IDENTIFICATION AND REMEDIATION OF SYSTEMATIC ERROR PATTERNS IN SUBTRACTION

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Abstract. **The present study investigated 90 elementary teachers' ability to identify two systematic error patterns in subtraction and then prescribe an instructional focus. Presented with two sets of 20 completed subtraction problems comprised of basic facts, computation, and word problems representative of two students' math performance, participants were asked to examine each incorrect subtraction problem and describe the errors. Participants were subsequently asked which type of error they would address first during math instruction to correct students' misconceptions. An analysis of the data indicated teachers were able to describe specific error patterns. However, they did not base their instructional focus on the error patterns identified, and more than half of the teachers chose to address basic subtraction facts first during instruction regardless of error type. Limitations of the study and implications for practice are discussed.**

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According to the Goals 2000: Educate America Act (PL 103-227), a high level of mathematics achievement for all students is a national priority. According to the National Research Council (2002), all students can and should achieve proficiency in mathematics. Additionally, mathematical skills are fundamental for individuals seeking occupational and educational advancement. Without proficiency in mathematics, students will likely experience difficulty completing other more advanced branches of mathematics (e.g., algebra) and be unprepared for many occupations. Mathematics education should enable students to understand and apply mathematical concepts. With this emphasis on conceptual understanding and higher-order problem-solving skills, teachers must not ignore computation.

Knowledge of basic computation skills cannot be separated from the overall conceptual understanding and forms the foundation for mathematical thinking (Wu, 1999). The National Council of Teachers of Mathematics (NCTM; 2000) emphasizes computation over overall performance in mathematics. According to the NCTM (2000), it is critical for students to know the basic number facts for addition, subtraction, multiplication, and division. Students' fluency and accuracy in methods of computation are equally important. The National Research Council (2002) further articulates the importance of computation by listing computation as the second of five main strands in mathematics. Yet, many students do not learn the basic mathematics skills required for success.

Even more troubling is the mathematics performance of students with learning disabilities (LD). Students with LD experience difficulties learning math, with problems surfacing early and continuing throughout their education (Bottge, 1999; Mercer & Miller, 1992). Deficiencies in mathematics performance are not limited to basic skills. Higher-order thinking skills such as problem solving are also a major challenge for these students (Jitendra, DiPipi, & Perron-Jones, 2002). The average mathematical performance of 16- and 17-year-old students with LD is approximately at the fifth-grade level (Cawley & Miller, 1989). Furthermore, students with LD have documented deficits in the areas of (a) basic facts, (b) subtraction, (c) solving word problems, (d) acquiring concepts, and (e) problem solving (e.g., Garnett, 1992; Miller, Stawser, & Mercer, 1996; Montague & Brooks, 1993).

Many students who are not proficient in the basic mathematics skills demonstrate numerous mathematics misconceptions (Marchand-Martella, Slocum, & Martella, 2004). For example, subtracting the smaller number from the larger number regardless of position is a common misconception not just among low-performing students or students with disabilities (Resnick & Omanson, 1987). When students make errors and formulate mathematical misconceptions, teachers should recognize the errors, prescribe an appropriate instructional focus, and implement an effective and efficient reteaching plan. The first step in this process, recognizing the errors, is completed through a systematic examination of students' mathematics work (Ashlock, 2002).

Error Analysis

Educators typically analyze students' mathematical errors with the intent to improve instruction and correct misconceptions (Mastropieri & Scruggs, 2002). Evaluating students' work to determine an appropriate instructional focus to correct errors is one of the main tenets of remedial or corrective education for all students, but especially for students with LD and low-performing students (Fuchs, Fuchs, & Hamlett, 1994; Salvia & Hughes, 1990; Salvia & Ysseldyke, 2004). Identification and analysis of students' arithmetic errors has the potential to improve instructional planning and, ultimately, student performance.

Although educators and researchers debate the numerous types of errors and their causes, as well as instructional approaches and procedures to correct errors, extensive research, including computer analysis of students' work (Woodward & Howard, 1994), indicates that large majorities of students' errors are consistent and systematic (e.g., Brueckner, 1935; Clements, 1982; Cox, 1975a, 1975b; Newman, 1977; Roberts, 1968).

Subtraction is particularly problematic for many students. Several researchers report that students experience difficulty with problems requiring borrowing (e.g., Cox, 1975a; Drucker, McBride, & Wilbur, 1987; Resnick, 1982). Specifically, many students exhibit an error type called smaller-from-larger (SFL) (National Research Council, 2002; Resnick, 1982). When making this type of error, students subtract the smaller number from the larger number regardless of position (e.g., 326 – 117 = 211, with the SFL error 6 – 7 = 1). Another error documented in students' work involves borrowing across a zero digit (BAZ). The BAZ error occurs when a student attempts to borrow from a zero and does not continue to borrow from the column to the left of the zero (e.g., 602 – 437 = 265, with the student not continuing to borrow from the hundreds column). This type of error occurs less frequently than SFL errors (Resnick, 1982). Both the SFL and BAZ error patterns are classified as incorrect or defective algorithms (e.g., Ashlock, 2002; Resnick, 1982; Roberts, 1986).

In general, an examination of a student's completed subtraction work is important because once a student's errors are pinpointed, a teacher can gear remedial or corrective instruction directly for the specific error patterns. Although identification of errors in mathematics is an important first step for remedial or corrective instruction, there is little evidence to suggest that teachers are able to perform systematic error analysis of students' work.

The purpose of this study was to (a) determine whether teachers were able to identify specific error patterns exhibited in subtraction; (b) establish whether teachers were better able to describe a more commonly occurring subtraction error (i.e., smaller-from-larger or SFL) than a less commonly occurring subtraction error (i.e., borrow-across-zero or BAZ); (c) determine whether teachers were able to prescribe an appropriate instructional focus; and (d) examine the instructional focus that teachers selected to address first.

METHODS

Design

Data were analyzed using a 2 x 2 x 2 Latin square design with repeated measures. The design was partially replicated because of the incomplete balancing of effects for the two levels of questions for each error type. Participants were randomly assigned to one of two groups. Group 1 (G1) participants received the common error first and the uncommon error second, whereas participants in Group 2 (G2) received the uncommon error first and then the common error. Therefore, the design provides complete replication for type of error. However, each type of error had two levels of questions, Question 1 (Q1) and Question 2 (Q2). It made no conceptual sense to evaluate the effect of order for the two questions because Q2 logically must follow Q1. Thus, the order of question was not considered a meaningful effect.

Participants

General and special education elementary teachers in two large urban school districts participated in the study.

Teachers were eligible for participation if two criteria were met: (a) all participants were currently teaching in an elementary school (k-6); (b) all participants had a current certification in elementary and/or special education.

Ninety elementary teachers (11 males and 79 females), working in three elementary schools, volunteered to participate. The highest degree earned for 31 participants was a bachelor's degree and for 59 participants, a master's degree. Participants' ages ranged from 22 to 67 years, with a mean of 41.76. Twenty-nine had certifications in more than one area (e.g., elementary, reading specialist, special education). Eighty-three held elementary certifications, 30 held special education certifications, 2 held secondary certifications, and 11 held other certifications. Teaching experience ranged from 1 to 35 years, with a mean of 15.34 years; average number of years in the current position ranged from 1 to 29, with a mean of 7.39. At the time of the study, 62 teachers were primarily responsible for mathematics instruction; 28 were not.

Instruments

The researcher developed the experimental protocols. The first question directed the participants to describe the error in each incorrect problem as specifically as possible. Two examples of unacceptable error descriptions were provided (i.e., Don't say: the student got the problem wrong or the student does not know how to multiply). The second question directed the participants to select the type of error(s) to address first during instruction to reduce future errors. No examples were provided for the second question.

The experimental protocols consisted of two worksheets, each containing 20 completed subtraction problems. Seven problems assessed basic subtraction facts (e.g., 18 – 9 =, 7 – 4 =), 10 problems assessed computational skills (e.g., 407 – 23 =, 89 – 55 =), and 3 word problems assessed problem-solving skills. All 20 problems on each worksheet were representative of math problems encountered by both general and special education elementary students (Stein, Silbert, & Carnine, 1997).

Because the study was intended to assess participants' ability to identify error patterns, each set of 20 problems also contained six random distracter errors so the error pattern would be less obvious. The random errors consisted of one problem with a wrong operation error (e.g., 14 – 7 = 21) where the numbers were added instead of subtracted; one problem with a basic subtraction fact error (e.g., 9 – 4 = 6); three problems with basic fact errors occurring in the computational problems (e.g., 477 – 23 = 451 with the fact error 7 – 3 = 1); and one word problem with a basic fact error in the computational solution. The random errors were carefully controlled so no patterns were formed (i.e., no basic subtraction fact errors were repeated).

The two types of error patterns examined were identified as "smaller-from-larger" (SFL) and "borrow-across-zero" (BAZ) and classified as incorrect or defective algorithm errors (e.g., Ashlock, 2002; Resnick, 1982; Roberts, 1968). The SFL error occurs when a student subtracts a smaller digit from a larger digit in the same column regardless of the position (e.g., 326 – 117 = 211, with the SFL error 6 – 7 = 1). This type of error is common in students' subtraction work (Ashlock, 2002; National Research Council, 2002; Resnick, 1982) and represented the common error pattern. SFL errors occurred in three computational problems and in one word problem for a total of four occurrences.

The BAZ error occurs when a student attempts to borrow from a zero and does not continue borrowing from the column to the left of the zero (e.g., 602 – 437 = 265, where the student does not continue to borrow from the hundreds column). This type of error occurs less frequently than SFL errors (Resnick, 1982). The BAZ error occurred in three computational problems and in one word problem for a total of four occurrences.

In the packet of materials for Group 1, the first set of problems contained an SFL error. The second set contained the BAZ error. The packet of materials for Group 2 contained the same two sets of completed subtraction problems, but the set of problems with the BAZ error was presented first, followed by the SFL error. Complete directions for all required tasks were included in each packet.

Procedures

Data collection. Participants completed the experimental protocols during the school's regularly scheduled faculty meeting with the researcher in attendance. Participants were directed to complete the experimental tasks in the order in which they appeared in the packet and place all materials back into the envelope when finished. All teachers who received a packet submitted a completed packet directly to the researcher. Therefore, a 100% return rate was achieved.

Scoring procedures. The participant response sheets were scored in random order by the experimenter and a graduate student who was blind to the group assignment and purpose of the study. Separate scoring keys were used to assess the presence of the required elements in Q1 and Q2 for each error type. The scoring keys contained acceptable and non-acceptable answers. Responses were scored as acceptable if the error and instructional focus were described in a specific and concise manner. If a participant's responses were presented in general and non-specific terms, they were scored as non-acceptable. A total score was given for each of the questions: Q1 scores could range from 0 to 4 and Q2 scores could range from 0 to 2.

For Question 1, SFL and BAZ were scored in a similar fashion. One point was awarded for correctly identifying each occurrence of the SFL and BAZ errors using a variety of synonymous terms (e.g., SFL: subtracted smaller from larger number regardless of position, reversed/transposed smaller from larger, subtracted minuend from subtrahend). The total points for these problems were summed for one score. No points were awarded for identifying the basic fact and wrong operation errors because they were used solely for the purposes of making the main error pattern less obvious. The points were summed for one score for each error type (SFL and BAZ) and used in the statistical analysis.

For Question 2, both SFL and BAZ were scored in the same fashion. A separate scoring key was designed to award points for acceptable and unacceptable answers. Two points were awarded to responses matching the designated acceptable responses in the scoring key. Since both the SFL and BAZ error patterns are classified as defective algorithms (e.g., Ashlock, 2002; Resnick, 1982; Roberts, 1986), the instructional response was considered appropriate if the error pattern was specifically identified as the priority in the instructional focus (Stein et al., 1997). The response was awarded two points if the predominant error type (SFL or BAZ) was prescribed as the initial instructional focus. For the SFL error, the instruction should focus on the rule relationship and procedures for subtracting a larger number from a smaller number. For the BAZ error, the instruction should focus on the procedures for borrowing with or across a column with a zero.

Interrater agreement. A reliability check was performed on 50% (n = 45) of randomly selected teacher response sheets. A graduate student was trained and then independently scored the teacher response sheets without knowing the group assignment. Interscorer reliability was calculated using point-to-point agreement computed by adding the frequency of agreement for occurrences and the frequency of agreement for nonoccurrence, dividing by the total number of observations, and multiplying by 100. The percent of simple agreement ranged from 0.87 to 1.00, with a mean of 0.93.

RESULTS

This study investigated teachers' ability to identify two systematic error patterns in subtraction and the instructional focus prescribed based on the errors. To clarify the relationship between teachers' ability to identify an error pattern, identify a common versus an uncommon error, and prescribe an appropriate instructional focus, means and standard deviations of the teachers' performance scores were calculated (see Table 1). The data suggest that teachers were able to accurately identify and describe both error types (SFL Q1, M = 2.52, SD = 1.62; BAZ Q1, M = 2.59, SD = 1.59), but they were not able to prescribe the appropriate instructional focus for either error type (SFL Q2, M = 0.73, SD = 0.91; BAZ Q2, M = 0.61, SD = 0.91).

Table 1

Summary of Means and Standard Deviations for Error Type and Question by Group

		G1 *M (SD)*	G2 *M (SD)*	Total *M (SD)*
SFL				
	Q1	2.18 (1.64)	2.87 (1.55)	2.52 (1.62)
	Q2	0.73 (0.94)	0.73 (0.84)	0.73 (0.93)
BAZ				
	Q1	2.69 (1.58)	2.49 (1.62)	2.59 (1.59)
	Q2	0.73 (0.91)	0.49 (0.87)	0.61 (0.91)

Note. SFL = smaller-from-larger error; BAZ = borrow-across-zero error; Q1 = the error identification question; Q2 = the remediation question; G1 = Group 1; G2 = Group 2.

Table 2
Analysis of Variance with Repeated Measures

Source	*df*	*SS*	*MS*	*F*	*p*
Between Subjects	89	357.01			
Groups (A)	1	0.34	0.34	0.083	.05
Error	88	356.67	4.05		
Within Subjects	270	572.33			
Error Type (B)	1	0.06	0.06	0.064	.05
Question (C)	1	319.22	319.22	339.596*	0.05
Residual	268	253.05	0.94		
Total	359	929.34			

Note. Groups (A) represents the performance of the two groups (G1 and G2). Error type (B) represents the identification and description of the SFL error type and the BAZ error type. Question (C) represents the selection of an appropriate instructional focus. *$p < .05$.

There was very little difference between the groups for either question. For SFL Q1 and Q2, the means for G1 and G2 were 2.18 and 0.73 and 2.87 and 0.73, respectively. For BAZ Q1 and Q2, the means for G2 and G1 were 2.69 and 0.73 and 2.49 and 0.49, respectively. While all cell means and standard deviations were calculated, the overall means for question (Q1 and Q2) and error type (SFL and BAZ) are of particular interest.

The mean suggests that teachers were better at identifying the error patterns than they were at prescribing an appropriate instructional focus to remediate them. Further, there does not appear to be a difference in the ability of teachers to identify or prescribe an instructional focus according to error type (SFL = BAZ). To examine the difference in teachers' error identification and instructional focus prescribed, an analysis of variance was calculated.

Analysis of Variance with Repeated Measures

To ascertain if the difference observed in the sample could be inferred to the population, an analysis of variance with repeated measures (ANOVA-R) was applied. A summary of this analysis appears in Table 2. Because only partial interactions can be computed for this design, the interaction effects were not evaluated but were collapsed into the appropriate error terms. The difference between describing the error and prescribing an instructional focus (C Effect) was significant, $F(1, 270) = 339.596$, $p < .05$. Therefore, teachers were better able to identify the two error types than prescribing an appropriate instructional focus. The differences between groups and error types were not significant, $F(1, 89) = 0.083$, $p <. 05$, and $F(1, 270) = 0.064$, $p < .05$.

Instructional Focus

Table 3 summarizes the type of instructional focus prescribed by the participants. Thirty (33%) prescribed an appropriate instructional focus for the SFL error, whereas 46 (51%) participants identified subtraction facts as the main instructional focus. Another 14 participants (16%) identified some other instructional focus (e.g., place value, self-check strategies, attention problems). Twenty-five participants (28%) were able to identify an appropriate instructional focus for the BAZ error, whereas 48 (53%) identified subtraction facts as the main instructional focus. Another 17 participants (19%) identified some other instructional focus (e.g., place value, self-check strategies, attention problems). Overall, only 31% of the participants were able to select an appropriate instructional focus for both error types, 52% targeted basic subtraction facts as the instructional focus and 17% selected some other instructional focus (e.g., place value, self-check strategies, attention problems).

DISCUSSION

The purpose of this study was to (a) determine whether teachers were able to identify specific error patterns exhibited in subtraction; (b) determine whether teachers were better able to describe a more commonly occurring subtraction error (i.e., SFL) than a less commonly occurring subtraction error (i.e., BAZ); (c) determine whether teachers were able to prescribe an appropriate instructional focus; and (d) examine the instructional focus that teachers selected to address first during mathematics instruction. The following sections present an interpretation of the results and a discussion of the limitations and implications of this research project.

Error Identification

More than half of the participating teachers were able to identify and describe the specific error pattern displayed in the students' subtraction work. The combination of each similar mistake produced the error pattern (i.e., SFL or BAZ) and occurred four times in each set of 20 problems. Therefore, each teacher had four opportunities to identify and describe the error. For the entire sample, the average number of times that the teachers correctly identified the error was 2.56. However, only 45% (n = 82) of the teachers were able to identify and describe each instance of the error pattern and 14% (n = 26) were able to identify the error three out of four times. Therefore, 60% (n = 108) of the teachers were able to correctly identify both the SFL and the BAZ errors.

This finding is important because researchers have indicated that the initial step in the process of providing appropriate corrective feedback and instructional remediation is identification of the error, mistake, or misconception that the student is making (Ashlock, 2002; Gable & Cohen, 1990; Slavin, 2000; Stein et al., 1997). If teachers cannot accurately identify the error, it is impossible, or at least very difficult, for them to design and deliver instruction that is appropriate and effective. One possible explanation for this finding may be a lack of training in error analysis or in teaching mathematics in general (Ashlock, 2002; Malzahn, 2000).

Common Error vs. Uncommon Error

Teachers were not able to better describe a more common error than a less common error. Thus, the average score for the common error was 2.52 (SFL), whereas the average score for the less common error was 2.59 (BAZ); the difference was not significant. Only 44% (n = 40) of the 90 participants were able to identify the common error, and 47% (n = 42) were able to identify the uncommon error on all four occasions. Paired with the results for the first question, this finding is not surprising.

Instructional Focus

The likelihood of students making errors during the instructional process is high, especially for students with disabilities and students struggling in mathematics. It is important for all teachers to not only recognize problematic areas for their students, but also to be able to select an instructional focus that will address the students' specific problem area. Therefore, the teachers' instructional foci were of particular interest.

Surprisingly, teachers seldom focused on the pattern of errors. Most often, they addressed random fact errors

Table 3

Frequency and Percent of Error Pattern to Be Addressed First During Instruction

	Specific Student Error	Basic Subtraction Facts	Other
SFL Q2 (n = 90)	30 (33%)	46 (51%)	14 (16%)
BAZ Q2 (n = 90)	25 (28%)	48 (53%)	17 (19%)
Total (n = 180)	55 (31%)	94 (52%)	31 (17%)

Note. The value in the parentheses represents the percentage of teachers selecting an instructional focus. The "Other" category includes all other recommendations for instruction identified by participants. SFL Q2 = selection of an appropriate instructional focus for the smaller-from-larger error; BAZ Q2 = selection of an appropriate instructional focus for the borrow-across-zero error.

(52%). Occasionally (17%), they addressed problems that could not be observed in student performance, such as lack of attention and self-check strategies.

Overall, teachers did not base their instructional focus on student error patterns (SFL or BAZ). Only 31% selected the specific error pattern exhibited in the student's work to address first. Moreover, only 50% of teachers who could identify the error pattern three out of four times selected the appropriate instructional focus. Two thirds of the teachers who could identify the specific error pattern did not base their instruction on the students' error pattern.

Most teachers chose to address basic subtraction facts first during instruction. This is difficult to understand because the facts were randomly incorrect and only incorrect in 5 out of 48 instances. Nevertheless, it supports the frequently reported finding that teachers reteaching or attempting to correct student errors focus heavily on basic facts instruction (Babbitt & Miller, 1996; Bottge, 1999; NCTM, 2000; Woodward, Baxter, & Robinson, 1999) at the expense of procedural and conceptual knowledge. Attention to basic fact errors is important (Garnett, 1992; Mercer & Miller, 1992; National Research Council, 2002; Stein et al., 1997); however, teachers must not attend exclusively to fact errors, especially when students make the same type of procedural error or mistake every time they face a particular type of problem.

One possible explanation for the instructional focus on facts is that teachers are trained to teach mathematics in terms of general concepts (e.g., facts, subtraction, addition, place value). This type of training may impede teachers' ability to directly teach parts of concepts or parts of procedural steps. Additionally, the current instructional philosophy or holistic constructivism (Ellis & Fouts, 1997; Pressley & Rankin, 1994) does not encourage breaking down instruction into smaller parts. Teachers' failure to identify accurately the specific error pattern as the instructional focus may cause teachers to use their very limited instructional time inefficiently. Thus, time is wasted teaching, reviewing, and practicing skills and concepts students have already learned and that are unrelated to their errors.

A related possibility is that mathematics instruction is heavily influenced by the curriculum materials and textbooks used. Approximately 75% of what occurs during mathematics instruction comes from the curriculum materials and textbook (Parmar, 1992; Porter, 1989). Moreover, it is estimated that 90% of teacher decision making in the classroom is governed by the textbooks used (Muther, 1985). Textbook publishers have received a great deal of criticism for poorly designed instructional features (Carnine, Jitendra, & Silbert, 1997; Jitendra, Carnine, & Silbert, 1996; Jitendra, Salmento, & Haydt, 1999; Stein et al., 1997). If the curriculum materials and textbooks do not include specific suggestions for reteaching or strategies to help correct students' errors, it appears teachers are more likely to revert back to basic facts practice.

Other Instructional Foci

Most interesting is the finding that approximately 17% of the teachers selected other areas to address first during instruction. The most common of these other areas was student attention. Some teachers concluded the errors displayed in the students' work were due to inattention rather than a systematic error pattern or a misconception (e.g., the student is not paying attention while completing the problems). This finding is consistent with previous research showing that many teachers attribute student errors to attention and attitude issues (Ashlock, 2002; Clements, 1982).

This is very disheartening because the error patterns occurred each time the student had an opportunity to complete a specific problem type. Thus, for these errors to have been caused by a lack of attention, a very unlikely scenario would have occurred: The student's attention would have followed the specific pattern that just happened to coincide with this particular type of problem. Looking for causes within the student deflects attention away from the curriculum materials and the instructional design and delivery chosen by the teacher. Clearly, the curriculum materials and instructional design and delivery methods have a substantial influence on students' performance (i.e., errors and misconceptions).

The results relating to the instructional foci selected by the teachers are discouraging. Valuable instructional time is wasted when the instructional focus does not match students' deficits, and inefficient use of instructional time is potentially damaging for students with disabilities and low-performing students. These students generally require more effective and efficient instruction on problematic content because they already lag behind. Instructional goals requiring students to complete drill and practice on skills not relating to their specific problem area only continue to cause students to perform poorly.

Limitations of the Study

The present study presents four possible threats to external validity. First, the sample consisted of elementary schools teachers from three schools within two school districts. The school districts use the same curriculum and adhere to very similar educational philosophies. Although improbable, it is possible that the study would have led to different results with a more diverse population of teachers. Specifically, curriculum, instructional design variables, and teacher variables impact student achievement (e.g., Brophy & Good, 1986; Rosenshine & Stevens, 1986; Stein, Carnine, & Dixon, 1998), and student error patterns may mirror the instructional approach (Woodward et al., 1999). Moreover, it is possible that the teachers were unaccustomed to identifying and remediating procedural errors (i.e., defective algorithms) because the curriculum they used does not emphasize procedural knowledge. Second, the study focused solely on a single skill area (subtraction) and a single error type (procedural error). It is plausible that the effects would be different if another skill area or different error types were used.

Third, the classification system of errors used in the present analysis was based on the literature on student errors in mathematics (e.g., Ashlock, 2002; Cox, 1975a, 1975b; Resnick, 1982). These researchers systematically analyzed large numbers of students' math work to determine common and less common errors. However, errors are influenced by many variables (e.g., age, ability, content, curriculum, instructional methodology), and the classification system of common and less common was arbitrary. This arbitrary classification of errors could have accounted for the insignificant effect for error type.

Fourth, the error types were classified as procedural (e.g., Ashlock, 2002; Stein et al., 1997) and, therefore, require very specific and systematic remediation procedures to correct (e.g., Stein et al., 1997). A different classification of the error types (e.g., conceptual) may produce different findings. Examining the most efficient and effective instructional approaches for correcting student errors is an important topic for future research.

Finally, the study presented one possible threat to internal validity. First, the researcher specifically developed the protocols for this study. The error patterns were displayed four times in each set of 20 problems. This decision was based on the premise that an error must occur at least three to five times to constitute a systematic error (Ashlock, 2002; Gable & Hendrickson, 1990). Materials employing more or less frequent errors indicating a particular error pattern may produce different results.

Implications for Practice

The two types of subtraction errors displayed in the protocols were errors that are easily observed in the mathematics work of students. It is the responsibility of teachers to diagnose student errors and then make the appropriate correction. Teachers must have the content knowledge and ability to provide appropriate and focused instruction to correct students' misconceptions and errors. Improving the ability of teachers to recognize error patterns and plan more appropriate instruction can be addressed through (a) preservice programs, (b) professional development opportunities in math, (c) refining curriculum materials, and (d) continued research in mathematics for students with disabilities.

Preservice programs. A high degree of responsibility must fall with preservice teacher education programs. Teacher training programs must produce content proficient and effective mathematics teachers. Far too many teachers do not have adequate training to provide appropriate and focused instruction to reteach students who struggle or when learning does not occur. The lack of mathematics training for elementary teachers is highlighted in the results of a survey conducted with practicing elementary teachers (Malzahn, 2000). The researchers found only 1% of elementary teachers had undergraduate majors in mathematics, the majority had completed at most seven semesters of mathematics coursework, and only 54% felt "very well qualified" to teach mathematics. If elementary teachers teaching mathematics lack the necessary content knowledge and training to teach and reteach mathematics concepts and skills, the performance gap between low- and high-achieving students will continue to increase.

Professional development. It is clear from the results of this study that teachers' experience does not necessarily impact their ability to prescribe an appropriate instructional focus to remediate a student's errors; therefore, high-quality professional development in mathematics must increase. Practicing elementary teachers report low levels of participation in professional development activities focusing on mathematics; only about one third have participated in 16 or more hours of professional development in mathematics (Malzahn, 2000). This lack of professional development targeting mathematics is particularly discouraging paired with the fact only about half of practicing elementary teachers feel very qualified to teach mathematics. It is critical for school administrators and individuals responsible for professional development to recognize that teachers need increased opportunities for professional development in the area of mathematics.

Curriculum materials. In addition to the lack of teacher preparedness, many teacher manuals accompanying mathematics programs do not provide specific details and directions for correcting student errors (Stein et al., 1997). Publishing companies must design better elementary mathematic textbooks that include specific instructional directions for teachers and reteaching procedures to better assist teachers in correcting student errors. If not corrected, students will continue to make the same errors.

Textbooks should emphasize the five strands in mathematics outlined by the National Research Council

(2002). Further, mathematics educators should study and adopt trends currently seen in the area of reading (National Institute of Child Health and Human Development, 2000). Publishing companies and their reading textbooks have been subject to unprecedented scrutiny. As a result, many publishers have redesigned their reading programs to integrate scientifically based practices to provide teachers effective instructional design features and components that present students the best probability of learning to read. This type of focus is needed in the area of mathematics.

Future research. Educational researchers must focus their efforts around the five strands of mathematics (National Research Council, 2002) to develop effective curriculum (i.e., scope and sequence), instructional strategies, useful assessment techniques (i.e., guides instruction), and corrective procedures to provide students, especially those experiencing problems in mathematics, the greatest probability of achieving proficiency in mathematics. An emphasis on implementing scientifically based practices in mathematics instruction and textbooks, similar to what is currently occurring in reading (i.e., National Institute of Child Health and Human Development, 2000), is long over due in mathematics. Students' problems and deficiencies in mathematics will not improve or disappear unless educators, publishers, and researchers work together to improve the instructional design and delivery of mathematics.

CONCLUSIONS

In conclusion, teachers were able to recognize the error patterns when presented with specific errors in students' mathematics work. However, they were unable to prescribe an instructional focus for those errors. Instead, most fell back on teaching facts, which is a finite set and done through drill and practice. This has serious ramifications for struggling students and students with disabilities because, if appropriate remediation and instruction is not provided, these students are likely to continue making the same types of errors, discouraging students and lowering future mathematical performance.

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AUTHOR NOTES

1. Operational definitions of each error type included on the protocol may be obtained by contacting the author.
2. A table containing all of the teachers' instructional focuses identified may be obtained by contacting the author.

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