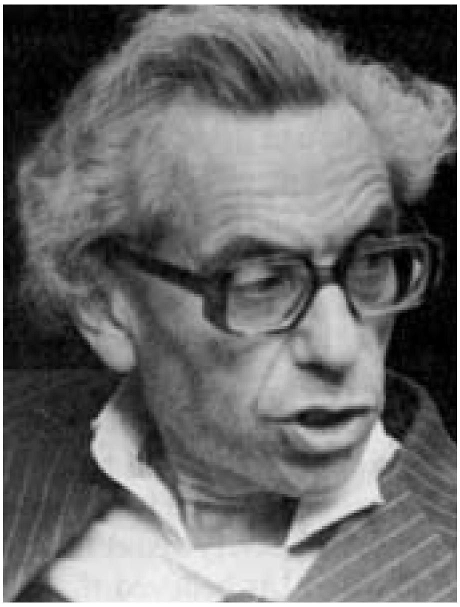


Paul Erdös

Helen Monroe & Paul Scott

Erdös the man

Erdös focused on problem-solving, particularly in the areas of number theory, combinatorics and graph theory. During his life he had no property, no family and no fixed address. He buttered his first piece of bread at age 21. He never cooked, nor ever drove a car. Another mathematician, Ron Graham, took care of his daily financial accounting.

He spent nineteen hours a day on mathematics, which explains why he needed the coffee. In 1951, he received the American Mathematical Society's Cole Prize. In 1984, he received $50 000 for winning the Wolf Prize. He gave all but $750 of this away to set up scholarships. Often, he presented mathematical problems to students and attached monetary awards for their solution. This reward could be anything from $5 to $10 000, depending on the difficulty of the problem. Living out of a suitcase, he went from country to country and continent to continent considering problems.

'A mathematician is a machine for turning coffee into theorems.'

Erdös and 'the Book'

Erdös liked to imagine that God had a book in which he wrote down all the most elegant and beautiful mathematical proofs. 'That's one for the Book,' was his greatest praise. *The Book* supposedly contains brilliant ideas, clever connections and wonderful observations that bring new insight and surprising perspectives on basic and challenging problems from number theory, geometry, analysis, combinatorics, and graph theory.

Erdös' approach to mathematics was as unique as his life. He invented a new kind of art: the art of raising problems. He said that mathematics is eternal because it has an infinity of problems; and in his view, the more elementary a problem is, the better.

Erdös was the consummate problem solver; his hallmark was the succinct and clever argument, often leading to a solution from 'the Book'. He loved areas of mathematics which did not require an excessive amount of technical knowledge but give scope for ingenuity and surprise.

Erdös and problems

Problems have always been an essential part of my mathematical life. A well chosen problem can isolate an essential difficulty in a particular area, serving as a benchmark against which progress in this area can be measured. An innocent looking problem often gives no hint as to its true nature. It might be like a 'marshmallow', serving as a

tasty tidbit supplying a few moments of fleeting enjoyment. Or it might be like an 'acorn', requiring deep and subtle new insights from which a mighty oak can develop.

Examples of Erdös problems

Here is a purely computational problem (this problem cannot be attacked by any other means at present). Call a prime p *good* if every even number $2r \leq p - 3$ can be written in the form $q_1 - q_2$ where $q_1 \leq p$, $q_2 \leq p$ are primes. Are there infinitely many good primes? The first bad prime is 97, I think. Selfridge and Blecksmith have tables of good primes up to 1037 at least, and they are surprisingly numerous.

I proved long ago that every $m < n!$ is the distinct sum of $n - 1$ or fewer divisors of $n!$. Let $h(m)$ be the smallest integer, if it exists, for which every integer less than m is the distinct sum of $h(m)$ or fewer divisors of m. Srinivasan called the number for which $h(m)$ exists practical. It is well known and easy to see that almost all numbers m are not practical. I conjured that there is a constant $c \leq 1$ for which for infinitely many m we have $h(m) < (\text{loglog } m)^c$. M. Vose proved that $h(n!) < cn^{1/2}$. Perhaps $h(n!) < c\,(\log n)^c$. I would be very glad to see a proof that $h(n!) < n^{\varepsilon}$.

A practical number m is called a *champion* if for every $m > n$, we have $h(m) > h(n)$. For instance, 6 and 24 are champions, as $h(6) = 2$, the next practical number is 24, $h(24) = 3$, and for every $m > 24$, we have $h(m) > 3$. It would be of some interest to prove some results about champions. A table of the champions < 106 would be of some interest. I conjecture that $n!$ is not a champion for $n >$ (some number) n_0.

'His peculiarities are so numerous it is impossible to describe them all.'

Stanislaw Ulam, 1976

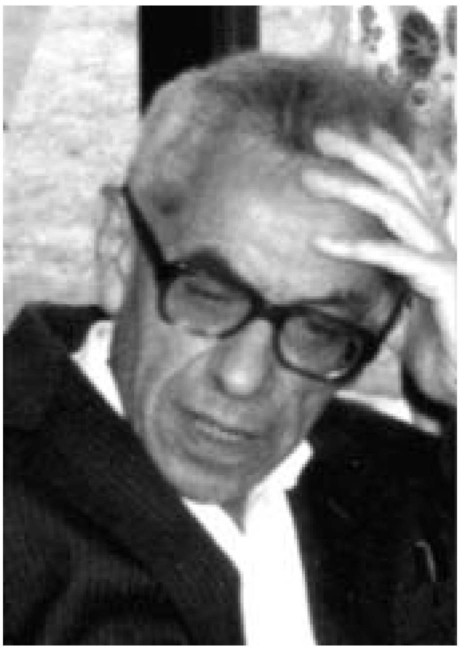
Erdös the mathematician

Although somewhat over the top, the following quote from Paul Hoffman in *The Man Who Loved Only Numbers*, shows the high regard in which Erdös was held by fellow mathematicians:

> Never, mathematicians say, has there been an individual like Paul Erdös. He was one of the century's greatest mathematicians, who posed and solved thorny problems in number theory and other areas and founded the field of discrete mathematics, which is the foundation of computer science. He also was one of the most prolific mathematicians in history, with more than 1500 papers to his name. And, his friends say, he was also one of the most unusual.

Erdös numbers

Erdös published papers with more than two hundred and fifty different co-authors; because of this prolific number a new term was coined: a person gets an *Erdös number* of 1 by having published a joint paper with Erdös, an Erdös number of 2 by publishing a joint paper with someone with an Erdos number of 1, and so on. Someone who published multiple papers with Erdös gains a fractional Erdös number; 2 joint papers earns you an Erdös number of 1/2, 8 joint papers earns you an Erdös number of 1/8, and so on.



In 2001, for the first time since 1935, no new co-authors were added to the list of mathematicians with an Erdös number of 1 (or less). The count stands at 507.

However, the number of mathematicians with an Erdös number of 2 increased by 230, to 6127. Erdös numbers range to 15, but the average is less than 5. The median number of papers is 2 and the mean is approximately 7.

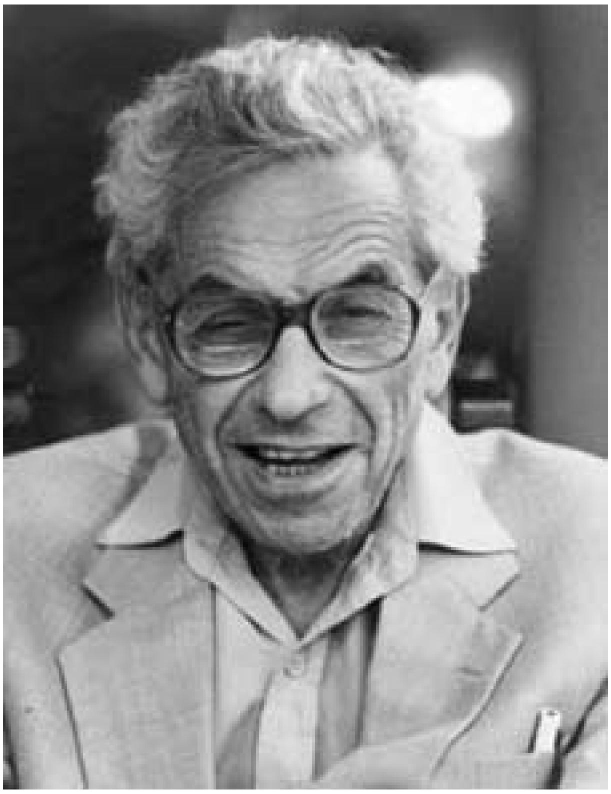
Erdös' philosophy

> You know, all those rules that may be perfectly correct for normal people, make no sense for prodigies. To say that Bach should pay any attention to how he was socially adjusted is just a bad joke. It is obvious that this is secondary.
>
> I never wanted material possessions. There is an old Greek saying that the wise man has nothing he cannot carry in his hands. If you have something beautiful, you have to look out for it, so I would rather give it away. I always say 'Private property is a nuisance'.

Erdösisms

Erdös created many of his own words to describe common nouns and verbs. Some examples are given in the box above.

Erdös-ese	English
My brain is open	I'm ready to do maths.
Boss	Woman
Slave	Man
Fascist	Anyone or anything annoying
Supreme Fascist	God
Preaching	Giving a mathematics lecture
Captured	Married
Liberated	Divorced
Recaptured	Remarried
To leave	To die
Trivial being	A non-mathematician



'Another roof, another proof'

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