Effects of Mathematical Word Problem–Solving Instruction on Middle School Students with Learning Problems

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This study investigated the differential effects of two problem-solving instructional approaches—schema-based instruction (SBI) and general strategy instruction (GSI)—on the mathematical word problem–solving performance of 22 middle school students who had learning disabilities or were at risk for mathematics failure. Results indicated that the SBI group significantly outperformed the GSI group on immediate and delayed posttests as well as the transfer test. Implications of the study are discussed within the context of the new IDEA amendment and access to the general education curriculum.

Mathematics is integral to all areas of daily life; it affects successful functioning on the job, in school, at home, and in the community. The importance of mathematics literacy and problem solving is emphasized in the Goals 2000: Educate America Act of 1994 and National Council of Teachers of Mathematics' *Principles and Standards for School Mathematics* (NCTM, 2000; Goldman, Hasselbring, & the Cognition and Technology Group at Vanderbilt, 1997). Increasing evidence suggests that high levels of mathematical and technical skills are needed for most jobs in the 21st century. Therefore, it is important to ensure that all students, not just those planning to pursue higher education, have sufficient skills to meet the challenges of the 21st century (National Education Goals Panel, 1997). In addition, one of the provisions of the 1997 amendments to the Individuals with Disabilities Education Act (IDEA) is that students with disabilities have meaningful access to the general education curriculum. In fact, these students are held accountable to the same high academic standards required of all students (No Child Left Behind Act, 2002).

As part of the mathematics reform and standards-based reform movements, the NCTM (2000) developed the *Principles and Standards for School Mathematics*. The focus of the NCTM standards is on "conceptual understanding rather than procedural knowledge or rule-driven computation" (Maccini & Gagnon, 2002, p. 326). This emphasis has significant implications for classroom practice because special education typically has focused on arithmetic computation rather than higher-order skills such as reasoning and problem solving (Cawley, Parmar, Yan, & Miller, 1998). Students with learning disabilities often manifest serious deficits in mathematics, especially problem solving (Carnine, Jones, & Dixon, 1994; Cawley & Miller, 1989; Cawley, Parmar, Foley, Salmon, & Roy, 2001; Parmar, Cawley, & Frazita, 1996). Specifically, these students perform at significantly lower levels than students without disabilities on all problem types, especially problems that involve indirect language, extraneous information, and multisteps (Briars & Larkin, 1984; Cawley et al., 2001; Englert, Culatta, & Horn, 1987; Lewis & Mayer, 1987; Parmar et al., 1996). While problems in reading and basic computation skills may account for these students' poor performance, difficulties in problem representation and failure to identify relevant information and operation may exacerbate their poor performance (Hutchinson, 1993; Judd & Bilsky, 1989; Parmar, 1992).

In addition, ineffective instructional strategies may explain the poor problem-solving performance of students with learning disabilities. One commonly used instructional approach is the "key word" strategy, in which students are taught key words that cue them as to what operation to use in solving problems. For example, students learn that *altogether* indicates the use of the addition operation, whereas *left* indicates subtraction. Similarly, the word *times* calls for multiplication, and *among* indicates the need to divide. However, Parmar et al. (1996) argued that "the outcome of such training is that the student reacts to the cue word at a surface level of analysis and fails to perform a deep-structure analysis of the interrelationships among the word and the context in which it is embedded" (p. 427). That is, the focus is on whether to add, subtract, multiply, or divide rather than whether the problem makes sense. Another commonly employed problem-solving strategy is the four-step (read, plan, solve, and

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check) general heuristic procedure. Unfortunately, this procedure may not facilitate problem solution for students with learning disabilities, especially when the domain-specific conceptual and procedural knowledge is not adequately elaborated upon (Hutchinson, 1993; Montague, Applegate, & Marquard, 1993).

For students with learning disabilities, explicit teaching for conceptual understanding is critical to establish the necessary knowledge base for problem solution. Recent reviews provide empirical support for problem-solving instruction, such as a schema-based strategy instruction, that emphasizes conceptual understanding of the problem structure, or schemata (Xin & Jitendra, 1999). Successful problem solvers typically create a complete mental representation of the problem schema, which, in turn, facilitates the encoding and retrieval of information needed to solve problems (Didierjean & Cauzinille-Marmeche, 1998; Fuson & Willis, 1989; Marshall, 1995; Mayer, 1982). Problem schema acquisition allows the learner to use the representation to solve a range of different (i.e., containing varying surface features) but structurally similar problems (Sweller, Chandler, Tierney, & Cooper, 1990).

Schema-based strategy instruction is known to benefit both special education students (e.g., Jitendra & Hoff, 1996; Jitendra, Hoff, & Beck, 1999) and students at risk for math failure (e.g., Jitendra et al., 1998; Jitendra, DiPipi, & Grasso, 2001) in solving arithmetic word problems. However, previous research on the effects of schema-based strategy instruction is limited, for the most part, to algebra problems (Hutchinson, 1993) and addition and subtraction (e.g., change, combine, additive compare) arithmetic problems. Although the effects of semantic representation training in facilitating problem solving have been demonstrated with college students with and without disabilities, the studies are limited to a sample of comparison problems only (Lewis, 1989; Zawaiza & Gerber, 1993). Furthermore, neither the study by Lewis nor the study by Zawaiza and Gerber emphasized key components (compared, referent, and scalar function) pertinent to the compare problem schemata. In addition, the rules for figuring out the operation (e.g., if the unknown quantity is to the right of the given quantity on the number line, then addition or multiplication should be applied) cannot be directly applied to solve multiplication or division compare problems when the relational statement involves a fraction or when the unknown is the scalar function (i.e., the multiple or partial relation between two comparison quantities).

A more recent exploratory study by Jitendra, DiPipi, and Perron-Jones (2002) employed a single-subject design to teach four students with learning disabilities to solve word problems involving multiplication and division using the schema-based strategy. However, one of the limitations of the study is that "the single-subject design employed in this investigation does not help clarify whether the study findings are attributable to specific schema-based nature of the instruction" (p. 37) or to the generally carefully designed one-on-one intensive instruction on two problem types. The purpose of the present investigation was to evaluate and compare the effectiveness of two problem-solving instructional approaches, schema-based and general strategy instruction, in teaching multiplication and division word problems to middle school students with learning disabilities or at risk for mathematics failure.

Method

Design

A pretest–posttest comparison group design with random assignment of subjects to groups was used to examine the effects of the two word problem–solving instructional procedures—schema-based instruction (SBI) and general strategy instruction (GSI)—on the word problem–solving performance of middle school students with learning problems.

Participants

Participants were 22 students with learning problems, including 18 who were school-identified as having a learning disability, 1 with severe emotional disorders, and 3 who were at risk for mathematics failure, attending a middle school in the northeastern United States. Specifically, participant selection was based on (a) teacher identification of students who were experiencing substantial problems in mathematics word problem solving and (b) a score of 70% or lower on the word problem–solving criterion pretest involving multiplication and division word problems. To determine sample size, a power analysis using an alpha level of .05 and an effect size based on existing schema-based instruction research studies (e.g., Jitendra et al., 1998) was conducted, which indicated that a minimum of 10 participants in each group is sufficient to obtain a power of .90 for a 2 × 4 repeated-measures analyses of variance (Friendly, 2000). Table 1 presents demographic information with respect to participants' gender, grade, age, ethnicity, special education classification, IQ level, and standardized achievement scores in math and reading. It is important to note that IQ and achievement data from school records were available for only nine students.

Procedure

Instructors were two doctoral students in special education and two experienced special education teachers. The two doctoral students taught the first cohort of 8 students (4 in each treatment group), and the two special education teachers taught the second cohort of 14 students (7 in each treatment group). Students in both cohorts were randomly assigned to the two treatment groups. To control for teacher effects, each pair of instructors (i.e., the two doctoral students or the two special education teachers) were randomly assigned to the two conditions, and they switched treatment groups midway

TABLE 1. Demographic Information

Variable	SBI group	GSI group
Gender		
Male	5	6
Female	6	5
Grade		
6	6	4
7	2	6
8	3	1
Mean age in months (*SD*)	153.8 (8.6)	156.7 (8.7)
Ethnicity		
Caucasian	4	3
Hispanic	5	7
African American	2	1
Classification		
LD	10	8
SEN	0	1
NL	1	2
IQ[a]		
Verbal		
M	95	93
SD	8.5	5.7
Performance		
M	92	92
SD	2.5	2.1
Full Scale		
M	92	92
SD	2.9	3.1
Achievement[b]		
Math		
M	84	88
SD	10.4	3.9
Reading		
M	90	93
SD	2.0	2.4

Note. SBI = schema-based instruction; GSI = general strategy instruction; LD = learning disabled; SED = seriously emotionally disturbed; NL = not labeled.
[a]IQ scores were obtained from the *Wechsler Intelligence Scales for Children-Revised* (Wechsler, 1974). [b]Achievement scores in math and reading were obtained from the *Metropolitan Achievement Test* (Balow, Farr, & Hogan, 1992), with the exception of scores for one student that were obtained from the *Stanford Achievement Test,* 9th ed. (1996). IQ and achievement scores were available for only 9 of the 22 students.

through the intervention. The first author developed the teaching scripts for both conditions and piloted them prior to employing them in the study. Instructors received two 1-hour training sessions to familiarize them with lesson formats, the suggested teacher wording, and lesson materials when implementing the two instructional approaches.

Students in both conditions received their assigned strategy instruction three to four times a week, each session lasting approximately an hour. The SBI group received 12 sessions of instruction, with 4 sessions each on solving *multiplicative compare* and *proportion* problems and 4 sessions on solving mixed word problems that included both types. Students in the GSI group also received 12 sessions of instruction, but they solved both types of problems in each session. Unlike the SBI group, students in the GSI group did not receive instruction in recognizing the two different word problem types. Students in the two groups solved the same number and type of problems.

Both Conditions

Across both SBI and GSI conditions, the teacher first modeled the assigned strategy with multiple examples. Explicit instruction was followed by teacher-guided practice and independent student work. Corrective feedback and additional modeling were provided as needed during practice sessions. It should be noted that students in both groups were allowed to use calculators during instruction and testing conditions, because computation skills were not the focus of this study. Table 2 summarizes the problem-solving strategy steps across two conditions.

Overall, both groups were taught to follow the four-step general problem-solving procedure of reading to understand, representing the problem, and planning, solving, and checking. However, the fundamental differences between the two conditions involved the second and third steps, with regard to how to plan and solve the problem. Specifically, the SBI group was taught to identify the problem structure and use a schema diagram to represent and solve the problem, whereas the GSI group learned to draw semiconcrete pictures to represent information in the problem and facilitate problem solving. A detailed description of the two instructional conditions, with an emphasis on how to "plan" and "solve" the problem is presented in the next section.

Schema-Based Instruction Condition

Instruction for the SBI group occurred in two phases: problem schemata instruction and problem solution instruction. During problem schemata instruction, students learned to identify the problem type or structure and represent the problem using a schematic diagram. In this phase, story situations with no unknown information were presented. The purpose of presenting story situations was to provide students with a complete representation of the problem structure of a specific problem type. In contrast, the problem solution instruction phase used story problems with unknown information. Below is a general description of instruction employed to teach the two problem types investigated in this study.

Multiplicative Compare Problems. When teaching the multiplicative compare problem schema, instruction emphasized several salient features. That is, students learned that a multiplicative compare problem always includes (a) a refer-

TABLE 2. General Problem-Solving Steps Employed in the SBI and GSI Conditions

Schema-based instruction (SBI)	General strategy instruction (GSI)
• Read to understand	• Read to understand
• Identify the problem type, and use the schema diagram to represent the problem	• Draw a picture to represent the problem
• Transform the diagram to a math sentence, and solve the problem	• Solve the problem
• Look back to check	• Look back to check

ent set, including its identity and its corresponding quantity; (b) a compared set, including its identity and corresponding quantity; and (c) a statement that relates the compared set to the referent set. In short, the multiplicative compare problem describes one object as the referent and expresses the other as a part or multiple of it. Students first learned to identify the problem type using story situations such as the following: "Vito earned $12 from shoveling snow over the weekend. He earned ⅓ as much as his friend Guy did. Guy earned $36 from shoveling snow." This story situation, because the amount Vito earned (compared set) was compared to what Guy earned (referent set), was deemed to be a comparison problem situation. Moreover, students learned that the comparison implies a *multiplicative* relationship (i.e., multiple or part) rather than an *additive* compare (more or less) situation.

A prompt sheet that contained information describing the salient features of the problem type and the five strategy steps was designed to facilitate problem solving. Step 1 of the strategy required identifying and underlining the relational statement in the problem. For example, students were taught that the relational statement in the above sample story was, "He earned ⅓ as much as his friend Guy did," because it describes the compared as a part of the referent. Step 2 involved identifying the "referent" and "compared" and mapping that information onto the multiplicative compare diagram. Students were instructed to examine the relational statement and note that the second subject or object, that following a phrase such as "as many as" or "as much as," indicates the referent (i.e., "Guy"), whereas the subject or object preceding the referent indicates the compared. Step 3 entailed finding the corresponding information related to the compared, referent, and comparison relation and mapping that information onto the diagram. Instruction emphasized rereading the story to find information about the compared (Vito earned $12), the referent (Guy earned $36), and their relation (⅓), as well as writing the corresponding quantities with the labels onto the diagram (see Figure 1).

In sum, students learned to identify the key problem features and map the information onto the diagram during problem schema analysis instruction. Next, they learned to summarize the information in the problem using the completed diagram. Instruction emphasized checking the accuracy of the representation by having students transform the information in the diagram into a meaningful mathematics equation for example,

$$\frac{12}{36} = \frac{1}{3}.$$

Students learned that when the representation does not establish the correct equation, for example,

$$\frac{36}{12} \neq \frac{1}{3},$$

they should check the completed diagram by reviewing the information related to each component (i.e., the referent, compared, and relation) of the multiplicative compare problem. In addition, the instructor provided a rationale for learning the problem schema. For example, the schematic diagram used to represent the story situation reflected the mathematical structure of the problem type, which could be used eventually to solve problems that involve an unknown quantity.

During the problem-solving instruction phase, students learned to solve for the unknown quantity in word problems. For example, in the following problem, "Vito earned $12 from shoveling snow over the weekend. He earned ⅓ as much as his friend Guy did. How much did Guy earn from shoveling the snow?" students were asked to solve for the unknown quantity. Instruction focused on representing the problem using the multiplicative compare schematic diagram, as in the problem schema instruction phase. The only difference was that students were taught to use a question mark to flag the unknown quantity (i.e., the amount Guy earned) in the diagram. Next, students learned to transform the information in the diagram into a math sentence and solve for the unknown (Step 4). That is, they derived the following math equation,

$$\frac{12}{?} = \frac{1}{3},$$

directly from the schematic representation. They then used cross multiplication to solve for the unknown (i.e., ? = 12 × 3 = 36). For Step 5, students had to write a complete answer on the answer line and check the reasonableness of their answer. Instruction required checking the accuracy of both the representation and the computation.

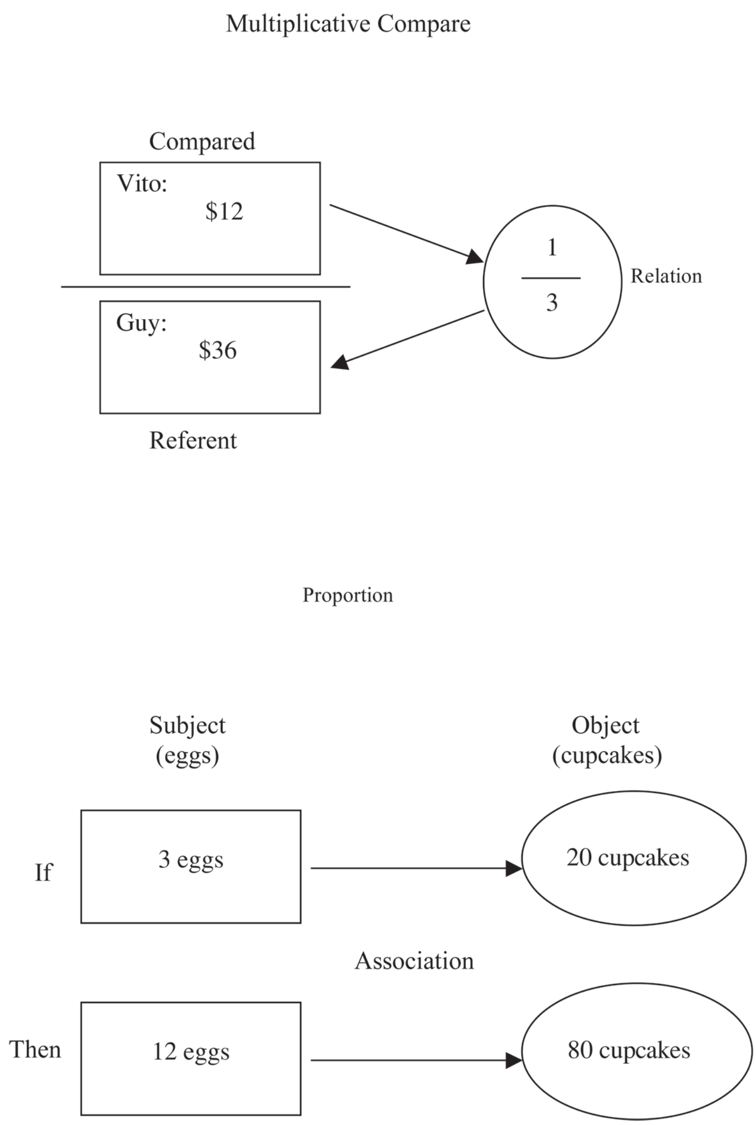

FIGURE 1. General problem-solving steps employed in the schema-based instruction and general strategy instruction conditions.

Proportion Problems. When teaching the proportion problem schema, the following salient features were emphasized: (a) The proportion problem describes an association (i.e., a ratio) between two things; (b) there are two pairs of associations between two things that involve four quantities; and (c) the numerical association (i.e., the ratio) between two things is constant across two pairs (see Marshall, 1995). Typically, the proportion problem involves an "if . . . then" relationship. That is, one pair of associations is the *if* statement, and the other is the *then* statement. The *if* statement declares a per-unit value or unit ratio in one pair, whereas the *then* statement describes the variation (enlargement or decrement) of the two quantities in the second pair. In addition, the unit ratio remains constant across the two pairs of associations (i.e., if 1 shelf holds 12 books, then 4 shelves will hold 48 books). Students first learned to identify the problem type using the following sample story: "A recipe for chocolate cupcakes uses 3 eggs to make 20 cupcakes. If you want to make 80 cupcakes, you need 12 eggs." Because this situation describes the association between eggs and cupcakes and involves two pairs of associations with the unit ratio unchanged, it is considered a proportion story.

The instructor provided a prompt sheet that contained information about the features of the proportion problem and included four problem-solving strategy steps. Step 1 required identifying the two things that formed a specific association or ratio in the story situation and defining one as the subject and the other as the object. In the sample story described above, "eggs" and "cupcakes" illustrate the ratio relationship. Students learned to identify one as the subject (i.e., "eggs") and the other as the object (i.e., "cupcakes") and to write them in the diagram under the "subject" and "object" dimensions, respectively (see Figure 1). Step 2 consisted of identifying the two pairs of numerical associations (involving four quantities) and mapping the information onto the proportion diagram. Instruction in representation and mapping emphasized the correct alignment of the two dimensions (subject and object) with their corresponding quantities. That is, while the first pair describes the association of "3 eggs" for "20 cupcakes," the second pair describes "12 eggs" for "80 cupcakes" rather than "80 cupcakes" for "12 eggs." Finally, instruction required checking the correctness of the representation by transforming the diagram into a math equation; that is,

$$\frac{3}{12} = \frac{12}{80}.$$

If the equation was not established in their representation, students were instructed to check the accuracy of their mapping (i.e., whether the two pairs of associations were correctly aligned). Instruction also highlighted the importance of the schematic diagram to solve proportion problems.

In the problem-solving instruction phase, problems with unknown information were presented. Students were instructed to first represent the problem using the schematic diagram as they did in the problem schema instruction phase. The only difference was that they used the question mark to flag the unknown. Next, they used Step 3 to transform the diagram into a math equation and solve for the unknown. Instruction emphasized that because the proportion problem schema entails a constant ratio across two pairs of association, the math equation can be derived directly from the diagram. For example, in the problem "A recipe for chocolate cupcakes uses 3 eggs to make 20 cupcakes. If you want to make 80 cupcakes, how many eggs will you need?" the math equation would be

$$\frac{3}{20} = \frac{?}{80}.$$

Students then used cross multiplication to solve for the missing value in the equation. That is, $? = (3 \times 80) \div 20 = 12$. The last step, Step 4, required writing a complete answer on the answer line and checking it. Students not only were taught to check the accuracy of the computation, they also learned to use reasoning and critical thinking to determine whether they correctly paired the quantities of the subject and object (i.e., "3 eggs" for "20 cupcakes" and "? eggs" for "80 cupcakes").

Initially, one type of word problem with the corresponding schema diagram appeared on student worksheets. After students learned how to solve both types of problems, mixed word problems with both diagrams were presented. When mixed word problems were presented, the sameness and difference between the multiplicative compare and proportion problems were discussed to help differentiate one type of problem from another.

General Strategy Instruction Condition

Strategy instruction for the GSI group was derived from that typically employed in commercial mathematics textbooks (e.g., Burton et al., 1998). A four-step general heuristic problem-solving procedure used in this study required students to (a) read to understand, (b) develop a plan, (c) solve, and (d) look back. The instructor employed a Problem-Solving Think-Along sheet to guide group discussion of the four-step problem-solving procedure. For the first step, *understand*, the instructor asked students, "What are you asked to find in the problem?" and "What information is given in the problem?" In addition, students were encouraged to retell the problem in their own words and list the information given to check their understanding of the problem. For the second step, *plan*, several strategies (e.g., draw a picture, make a table, make a model, write a math equation, act it out) were listed on the Think-Along sheet, and students were questioned as follows: "What problem-solving strategies could you use to solve this problem?" Because students in this study commonly selected the "draw a picture" strategy, teacher instruction explicitly focused on modeling use of pictures to represent information in the problem (see Figure 2) followed by counting up the drawings to get the solution. In addition, students were encouraged to use reasoning to predict their answer. For the third step, *Solve*, students had to show their drawing, articulate how they solved the problem, and write their answer in a complete sentence. For the last step, *Look Back*, students were asked to justify whether their answer was reasonable and to indicate whether they could have solved the problem using an alternate method. During the modeling and guided practice portions of instruction, practice worksheets included the four strategy steps. Although worksheets during independent practice did not contain the strategy steps, students were given a separate prompt sheet with the four steps.

Measures

Four parallel word problem–solving test forms, each containing 16 one-step multiplication and division word problems (i.e., multiplicative compare and proportion) were developed for use as the pretest, posttest, maintenance test, and follow-up test. Target problems were designed to include each of several variations of multiplicative compare and proportion problems that were similar to those used during the treatment. Multiplicative compare problems varied in terms of the position of the unknown. That is, the unknown quantity might involve the compared, referent, or scalar function. Proportion problems ranged from some in which the unit value was unknown to some in which the quantity of either one of two dimensions (i.e., subject or object) was unknown. The four tests differed only in terms of the story context and numerical values in the problem. The two types of word problems were presented in a random order on all four tests. In addition, a generalization test that comprised 10 transfer problems derived from commercially published mathematics textbooks and standardized achievement tests (e.g., the *Woodcock-Johnson Test of Achievement*) was designed. The test assessed students' transfer of learned skills to structurally

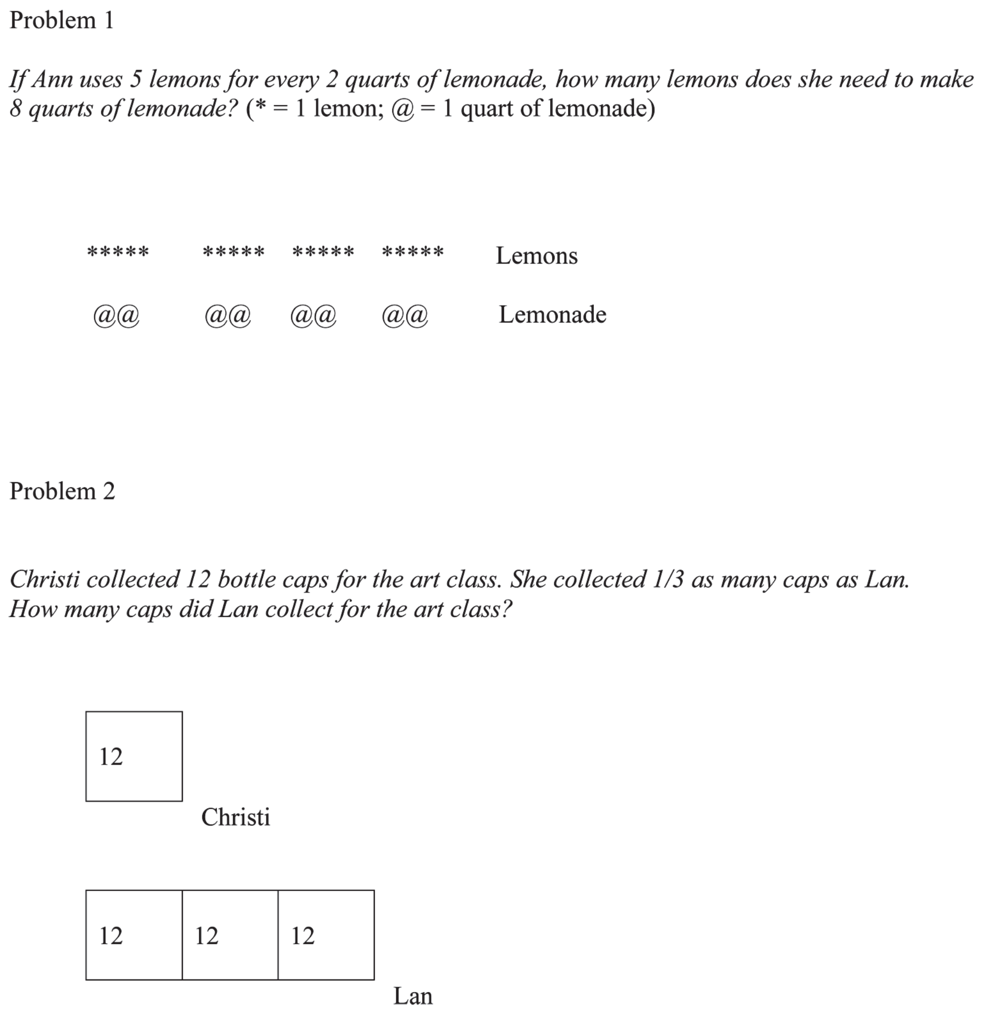

FIGURE 2. Schematic representations of multiplicative compare and proportion problems.

similar but more complex problems (e.g., problems with irrelevant information and multiple steps).

Reliability of the four parallel test forms was established by testing a group of eight sixth-grade students from the same school as the participants in the study. Students were divided into four groups. Each group received each of four forms of testing on four consecutive days. The order in which each group received the forms of testing was counterbalanced to control for the effects of testing sequence. The mean parallel form reliability of the four tests was 0.84 (range = 0.79–0.93). To demonstrate equivalency of the four forms, mean scores were calculated. The scores for forms 1, 2, 3, and 4 averaged 60%, 55%, 63%, and 56%, respectively. In addition, the internal consistency reliability of the generalization test was 0.88 (α).

Testing and Scoring

All testing was conducted in small groups in a quiet room. The instructors had students read each problem and encouraged them to do their best. Students were assisted if they had difficulty reading words on the test. Instructions also required students to show their complete work. No feedback was given regarding the accuracy of their solution or work. All students were provided with sufficient time to complete the tests. Students in both groups completed (a) the pretest and generalization test prior to their respective strategy instruction; (b) the posttest and generalization test immediately following instruction; (c) the maintenance test, with a 1- to 2-week lapse after the termination of instruction; and (d) the follow-up test, with a lapse time ranging from 3 weeks to about 3 months. To ensure that they used their assigned strategy during the maintenance and follow-up testing, students in each condition were provided with a brief review of the respective strategy immediately before the tests.

Items on the word problem tests were scored as correct and awarded 1 point if the correct answer was given. Partial credit (i.e., one half point) was given if only the mathematics sentence or equation was correctly set up. Percentage correct was used as the dependent measure for word problem–solving performance and calculated as the total points earned divided by the total possible points (i.e., 16 for pre- and posttreatment tests and 10 for the generalization test). A graduate student who was naive to the purpose of the study scored all tests using an answer key. A second rater rescored 30% of the tests. Interrater reliability was computed by dividing the number of agreements by the number of agreements and disagreements and multiplying by 100. Mean scoring reliability was 100% for all tests across the two independent raters.

Treatment Fidelity

For each instructional condition, a checklist that contained critical instructional steps was developed to assess the instructor's adherence to the assigned strategy instruction. A doctoral student in special education observed and evaluated the completeness and accuracy of instruction. The adherence of the instructor's teaching to the assigned instructional strategy was judged on the presence or absence of each critical component. Fidelity of implementation was assessed for about 30% of the lessons in both conditions. Fidelity observations were followed by feedback to the instructors, whenever procedural implementation was less than 85% accurate or complete. Overall, fidelity was 100% for the GSI group and 94% for the SBI group (range = 76%–100%).

Results

Given that our sample included a group of students with and without disabilities, we conducted a two-step analysis process in which we first analyzed the data for all 22 participants. To identify potential mediating effects of the presence of a disability, we conducted subsequent analyses of only the data for the 16 students with learning disabilities who completed all tests in the study. Because results of the analyses for the sample of students with learning disabilities revealed the same findings as those for the entire sample of students with and without learning disabilities, we report only the results of the primary analyses for the entire sample of 22 students. Table 3 presents means, standard deviations, and effect size indexes for all pretests and posttests on target and transfer problems for all participants in the two conditions (SBI and GSI).

Pretreatment Group Equivalency

Separate one-way ANOVAs were performed to examine pretreatment group equivalency on target and transfer problems. Results indicated no significant differences between the two groups on either target problems, $F(1, 20) = .237$, $p = .632$, or transfer problems, $F(1, 20) = .736$, $p = .401$.

Acquisition and Maintenance Effects of Word Problem–Solving Instruction

A 2 (group) × 4 (time of testing: pretest, posttest, maintenance test, and follow-up test) ANOVA with repeated measures on time was performed to assess the effects of instruction on students' word problem–solving performance. It must be noted that 2 participants in the SBI group did not complete the maintenance and follow-up tests, and 1 in the GSI group did not complete the follow-up test. As such, this analysis was based on the data for the 9 students in the SBI group and 10 students in the GSI group who completed all four times of testing. Results indicated significant main effects for group, $F(1, 17) = 14.906$, $p < .001$, and time of testing, $F(3, 15) = 33.276$, $p < .001$. In addition, results indicated a statistically significant interaction between group and time, $F(3, 15) = 9.507$, $p < .01$. Furthermore, post hoc simple-effect analyses indicated significant group differences on posttest, $F(1, 20) = 15.747$, $p < .01$; maintenance test, $F(1, 18) = 31.755$, $p < .001$; and follow-up test, $F(1, 17) = 35.032$, $p < .001$, all favoring the SBI group (see Table 3).

Post hoc results based on the data for those who completed all four times of testing using paired-samples tests indicated that the SBI group significantly improved its performance (mean difference = 54.22, $SD = 17.17$) from pre- to posttest, $t(8) = 10.473$, $p < .001$; maintained the improved performance (mean difference = 5.17, $SD = 12.10$) from post- to maintenance test, $t(8) = 1.281$, $p = .236$; and further improved its performance (mean difference = 9.56, $SD = 12.52$) from posttest to follow-up test, $t(8) = 2.290$, $p = .051$. The GSI group improved its performance (mean difference = 17.590, $SD = 12.91$) from pre- to posttest, $t(9) = 4.791$, $p < .01$, and maintained the improved performance (mean difference = −2.94, $SD = 10.29$) from post- to maintenance test, $t(9) = -.903$, $p = .390$ and to follow-up test (mean difference = −4.37, $SD = 19.22$), $t(9) = -.719$, $p = .490$.

Transfer Effects of Word Problem–Solving Instruction

A 2 (group) × 2 (time of testing: pretest and posttest) ANOVA with repeated measures on time was performed to examine the two groups' performance on the generalization test. Results indicated that the main effect for group, $F(1, 19) = .054$, $p = .819$, was not significant. However, there was a significant main effect for time, $F(1, 19) = 18.465$, $p < .001$, and a statistically significant interaction between group and time, $F(1, 19) = 8.579$, $p < .01$. Post hoc paired-samples tests indicated that the SBI group significantly improved its performance (mean difference = 36.97, $SD = 24.82$) from pre- to posttreatment, $t(10) = 4.940$, $p < .01$, whereas the GSI group's performance on the generalization test did not show a statistically significant change (mean difference = 7.00, $SD = 21.76$) from pre- to posttreatment, $t(9) = 1.017$, $p = .336$.

TABLE 3. Percentage of Correct On-Target and Transfer Problems by the SBI and GSI Groups

	M		*SD*		*n*		
Test	SBI	GSI	SBI	GSI	SBI	GSI	ES[a]
Pretest	25.19	29.85	22.52	21.36	11	11	–0.21
Posttest	79.41	47.55	13.92	22.70	11	11	+1.69
Maintenance	87.29	45.45	14.51	17.97	9	11	+2.53
Follow-up	91.68	46.06	13.79	19.04	9	10	+2.72
Gen. pretest	25.45	35.00	29.11	22.69	11	11	–0.37
Gen. posttest	62.43	45.50	21.52	15.89	11	10	+0.89

Note. SBI = schema-based instruction; GSI = general strategy instruction; ES = effect size; Gen. = generalization.
[a]Effect size was calculated as the two conditions' mean difference divided by the pooled standard deviation (Hedges & Olkin, 1985). A positive ES indicates a favorable effect for the SBI condition; a negative ES indicates a favorable effect for the GSI condition.

Discussion

The present investigation compared the differential effects of schema-based and general strategy instruction on the mathematical problem–solving performance of middle school students with learning problems. Results showed that students in the SBI group performed significantly better than students in the GSI group on all measures of acquisition, maintenance, and generalization. These findings support and extend previous research regarding the effectiveness of schema-based strategy instruction in solving arithmetic word problems (e.g., Hutchinson, 1993; Jitendra & Hoff, 1996; Jitendra et al., 1998, 1999, 2002).

In general, results of this study indicated significant differences between the SBI and GSI groups on the posttest, maintenance, follow-up, and generalization tests. The effect sizes comparing the SBI group with the GSI group were 1.69, 2.53, 2.72, and .89 for posttest, maintenance, follow-up, and generalization tests, respectively. These effect sizes are much larger than the effect sizes reported in the Jitendra et al. (1998) study (.57, .81, and .74 for acquisition, maintenance, and generalization, respectively). In that study, elementary students with mild disabilities or at risk for mathematics failure learned to use schema diagrams to represent and solve addition and subtraction problems (i.e., change, group, and compare). After students represented the problem using schema diagrams, they had to figure out which part in the diagram was the "total" or "whole." Next, they had to apply a rule (i.e., "When the total [whole] is not known, we add to find the total; when the total is known, we subtract to find the other [part] amount"; p. 351) to decide whether to add or subtract to solve the problem. It might be the case that the schema diagrams for multiplication and division problems (i.e., multiplicative compare and proportion) in our study made a more straightforward link between the problem schema representation and its solution than those in the Jitendra et al. study and that errors were minimized once students correctly represented the problem in the diagram.

Although these findings confirm prior research on schema-based instruction by Jitendra and colleagues, an explanation regarding the more positive findings in our study when compared to previous research on semantic representation only (Lewis, 1989; Zawaiza & Gerber, 1993) is warranted. In the studies by Lewis and Zawaiza and Gerber, the diagram strategy was effective in reducing students' reversal errors, but it did not improve their overall word problem–solving scores. Participants in those studies were taught to use the diagram (i.e., a number line) as an external visual aid to check the operation for the purpose of preventing reversal errors.

Unlike the Lewis (1989) and Zawaiza and Gerber (1993) studies, this study used a schema-based instruction to systematically teach the structure of different problem types and directly show the linkage of the schematic diagram to problem solution. An examination of students' pretest performance in both groups indicated a lack of conceptual understanding: Students typically grabbed all the numbers in the problems and indiscriminately applied an operation to get the answer, regardless of the nature of the problem. Following instruction in the assigned strategy, most students in the GSI group drew pictures to represent information in the problem and then counted their drawings to get the solution. However, when numbers in the problems got larger or the problem relation was complex (e.g., the scalar function in a multiplicative compare problem was ⅔ or ¾), students found their drawings and counting to be cumbersome, and their work was prone to errors. In contrast, the performance pattern of students in the SBI group reversed following instruction. Those students used higher-order thinking, such as identifying problem structure or type and applying schema knowledge to represent and solve problems.

The intensive training in problem structure in the current study may have contributed to students' conceptual understanding and maintenance of word problem–solving skills. At the same time, it should be noted that students in both the SBI and GSI groups were reminded about the assigned strategy before completing the maintenance and follow-up tests. Specifically, students in the SBI group were shown the two schemata diagrams and asked to use them to solve problems. Students in the GSI group were provided with a review of the four-step strategy of reading to understand the problem, drawing a picture to represent the problem, solving the problem using the selected strategy ("draw a picture"), and looking back to check the solution. This review ensured that students in both groups used the assigned strategy and served to validate the differential effects of the two problem-solving strategies on students' performance. The further boost in students' performance in the SBI group on the follow-up tests compared to the posttests may be attributed to the coherent representation of the word problem and subsequent internalization of the schema-based strategy that was lacking in the general strategy. In contrast to the findings regarding maintenance in our study, only four of the six students in the Zawaiza and Gerber (1993) study maintained their posttest performance. One plausible explanation for the large effects found in our study is that participants in the SBI condition systematically learned the problem schemata and problem-solving procedures in twelve 1-hour sessions. However, community college students with learning disabilities in the Zawaiza and Gerber study received semantic structure representation training to solve compare problems during two 35- to 40-min sessions only.

The results of this study also indicated that only the SBI group significantly improved their performance on the generalization measure after the schema-based instruction. This finding confirms previous research (e.g., Hutchinson, 1993; Jitendra et al., 1998, 1999, 2002; Jitendra & Hoff, 1996), in that students in the SBI group transferred the learned skills to new tasks that included structurally similar but more complex problems when compared to the target problems. It may be that the emphasis of the schema strategy on conceptual understanding of the problem structure in conjunction with the diagram mapping helped students differentiate relevant from irrelevant information during problem representation and planning to accurately solve novel problems (Schoenfeld & Herrmann, 1982).

In summary, findings document the efficacy of the schema-based instruction over general strategy instruction in enhancing problem-solving performance for middle school students with learning difficulties. Given that "learners' 'true' math deficits are specific to mathematical concepts and problem types" (Zentall & Ferkis, 1993, p. 6), this study provides further support for schema-based instruction in enhancing students' conceptual understanding of mathematical problem structures and problem solving in general.

At the same time, several limitations of the study require cautious interpretation of the findings. First, due to missing data in school records, we did not have complete descriptive information for all participants. This presents problems in terms of accurately identifying the sample in the study, a common struggle that researchers encounter when conducting applied research in the classroom. Second, the participant sampling procedure in this study did not control for students' reading levels. Reading comprehension is an important contributing factor to students' word problem–solving performance (Zentall & Ferkis, 1993). As such, it is not clear to what extent reading comprehension skills contributed to the findings in this study. While we ensured that the two groups' entry skills with respect to problem-solving skills were comparable, it is important that future research investigate the effects of the two instructional strategies while controlling for students' reading skills.

Third, the large standard deviation scores indicate great variation in pretest performance within each group on both target and transfer problems. Pretest scores for students in both groups ranged from 0%–68% correct. Therefore, future research should employ more homogeneous groups (e.g., students with learning disabilities, students with mathematics disabilities only) and increase the sample size to examine the differential effects of strategy instruction.

Fourth, the "pull-out" nature of instruction employed in this investigation may be a limitation. Because instruction did not occur during the regularly scheduled math period in the school, there is a possible disconnect between the strategy instruction provided in this study and regular classroom instruction. It is important for future research to examine the effects of schema-based instruction in regular classroom settings and to facilitate students' broad application of the strategy. A fifth limitation is the use of standard text-based word problems rather than real-world problem-solving tasks. An area for future research is to investigate the effects of the schema-based instruction to solve real-world problems. Finally, our study did not address the relative efficacy of the schema-based strategy when compared to problem-solving treatment procedures that employ manipulatives and other empirically validated strategies (e.g., cognitive–metacognitive strategy) described in the literature (Jitendra & Xin, 1997).

Implications for Practice

Overall, findings from this study have several implications for practice. First, the effectiveness of the schema strategy instruction in this study suggests that classroom instruction should emphasize systematic domain-specific knowledge in word problem solving to address the mathematical difficulties evidenced by students with learning disabilities (Montague, 1992, 1997). Schema-based instruction teaches conceptual understanding of problem structure, which facilitates higher-order thinking and generalizable problem-solving skills. Although the "draw a picture" strategy in the GSI condition emphasized understanding of the problem, the representation step of the strategy focused more on the surface features of the problem and did not allow students to engage in the higher-level thinking necessary to promote generalizable problem-solving skills. The general heuristic strategy, such as the four-step approach (e.g., read, plan, solve, and check) typically found in commercial mathematics textbooks may be "limited unless it is connected to a conceptual knowledge base" (Prawat, 1989, p. 10). One of the key differences between the schema-based strategy and traditional instruction is that only the former emphasizes pattern recognition and schema acquisition. The schema-based strategy in this study provided students with explicit instruction in problem schemata and problem solving.

Second, the effectiveness of the schema strategy instruction in this study suggests that students with disabilities are able to learn strategies involving higher-order thinking to improve their problem-solving skills. Many students with learning disabilities are often cognitively disadvantaged because of attention, organizational, and working memory problems (Gonzalez & Espinel, 1999; Zentall & Ferkis, 1993). Furthermore, they experience difficulties in creating complete and accurate mental problem representations (Lewis, 1989; Lewis & Mayer, 1987; Marshall, 1995). It is essential that teachers provide students with learning disabilities with scaffolds, such as schemata diagrams, when teaching conceptual understanding of key features of the problem. The schematic representation should be more than a simple semantic translation of the problem and should emphasize the mathematical relations in specific problem types to allow students to directly transform the diagrammatic representation into an appropriate math equation. Such representations may be useful aids to organize information in word problems, reduce students' cognitive load, and enhance working memory by directing resources to correctly set up the math equation and facilitate problem solution.

Overall, the findings of this study indicate the effectiveness of schema-based instruction in enhancing word problem–solving performance of middle school students with learning disabilities. Given that an increasing number of students with disabilities are currently served in general education classrooms (Cawley et al., 2001), providing them with effective strategies to access the general education curriculum as mandated by the amendment to the Individuals with Disabilities Education Act (IDEA, 1997) is critical. Schema-based instruction, with its emphasis on conceptual understanding, facilitates higher-order thinking and may be an effective and feasible option for teachers. It provides students with a tool to be successful problem solvers and to meet the high academic content standards. This has particular importance given current legislation's emphasis on "scientifically-based instructional programs and materials" (No Child Left Behind Act, 2002).

AUTHORS' NOTES

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