Notes for applied mathematics in trigonometry and Earth geometry/navigation

Peter Faulkner
Launceston College

As time has progressed, the role of applied mathematics has become increasingly important. Indeed there are now more students enrolled in applied mathematics courses in senior high schools and colleges than in pure mathematics. Such courses become more relevant both to the student and to future employers, if the same constants and equations that are used in industry are reflected in the student's texts. The appropriate formulae for such texts are featured in bold-face in what follows below.

In trigonometry, problems involving planes and ships are very much part of any exercise. When calculating speed for either a commercial plane or ship, the unit is the *knot* (kn), and for distance, the unit is the *nautical mile* (nm) (Bowditch, 1995). In 1929 the International Hydrographic Bureau set the distance of 1 nautical mile at 1852 m. This is the figure that is used today in Australia. Bearings and courses have been expressed in three figure notation (234°T) in the services and commercial navigation for more than thirty-five years (Moore, 1967). Quadrantal notation (4°W) is only used with reference to compass error for variation and deviation.

Problems involving ships and planes in Earth geometry or navigation should use the same or similar formulae as those used by professional navigators. The methods have not changed since the days of sail and offer an interesting historical perspective for any applied mathematics course. The following formulae for distances, adapted from Frost (1983), were applied at the Australian Maritime College in March 2003 (personal communication).

East/west passages along a parallel of latitude

Since DEP = D.LONG cos LAT

and for east/west routes,

DEP = DIST,

where DEP or departure, is the distance east/west along a parallel of latitude.

DIST = D.LONG cos LAT

where D.LONG = difference of longitude in minutes (0.1 min),
LAT = latitude in degrees and minutes (0.1 min) and
DIST = distance is rounded to the nearest nautical mile.

North/south passages along a meridian of longitude

$$\mathbf{DIST = D.LAT}$$

where D.LAT = difference of latitude in minutes (0.1min).

Example

Find the distance in nautical miles between

(a) 42° 12'S 167° 23'W and 42° 12'S 150° 17'W and
(b) 42° 12'S 150° 17'W and 13° 23'S 150° 17'W

(a) The passage here is due east along a parallel of latitude so we apply the formula

DIST = D.LONG cos LAT

Lat = 42°12'S.
D.Long = 167° 23'W – 150° 17'W,
= 17° 6'E or 1026'E [$17 \times 60 + 6$]
DIST = D.LONG .cos LAT,
= $1026 \times \cos 42°\ 12'$,
= 760 nm.

(b) The passage this time is due north along a meridian of longitude. Here the formula is

DIST = D.LAT

D.Lat = 42° 12'S – 13° 23'S,
= 28° 49'N or 1729'N.
DIST = D.LAT,
= 1729 nm.

If it is wished to keep D.Long and D.Lat in degrees and minutes the first two formulae become:

$$\text{DIST} = 60\ (\text{D.LONG})\ \cos \text{LAT}$$

$$\text{DIST} = 60\ (\text{D.LAT})$$

When the distance is required in kilometres, the distances calculated in nautical miles must be multiplied by 1.852.

Example

The distance (kilometres) in example (b) above is simply

$$1729 \times 1.852 = 3302 \text{ km.}$$

Since the nautical mile is not determined from the mean radius of Earth, errors arise if an application uses the arc distance formula

$$A = \frac{2\pi r\theta}{360}$$

with the mean radius of 6371 km for Earth (Yoder,1995).

Example

Applying example (b) again, but with the distance in kilometres from 42° 12'S 150° 17'W to 13° 23'S 150° 17'W.

Using the arc distance formula with the radius of Earth as 6371 km,

$$A = \frac{2\pi r\theta}{360}$$

D.Lat	= 42° 12'S – 13° 23'S,
	= 28° 49'N.
A	$= 2\pi r\theta/360$,
A	$= 2\pi(6371)(28°49')/360$,
A	= 3204 km.

This application implies a distance of 1853.23 m per nautical mile, which is very nearly the *geographic mile* (6080.15 ft) measured along one minute of arc on the equator.

One might raise an argument to apply a figure of 6367 m for Earth's radius, this being the truncated average of the polar axis and the equatorial radius (Yoder,1995). A satisfactory relationship between the metre and the nautical mile could then be established.

For distances less than 600 nm, Earth may be considered flat and simple plane triangle trigonometry used to solve the problem. Distances greater than these following a great circle route were originally calculated applying spherical trigonometry and using haversines $[\frac{1}{2}(1 - \cos\theta)]$ to allow for the use of log tables. A more convenient approach for today's electronic age is (The Ministry of Defence, 1997):

$$\cos(\text{DIST}) = \sin(\text{LAT A})\sin(\text{LAT B}) + \cos(\text{LAT A})\cos(\text{LAT B})\cos(\text{D.LONG})$$

The distance is expressed initially in angular measure and has to be converted thus:

$$\text{DIST (nm)} = 60 \cos(\text{DIST})$$

Combining the two we have:

DIST = 60cos^{-1}[sin(LAT A)sin(LAT B) + cos(LAT A)cos(LAT B)cos(D.LONG)]

where DIST = distance is rounded to the nearest nautical mile,
D.LONG = difference of longitude in degrees and minutes (0.1 min) and
LAT = latitude in degrees and minutes (0.1min).

Example

Find the great circle distance between Port Elizabeth 34° 15'S 25° 30'E and Fremantle 31° 50'S 115° 30'E to the nearest nm.

LatA 34° 15'S	LongA 25° 30'E
LatB 31° 50'S	LongB 115° 30'E
	D.Long 90° 0'E

Dist = 60cos^{-1}[sin34° 15'sin31° 50' + cos34° 15'cos31° 50'cos90° 0']
Dist = 60cos^{-1}[0.296851]
Dist = 60 × 72° 43.88'
Dist = 4364 nm

If formulae that include latitude and longitude are to be programmed into a computer or calculator, by convention north and east are positive but south and west are negative (Harris, 1989).

When exercises in local mean time and zone time are part of an Earth geometry unit, problems involving the sun at noon are best omitted unless a closer study of apparent time and the equation of time (Ministry of Defence, 1997) is included.

References

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