, from Mukherjee (1995, January 25, p. 3).

2. $x^2 + y^2 = \frac{3}{4}$

This equation is the mathematical solution of the first set of equations, and it can be reduced to $x + y = 0.866$. But it behaves differently. Because z is not specified, it can be anything, and the result is the cylinder shown below, which has its axis on the z-axis and a radius of 0.866, the square root of $\frac{3}{4}$ (Mukherjee, 1995, January 25).

2. The equations (as far as I can make them out from a video and a picture of Bayer in front of a blackboard from Mackenzie's paper) are:

$$V = \{F : R^3 \setminus X \to R^3 \text{ so } \nabla \times F = 0\}$$

$$W = \{F = \nabla_g\}$$

$$d : m\left(\frac{V}{W}\right) = \delta$$

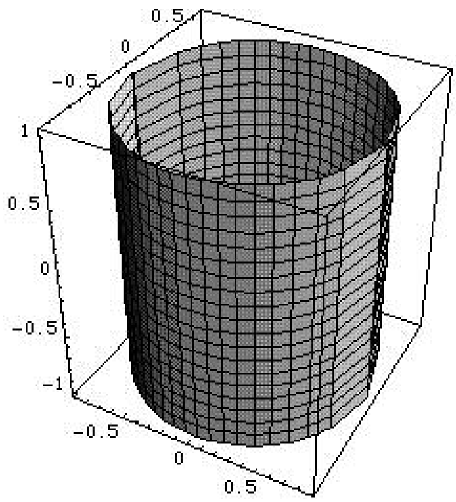

Figure 2. A cylinder with its axis on the z-axis and a radius of 0.866 generated by Mathematica, from Mukherjee (1995, January 25, p. 3).

Conclusion

There are many kinds of mathematical fiction and several ways it can be used. It can be given to students before the mathematics is taught, then as you teach it you can refer back to the story, play or film; you can give out the story after the mathematics has been taught as a means of reviewing the concepts; a third way would be to give the story to the students to research the mathematics and answer questions independently; and, finally, you may tell students to write stories that involve the mathematics being taught (Smith, 2003, Mower, 1999).

Some of today's authors are intrigued by mathematics. Intentionally (or unintentionally) they introduce and discuss mathematical ideas in their short stories, novels, plays and screenplays. Some of this literature is suitable for the introduction of mathematical topics into the classroom or lecture hall. Indeed, used properly, these works can: motivate students; introduce mathematical ideas in an informative context; elaborate on topics; supply imaginative applications; and help clarify mathematics, with or without the collaboration of science and humanities teachers.

These works are useful propaganda for mathematics, encouraging an appreciation of it. They are capable of captivating an audience, your audience, a group of students, with some challenging mathematics.

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