The CAS classroom

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The Victorian Curriculum and Assessment Authority (VCAA) Computer Algebra System (CAS) Pilot study (2001–2005) is monitoring the use of CAS in senior secondary mathematics. This paper explores my experiences in the CAS classroom and delineates changes in my teaching style, as a result of the introduction of CAS into my senior mathematics classrooms at Ballarat Grammar School. It also explores reactions from colleagues teaching with CAS and reactions of students in the CAS classroom. Handheld CAS calculators that are able to perform algebraic as well as graphic and numeric calculations have been assumed in teaching, learning and assessment at Ballarat Grammar from 2000.

The introduction of CAS into Victorian schools

CAS stands for Computer Algebra Systems, which, in 2002, were introduced into the Victorian Certificate of Education (VCE) external examinations for the subject Mathematical Methods CAS. The systems are available on handheld calculators or are computer based. The more familiar CAS used in Victoria are the handheld Texas Instruments TI89, TI92+ and Casio FX 2.0+, and the computer based *Derive* and *Mathematica*. CAS in the classroom allows teachers and students alike to explore algebraic functions in ways that were previously unattainable at secondary school level, and allows teachers to introduce mathematics in different ways. Students can solve, factorise, differentiate and integrate functions with a CAS, hence, they are not limited by their by-hand algebra skills.

The new VCE subject, Mathematical Methods CAS Units 1–4, was accredited in February 2001 (www.vcaa.vic.edu.au/vce/studies/MATHS/caspilot.htm). The pilot implementation initially involved three volunteer schools and commenced with Units 1 and 2 in Year 11 in 2001, followed by Units 3 and 4 in Year 12 in 2002. Units 3 and 4 were externally examined in November 2002. The pilot program has been progressively expanded, with students from 16 schools enrolled in Mathematical Methods CAS Units 3 and 4 in 2003. The

numbers of students enrolled in the new subject will increase over the years from 2002 to 2005, until the next accreditation period starting in 2006. The initial stage of the pilot (2000–2002) proceeded in conjunction with an Australian Research Council grant and involved a partnership between the Department of Science and Mathematics Education at the University of Melbourne, the Victorian Curriculum and Assessment Authority (VCAA), and three calculator companies, Casio, Hewlett-Packard and Texas Instruments (www.edfac.unimelb.edu.au/DSME/CAS-CAT). The VCAA continued and expanded the pilot, with the decision made at the VCAA Board meeting (October 2003) that:

> …the current VCE Mathematics study structure and relationship between VCE mathematics studies continue for the next accreditation period, with the inclusion of Mathematical Methods (CAS) as a parallel and alternative study to Mathematical Methods, available for all schools from 2006. Thus, from 2006, Mathematical Methods (CAS) Units 1–4 can be implemented by all schools at a time suitable to them. (VCAA, 2004, p. 1)

The expanded VCAA pilot is investigating the use of CAS in all aspects of teaching and learning in the VCE mathematics classroom. Ballarat Grammar School, where I teach, was one of the three original project schools and is now part of the VCAA expanded pilot.

Prior to the Mathematical Methods CAS subject, work with CAS in Victorian schools was based on existing curriculum and assessment structures. CAS use was permitted in coursework and internally assessed work, but external examinations were conducted without CAS. As a result, teachers needed to balance developments in pedagogy that were stimulated by the use of CAS against assessment structures that did not allow CAS technology. This led to the tendency for teachers, myself included, to ignore CAS capabilities, in the pressure of preparing students for examinations. In contrast, the subject, Mathematical Methods CAS, assumes CAS in all aspects of internal and externally assessed material, therefore provides the opportunity for the extended use of Computer Algebra Systems.

As the teacher heading up the initial and expanded pilot projects at Ballarat Grammar, I have observed many changes in my CAS classroom, and have unwittingly changed my teaching style as a result of using CAS in all teaching and assessment. The shift to new pedagogies with CAS has also been reported by others (see Ball, Stacey & Leigh-Lancaster, 2001, Leigh-Lancaster 2002, Tynan, 2003) and is explained by Asp and McCrae (2000):

> This widespread availability of symbolic manipulation by computer algebra… will result in changes to existing curricula, assessment and teaching styles because it challenges the algorithmic algebra and graphing that forms the central thread of secondary mathematics. (p. 138)

My colleagues and I have discussed changes to existing curricula, assessment and teaching style, and, as well, I have shared my reactions with the

wider educational community (e.g., Garner, 2002, 2003a, 2003b; Garner & Leigh-Lancaster, 2003). I have been interested to learn of the similarities, as well as the differences, experienced in this journey of change. The responses of the students who are sharing this learning process with me have also been fascinating. It is in these students' reactions and their quick uptake of the facilities of CAS that we see where the future of this technology lies.

So, in summary, in this paper I report on some of my own experiences, the responses to the pilot of two colleagues, and selected responses of students. I draw heavily on interview data, from when I was interviewed as part of the CAS-CAT project. The paper is in six sections. A short section on the Mathematical Methods CAS subject follows this introduction. The other sections are headed:

- My experience: Teaching with CAS
- Colleagues' experience: Teaching with CAS
- Student experience: Learning with CAS
- Where do we go from here?

What is this new subject?

The new VCE subject in Victoria is called Mathematical Methods CAS Units 1–4. Units 1 and 2 are traditionally covered in the first year of the VCE (Year 11), while Units 3 and 4 are traditionally covered in the second year of the VCE (Year 12), with corresponding external examinations. The subject Mathematical Methods CAS Units 1 and 2 is the most difficult algebraic mathematics subject taken by Year 11 students, while Mathematical Methods CAS Units 3 and 4 is the algebraic 'middle' between the easier Further Mathematics subject and the more difficult Specialist Mathematics subject: the CAS course includes Calculus and its associated algebra, but not at the difficult level of algebra that is required for Specialist Mathematics. A proportion of the Mathematical Methods CAS students also concurrently study Specialist Mathematics. Hence, the use of CAS in all aspects of teaching, learning and assessment adds a subject that not only uses CAS, but also introduces new and changed material. From my experience, two teaching techniques emerge side by side in this new subject, that of:

- teaching old content in new ways, and
- teaching new content, some of which can be remembered from Year 12 curriculum in previous years.

My experience: Teaching with CAS

Involvement by Ballarat Grammar School in the VCAA pilot study and the CAS-CAT project was the catalyst for us to consider issues associated with the use of CAS in the classroom. In the first phase of the pilot study it was found that teaching and learning in the senior mathematics classroom changed substantially when students used CAS calculators on an everyday basis.

However, it is interesting to ask whether teaching with CAS necessarily entails significant new pedagogy rather than an extension of existing approaches. Leigh-Lancaster (2002) writes:

> CAS use naturally draws teachers into further development of discipline knowledge, especially where they are used in the course of inquiry to support general analysis of particular aspects of mathematics. This has certainly been a positive reflection of teachers involved in the professional development workshops for the pilot, and teachers involved in the initial phase of the pilot study have reported similar effects anecdotally for students. (Leigh-Lancaster, 2002, p. 26)

Some intriguing evidence is emerging from the CAS-CAT project and the VCAA pilot study:

> Teachers will choose to highlight attributes of CAS which support their own beliefs and values about mathematics. Teachers who value routine procedures can find on a CAS a plethora of routine procedures to teach students; teachers who value insight can find many ways in which they can demonstrate links between ideas better than ever before. (Kendal and Stacey, cited in Stacey, Asp & McCrae, 2000, p. 248)

The following extracts of my interview, as teacher, in the CAS-CAT project, when I was reflecting on a sequence of lessons towards the end of 2002, highlights the above quote.

Interviewer: By teacher-led you mean you direct the students?

Teacher: Yes, yes. I think that's just who I am … So when you've got this technology its going to affect my teaching in a totally different way than its going to affect someone else because we've got different personalities and I think we'll see that across the project. But the fact that they [the students] voted that they were going to do it [a skills question] in three different ways, it surprised me but I was **absolutely** delighted. And I said to them. 'This is **terrific**'. Because what this means now is that teaching has moved from me telling you what to do into you making your own decisions that suits you … and the way you think about things. And that each was equally valid. (Bold indicates emphases given verbally during the interview)

The context of the quote was a lesson teaching a circular functions question, where students would be faced in an analysis exam with the choice of solving a trigonometric equation in three ways: that of using a graphical, numerical or algebraic solution. I would say that in this particular experience I was able to demonstrate links between ideas better than ever before.

Towards the end of the CAS-CAT Project, the four teachers involved discussed their overall experience. It emerged that we agreed the success of a CAS-active curriculum 'sinks or swims' with the teacher. We also stated the

importance of one teacher in a school having an overview and teaching at both Year 11 and Year 12 in the CAS subject. Success also requires reflective practice on behalf of the teachers involved. Another extract from my interview as a teacher in the CAS-CAT project illustrates this point of reflective practice.

Interviewer: So what criteria would you have been using at the beginning that you changed to now?

Teacher: I preferred by-hand solutions … Whether that's security, and I know I'll get it right … Security, safety, and familiarity in doing what I've always done … The students and I have moved to a feeling that the calculator gives you something that has got an intuitive feel to it, rather than just churning out a result that you use with no understanding.

Interviewer: Is the CAS result triggering a mathematical learning situation?

Teacher: Yes, yes. And I didn't think it was going to. I thought it was going to be just 'This is just an answer, and manipulate it, and don't understand'. And the fear is all the time that you're going to lose your algebraic skills … I don't think that's the case because I think the answer means so much more when you've got the algebraic understanding underneath what's happening.

A question I found myself asking early in the CAS-CAT project was: to what extent, and in what ways, are changes in teaching, stimulated by the use of CAS, a function of values and previous experiences of the particular teacher? There appeared to be a new direction in student and teacher behaviour, especially at the stage when CAS technology was first introduced, but the questions is whether that experience will be shared by all?

The early journey

For me, teaching with CAS appeared to engender, unexpectedly, a new way of teaching. In looking for a way to articulate my experience I have described (Garner, 2003a) my teaching as teaching the ends and sides of a topic (TENDSS) rather than proceeding in a linear fashion. Extracts from my interview as part of the CAS-CAT project illustrate the idea of TENDSS.

> If I'm going to start a topic, previously I would probably have started it by what you might call linearly. I would start from the beginning and go step by step to the end. Now I don't at all. If I'm going to start a topic, I very often start at the end. (Garner, interview, 2002)

> With the Year 11s I talked about instantaneous rate of change and then I looked at it on the graph. I looked at tangents. I traced the gradient. I looked at gradients in tables. I looked at gradients in the RUN menu and looked at

> gradients in the CAS menu. So I would have jumped to what might be perceived to be the end, to give an understanding of where we're going. It's almost like you don't need to do the rules as such any more. It's interesting isn't it? The rules just develop as you go along. The understanding of the rules and what you need develop amidst it. I don't feel I need to go step by step any more. And I think the kids have a better understanding of it. (Garner, interview, 2002)

In starting with the CAS-CAT project, I expected an interesting and valuable professional experience, not one that would stimulate my thinking in the way that it has. Nor did I expect it to turn into the reflective, learning experience that it has. I have said previously:

> Rather than a slow change in methods taught, there seems to have been a roller coaster ride of fresh insights and new experiences [for me], and a huge level of new knowledge gained, both in calculator use and in the mathematics itself. (Garner, 2003a)

The journey continues

The dynamics of my classroom have changed with the calculator viewscreen a central focus. The viewscreen has been evident in Victorian classrooms for a long time, but in my experience the use of CAS has been the catalyst for us at Ballarat Grammar to have it available in all our senior Mathematics classrooms. Teaching with the viewscreen allows an easy exploration of functions at all levels of difficulty. An unexpected by-product of the viewscreen in the classroom at all times is the ease with which students can demonstrate their working to others.

Another outcome is that multiple representations (i.e. numeric, graphic and symbolic forms) are more easily demonstrated in the CAS classroom. It is hoped also that the students move more easily between these representations because of exposure to them, and thereby develop a deeper and broader understanding of the concepts. A common experience of Victorian teachers is that in the early days with the introduction of graphic calculators, there was an explosion in dealing with the graphs of functions previously not covered in VCE Mathematics. With the introduction of CAS, these functions can now be explored from a numeric, graphic and symbolic point of view, thus completing the continuum of multiple representations of functions (see Garner, McNamara & Moya, 2003). It is in this multiple representations of functions that teaching the ends and sides rather than teaching linearly have become evident in my classroom.

The following are more excerpts from my interviews with the CAS-CAT project where I talked about using the various representations and using VARS, a calculator memory, to store functions in algebraic form:

Interviewer: Would it make any difference which representation you're using?

Teacher: No, I flip between them all the time. That's what's changed for me and what I mean about the holistic stuff.

Teacher: There probably would have been lessons earlier in the year where I would have said, 'Let's actually look at this because this is going to be useful.' But I find, if I teach a bit of technology, a bit about the calculator that is out of context, they're actually not interested. They're only interested in looking at it when a problem comes up as we go along. So … the thing about VARS to store the function … When I showed it to them they said, 'Ho hum, yeah that's fine'. But, during the application task, I could hear them all saying 'Use VARS, its much better. Use VARS to store the whatever, then it'll be in the GRAPH.' And I found out, marking last night that they've actually adopted it and used it extremely well. (Garner, interview, 2002)

Ball, Stacey and Leigh-Lancaster (2001) describe this aspect of CAS use: they expect teachers to find that students will not be particularly interested in the capabilities of the calculator until a specific need for their use arises.

I have also found that teachers need to be flexible while students are discovering, often more quickly than the teacher, new ways to tackle a problem. As Tynan (2003) succinctly states:

> We were often surprised how quickly students learnt the syntax, and adapted to most of the idiosyncrasies of the calculator operations. Often students delighted in revealing to the class another discovery that they had made about a short cut or new method, or explain(ing) why a particular procedure might not work. (p. 256)

Comments from a group of my Year 12s in 2003 illustrate the points that Tynan makes. They relate to a revision class on exponential and logarithmic functions:

Student A: We're on a roll here! Change of base is wonderful.

Student K: Is it cheating?

Student A: How can I do log to the base 10?

Student K: It's OK; this is just used to get it on the calculator so we can use it for base 10 as well as *e*.

Student A (to another student): I just did change of base and just plugged it into the calculator. I am hooked on it now!

(Students K and A, August 2003)

Interestingly, change of base for logarithms is not part of the VCE Mathematical Methods course, but I have taught it in the last three years to get around the fact that the calculator only deals with base 10 and *e*. I would never have considered teaching this method, before using CAS.

The question of 'by-hand' skills

The question I am most frequently asked, in the educational and wider community, is 'what happens to the students' by-hand skills?' The same question can be asked about numerical long division as it can be asked about polynomial long division, and teachers, by the very nature of their profession will differ in opinion. Use of CAS in the classroom, and its impact on by hand skills, is expected to be a 'catalyst for debate over the future goals for mathematical proficiency in the CAS age' (Flynn, Berenson & Stacey, 2002, p. 7). Dunham (2000) clearly articulates the substantial discussion of controversial issues in graphics calculator and CAS research. She gives a response to what, for many, is an important issue surrounding the introduction of any new technology into mathematics education:

> Students taught with technology, but tested without it, perform as well or better than students taught with more traditional methods. (Dunham, 2000).

> Students in my classes have performed as well or better than their non-CAS peers after being taught with CAS technology, and tested without it, on important algebra and calculus content (Garner, 2003a).

Colleagues' experience: Teaching with CAS

The question has been asked: 'to what extent, and in what ways, change in teaching approaches stimulated by the use of CAS is a function of the beliefs, values and preferences of the particular teacher?' (Garner & Leigh-Lancaster, 2003, p. 374).

It has been suggested that:

> The multiple representation capability of CAS will lead to a diversity of teaching methods … [Is] there a privileging of teaching style that is most advantageous for students? (Asp & McCrae, 2000, p. 140)

Is there a way of teaching that privileges the use of CAS, or is it merely assumed that an excellent teacher will be an excellent teacher in all contexts with all technologies? And does any teacher's own pedagogy ultimately reflect their own value preferences? The case of two teachers highlights this dilemma.

The case of Teacher X

Teacher X has taught Mathematical Methods CAS Units 1 and 2 for two years, but in the third year opted out of VCE CAS Mathematics teaching. Teacher X is a most experienced mathematics teacher, and his decision not to teach this course anymore is a loss to the VCE students at this school. Teacher X has made the decision to leave the CAS teaching to his colleagues:

> I'm totally supportive of CAS Maths — this is the way maths is going and I particularly appreciate the fact that CAS enables students with unreliable skills, but good understanding, to continue to use maths as a tool for problem solving. The only reservation about CAS Maths was … we have asked too much of our Year 11 students i.e. too big a jump, too early … Whilst the Year 12 results last year proved that an outstanding job was done for the best of the mathematicians, I wonder whether there were some of the less able students who could have lasted the distance with a gentler approach. (personal communication, 2003)

Teacher X, however, is using his experience of teaching with CAS in the VCE. His Year 10 students have, on loan, the Casio FX 2.0 while they are learning Year 10 algebra. His comment to me via email, printed below, shows evidence of his move towards using CAS in all aspects of his teaching, despite the ideological dilemma he faces.

> P at the start of the year had very weak algebraic skills but has become a CAS junkie … P is a good example of how CAS gives students with weak algebra skills, the ability to cope with Methods — I think this is a good thing?

The case of Teacher Y

Teacher Y is an experienced VCE Mathematics teacher and has shown a willingness to learn the use of the CAS while actually teaching the Mathematical Methods course for the first time in six years. There is flexibility in this teacher's approach that openly reveals to students and colleagues the learning process of using the calculator. The approach that Teacher Y uses requires a confidence in revealing her vulnerability as a learner, and a willingness to visit colleagues' mathematics classes. In the culture of some schools, where the teacher's domain and classroom is sacrosanct and private, this approach would not be successful. In an interview with me in mid-2003, Teacher Y said of the experience of teaching with CAS:

> It allows the fun back into Maths. Before we were always restricted by the algebraic techniques that we, and the students, could do by-hand and we had to stick to that. Now, sometimes I even don't know the answer, but in the exploring we get to find out some wonderful things!

This freedom of approach reflects my own experience:

> In teaching topics with the students freely using CAS, it was immediately obvious that the approach would be different … It seems that the students can 'own' the material far earlier than in previous years without this technology. (Garner, 2002, p. 393)

Student experience: Learning with CAS

Students during the 1990s in Victoria will have experienced the importance of word processing and computer-generated graphs in the days of the Common Assessment Tasks (CATs). My 1996 study (Garner, 1997) of Year 12 students completing CAT 1 (Common Assessment Task 1) in Mathematical Methods Unit 3 describes the stress students experienced in handling the differing levels of mathematical and technological competence in teachers and fellow students alike. The following extract is from a student describing her experience:

> I hated it. I didn't hate it to start with and I expected it to be a challenge. But I wasn't happy with my work at the end … I was stuck on Part d for 2 weeks. I have never cried about any school till now but we all cried … I wasted 2 hours with a tutor, but he had no idea. And I could have spent those 2 hours seeing what I did wrong. The tutor suggested Derive…I couldn't tell him I … (didn't) … have access to Derive. In the Cat I was able to use Derive to do Part d. I thought I knew what I was doing but I didn't get part e done …unbelievably awful experience. (Garner, 1997, pp. 47–48)

This situation has changed for current students in School Assessed Coursework (SACs), which have replaced the CATs. In SACs, the emphasis is on hand written solutions, using a handheld technology (CAS or non-CAS). CAS on computers (Derive5 and Mathematica) is also available to VCE students as part of the VCAA Pilot Project. Derive and Mathematica have been used in some classes for years (for the non-CAS subjects), but their use remains quite uneven across teachers and schools. The issue with SACs is not what technology is used, but the efficient use and selection of the aspects that the particular calculator or computer program provides, within the shorter timeframe allowed for the SACs.

The use of CAS in SACs and other internal assessments causes new stresses and new challenges. The following example relates to a 2002 Year 12 CAS student:

> Initially in the project I had expected that simple questions at the beginning of typical Maths Methods exam question would be rendered redundant by the use of CAS in assessments (see Flynn, 2001). Asking for the derivative of an expression like $\sqrt{x^2-1}$ would seem trivial with the use of a CAS that can directly give the answer. The question changed to 'Write down an expression for the derivative of $\sqrt{x^2-1}$' may still appear trivialized. However Student K, in a SAC test, entered the brackets of this expression incorrectly and discovered that her calculator gave her the answer signum(x). She panicked and even though this was unfamiliar to her, wrote this answer down verbatim. She naturally had difficulty answering the subsequent parts of the question. (Garner, 2002, p. 398)

An efficient use of CAS in SACs can impact on students' confidence and

achievement in that assessment task. Other student responses and outcomes that I have observed include students who are using CAS in all classes develop a personal relationship with their CAS, and praise or blame it as a 'person-object' accordingly. One student wrote in his journal:

> During our time together, the CAS calculator and myself have held very much of a love/hate relationship — there are many things about the CAS which are extremely useful, yet other things which are very frustrating and misleading. (Garner, 2002, p. 390)

Another was recorded in focus groups as saying: 'I still want to marry my CAS' (Student M, 2003).

A student's expectation of the traditional form of an algebraic expression or equation has emerged in discussions from the beginning of the project. The students also appear to be able to deal with more complicated algebraic expressions than ever before, as they are used to dealing with the algebraic steps in handling expressions on their calculator. However, the most efficient use of the calculator arises with the student who is confident about the results gained according to a framework of algebraic knowledge that already exists. Two eloquent quotes from Year 12 students during class discussion on using CAS display this idea:

> I think to use the CAS effectively, a sound knowledge of algebra and any processes such as differentiation and integration that will be used is needed, lest the student be confused or put off track by the dissimilar and sometimes quite complicated answers produced by the calculator's algorithms. (Student A)

> It can't (usually) be used to solve problems that the user doesn't have an algebraic understanding of…the CAS may be used in the gaining of that understanding. (Student C)

These comments are in line with the idea of the CAS 'scaffolding students as they undertake unfamiliar processes' (Stacey, Ball, Asp, McCrae &Leigh-Lancaster, 2000, p. 62). In my observations it is the students and teachers who have no experience of using the CAS in the classroom who are critical and fearful of a student losing algebraic skills. Teaching with CAS, appears to be a whole new way of teaching, rather than an adjunct that will spoil previously honed skills or stop the gaining of new ones.

One unexpected early outcome of the CAS-CAT project was the student for whom the use of CAS allows the step into the world of formal symbolic mathematics. Another excerpt from my CAS-CAT interview demonstrates this point:

> I asked him (a student), 'Can you do maths better this year?'
> He said to me, it was one of those crowning moments: 'I can do maths. Last year I couldn't do it. I nearly didn't do Methods… This year I can do it'.

> I said to him, 'Do you mean that you are using your calculator to do the maths instead of the maths. Is that what you are saying to me?'
> And he said, 'No, I understand what you're talking about and I am so excited. And I'm getting 8/10 for my SACs and I've never, ever been able to do maths before.'
> And he's just a changed boy. It's not because he's using the calculator all the time. It's given him security to jump the fence to say, 'I can now do the maths'… it's sort of like it [the CAS] has empowered them to do maths when they thought they couldn't. That still keeps surprising me.

Since early in the pilot project, I have observed two clearly different groups of students:

1. mathematically-able students who naturally integrate CAS into their mathematical framework (see Pierce & Stacey, 2002); and
2. students for whom CAS helps them jump the hurdles that arise with formal symbolic mathematics, and enables the step into senior mathematics (see Garner, 2002).

Similar categories have continued to be evident in subsequent groups of students. However, contrasting with the positive responses, one of the differences that I noticed in 2003, as compared with 2002, is that for the student who has adopted a full and complete use of CAS, there appears to be a stage where the CAS can take over, perhaps reducing algebraic flexibility. In a Year 12 class in 2003, students were heard to say: 'I don't know how I would manage without my CAS' (Student A).

For student M (I can do Maths) discussed in the previous quote: 'I am really struggling with these trig equations. I had to bite the bullet, and do these by-hand using the Unit Circle, to fully understand' (Student M).

As teachers 'on the ground' we must continue to monitor these changes in our own mathematics classrooms.

In schools involved in the pilot and the project, it has been found that, generally, after the initial hesitation in CAS calculator use, students used CAS in a powerful way. Tynan (2003, p. 258) explores the transition in students' attitudes, describing changes from early students' responses to later students' responses as students moved through Units 1–4 across Year 11 and 12. In summary he describes the early frustration of students making way for a clearer definition of problems; the change from inadequate use of brackets to effective, and even overuse, of brackets; from single steps in CAS computations to multiple steps; and from the use of calculator syntax in written solutions to more cohesive and mathematically accurate solutions. Leigh-Lancaster (2002) writes:

> CAS provide students with a powerful tool for undertaking their own mathematical inquiry in relation to areas involving the study of coordinate geometry, functions, algebra and calculus, in a similar way to which dynamic geometry systems can be used to support mathematical inquiry in classical geometry and affine transformations of the plane. (Leigh-Lancaster, 2002, p. 25)

Where do we go from here?

The use of CAS in internal and external assessment tasks for the VCE has changed how my students view and do their mathematics. It has also changed how I view my teaching. As Leigh-Lancaster (2000) has observed, the congruence of CAS use in pedagogy, curriculum and assessment leads to changes in teaching and learning in senior mathematics, which is certainly my experience.

The Victorian CAS pilot study is unique in its composition, in comparison with similar international studies:

> Several systems have responded to the emergence of readily accessible and available graphics calculators, and more recently CAS, by retaining essentially the same curriculum but modifying part of the assessment, typically examinations. The Danish Ministry of Education recently completed its CAS pilot of several years and now offers Danish Baccalaureate Mathematics examinations for both CAS and non-CAS based implementations of its course. To achieve this it varies some questions on the examination, with CAS users and non-CAS users tackling several common questions, and some distinctive questions (CAS and non-CAS alternatives). CAS questions are typically more complex and involve the use of parameters. There are no restrictions on the CAS used, nor its memory, so for example, Maple could be used with stored programs. There is also a component of the examination that is technology free. (Leigh-Lancaster, 2002, p. 26)

The technology free component mentioned above points to one aspect of the current VCAA consultation about the structure of future assessments in Victoria. According to decisions made at the VCAA Board meeting in October 2003, it is expected that the assessments from 2006 will have a technology-free component.

> The VCAA also approved, in principle, the recommendation from the Mathematics Expert Studies Committee that each of Mathematical Methods, Mathematical Methods (CAS) and Specialist Mathematics have a technology free examination and a technology assumed access examination (approved graphics calculator or CAS as applicable) … subject to consultation on models for these examinations with key stakeholders. (VCAA, 2003a, p.7)

Subsequent to the announcement of this decision, the VCAA has published a consultation paper (VCAA, 2003b) that provides a historical overview, issues for discussion, models for examinations and invites open response from all interested parties. The key consideration in this consultation process is that there are some technology independent skills that are valued in mathematics teaching, learning and assessing.

In conclusion, the use of CAS in the Mathematical Methods CAS subject has provided a unique opportunity to assess the impact of CAS use on all aspects of teaching, learning and assessment. As reported in a keynote

address (Garner, McNamara & Moya, 2003) at the 2003 Mathematical Association of Victoria (MAV) annual conference:

> The unrestricted use of CAS has led to dramatic changes in pedagogy and assessment in each of the three schools (Ballarat Grammar School, Frankston High School, MLC). CAS has proved to be a powerful learning tool that allows students to move between numeric, graphical and symbolic representations of a problem. It also allows students to observe patterns and explore concepts … Traditionally, the focus of senior mathematics courses has been on how to carry out mathematical procedures, such as solving equations and differentiating expressions. CAS has made it possible to increase the emphasis on the understanding of concepts and on helping students to decide when and why it might be appropriate to apply a particular procedure. Unrestricted access to CAS has challenged us, as educators, to start inventing new paradigms for the teaching and learning of senior mathematics. (p. 271)

This is certainly the current challenge for senior secondary mathematics teachers across Australia. The education profession is renowned for its contrasts in adopting new strategies of learning. There will be the conservatives at one end of the spectrum, stretching to the radical teachers who will easily adopt and absorb any new ideas and processes. The interesting thing for me is that this particular experience has moved me along the spectrum, from being a timid technology user to being, unexpectedly, one of the 'radicals'.

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