Sparking interest in trigonometry

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Over the years, pre-service mathematics education students, when asked to jot down their ideas for introducing trigonometry to Year 9, most commonly flood their notes with hypotenuse, opposite, adjacent, sin, cos, tan, and so on. They receive a mark of 1/10. Some do much better by toying with ideas of doing some measurement activity.

Surely, the most important objective when starting a class on a previously unknown branch of mathematics is to ensure that they enjoy and appreciate the significance of this new aspect of mathematics. Inundating them with a flood of technical terms and definitions without any reality experience is a fast way to send the students out of class perplexed and developing an immediate distaste for trigonometry.

My recommendation for starting any new aspect of mathematics is to immerse the students in the context of the new work as a first step. Technical terms and processes can be gradually treated in succeeding stages of the unit of work envisaged.

First stage

A great way to involve the class in group work at the very start of a unit on trigonometry is to provide each group with a 45-45-90 set-square, a drinking straw, some Blu-Tack, and a metre rule, and direct them to work out some way to use all these materials to measure the height of the classroom. Be specific about where the height is to be registered, such as indicating clear junction lines for wall and ceiling. No technical terms or demonstrations are to be used — just step back and watch what groups come up with.

If, after about ten minutes, none of the groups tries fixing the drinking straw to the longest side of the set-square to create a viewing instrument, try quietly suggesting this to one group and the idea will spread. Usually first trials are attempts to view the top of the wall while lying on the floor. This might become practical if the drinking straw is placed towards one end of the hypotenuse, as shown by the Year 10 boys in Figure 1 when they thought of the move themselves.

Figure 1. Year 10 students.

Viewing becomes simpler when the viewing instrument is placed on a horizontal surface such as a desktop and the position of the desk is adjusted until a suitable line of sight is established, as shown in Figure 2 by the pre-service students who had the brainwave to try that move.

Figure 2. ACU pre-service students.

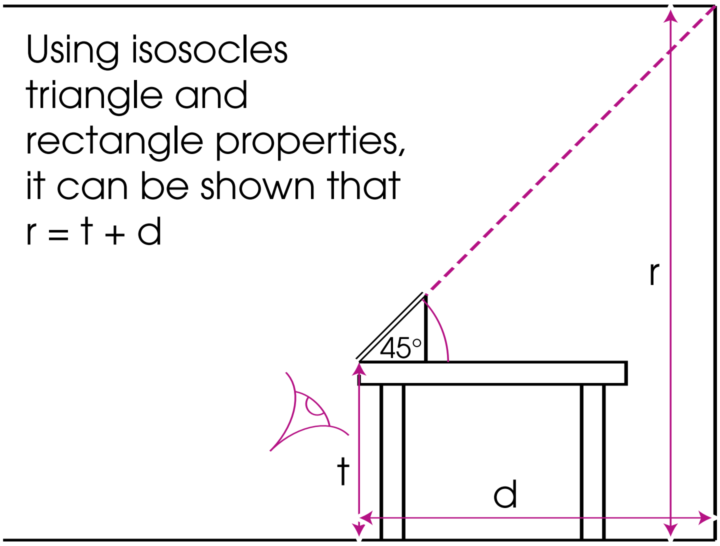

Figure 3. First Method for finding height of room.

The most common next move is to take two measurements. These are:

1. the height of the table, and
2. the distance to the wall from the point on the floor below the bottom end of the view-finder.

See Figure 3 to note these measurements in diagram form. Here,

t = height of table,
d = distance to wall,
r = height of room.

After using this idea for several years, one group a few years ago improved dramatically on this first method. After positioning the set-square for viewing up to the top of the wall, the group found the point on the floor where the line of sight downwards hit the floor. Then they needed just one measurement, namely the distance from this latter point to the wall, as shown in Figure 4.

In each case, the height of the room can be checked by actually measuring up the wall. Group results should be assembled and errors of measurement can be acknowledged and classified as something to be expected whenever measurements are undertaken. Nevertheless, some discussion of how errors might have occurred is recommended.

The students could be told, after all the action, that they have made a start on coming to appreciate and understand a whole new branch of mathematics known as ‘trigonometry’.

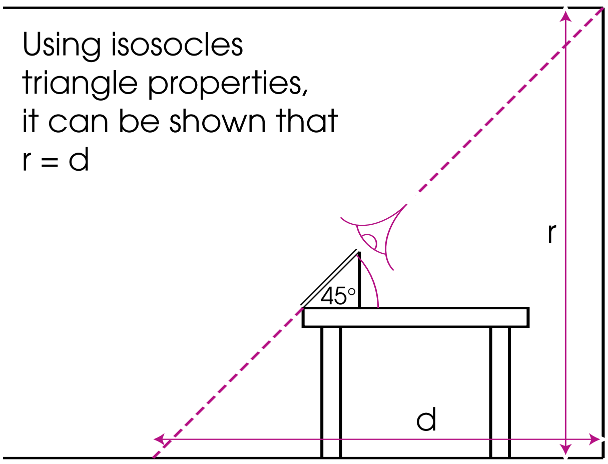

Figure 4. Second method for finding height of room.

Second stage

Next, challenge the groups to repeat their measuring of the height of the room using 30-60-90 set-squares instead of 45-45-90 set-squares.

The students should quickly realise that the set-square no longer provides an isosceles triangle such that the ratio $r : d$ equals 1 : 1. Here are some possible student activities.

- Have students explore the $r : d$ ratio (as shown in Figure 6) using different sized (but similar) set-squares.
- Have students draw a series of similar 30°-60°-90° triangles on graph paper and compare the $r : d$ ratios.
- Have students find the (new) $r : d$ ratio using the 30° angle of the set-square instead of the 60° angle. How are the two $r : d$ ratios related? Why?
- Help students to obtain the diagram in Figure 5 to answer the above question, and obtain a factor of 3.

All that the students need is a knowledge of isosceles triangles and, if they have not studied congruency (for triangles ABD and ACD), they can be convinced that the altitude of triangle bisects the base by just folding triangle ABC in half. The values for tan 30° and tan 60° can be checked using a calculator and, later, formally derived by applying Pythagoras' Theorem to triangle ABD.

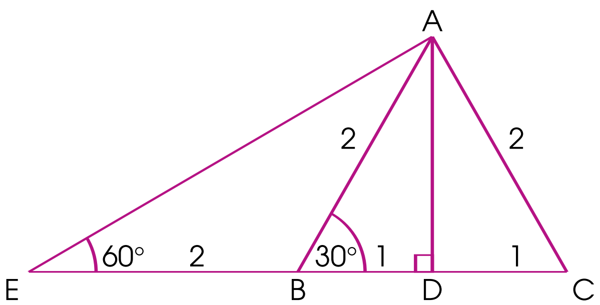

Figure 5. Comparing 60 and 30 degree sighting angles.

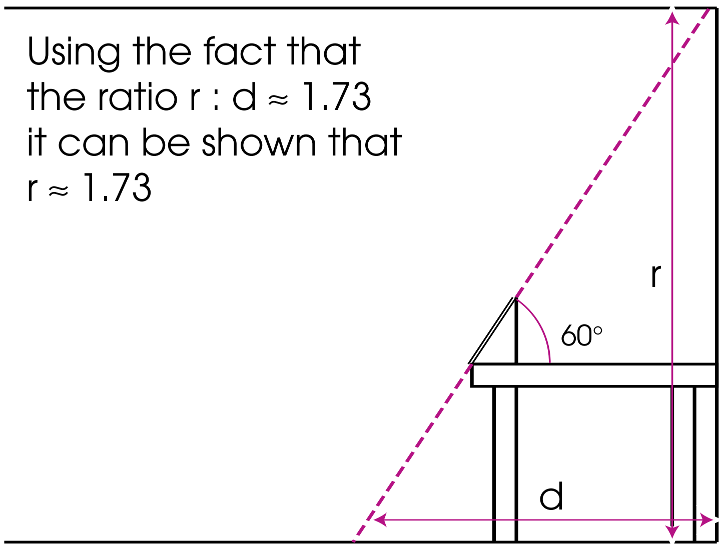

Figure 6. Second stage method for finding height of room.

It is time now for the introduction of the technical terms opposite, adjacent, and tangent ratio and the definition of tangent of an angle. If the students meet such terms in this reality context, there is more chance that they will appreciate the relevance and usefulness, especially if they are led to think about the freedom obtained by not being restricted to the angles of set-squares.

Third stage

Many more problems involving only the tan ratio should be used to settle the students into doing trigonometry. Stay with tan for some time before introducing problems which require sin or cos. A few practical cases involving tan are the following.

1. Go to a basketball or other sporting field; measure one side; sight along a diagonal and measure the angle between that diagonal and the side just measured; then use trigonometry to calculate the length of the adjoining side; and finally measure the latter to check the calculation.
2. Find the height of the window-sill of a window facing outwards across level ground by using a method similar to that shown in Figure 6, although allowing freedom as regards the angle involved. In fact, have different groups use differing angles! The trigonometric

calculations can be checked by measuring a plumb-line from the window-sill to the ground below the window.

Concluding comment

This approach demonstrates the following basic principles of pedagogy. The first two of these I distinctly remember from my own teacher education days in 1951.

1. Go from concrete to abstract. Avoid starting with definitions.
2. Go from particular to general. Here, we started with a few special angles before going on to general acute angles and the need for finding the values of the tangent of such angles by construction or by using a calculator or other means.
3. Immerse students in the context of any new concept before explicating its technicalities and intricacies and mathematical jargon. The above approach illustrates the truth that students can be using the tangent function before they have even heard of the term!
4. The lesson introducing a new concept should be one that results in favourable reactions from the students. The Year 10 students shown in Figure 1 dramatised their delight with their first experience of anything trigonometric. Herman Tay who ran their first lesson (while a pre-service student) reported that the boys were 'enthusiastic', 'excited', and 'thrilled'. He went on to report that 'the whole class was awash with enthusiasm, once one group attached the drinking straw to the set-square,' and, 'We were oblivious of the bells at lunch break; nobody was anxious to leave the classroom.'

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SIMPLE for

HELEN CHICK

Most students start their statistical experiences in primary school with simple data handling techniques such as tallies and bar charts. These situations often involve single variable data, so that a typical activity might involve producing a frequency graph showing the favourite football team of students in the class.

Data analysis becomes much more interesting when the data set involves multiple variables. This is because relationships among the variables can be explored. Data exploration now might involve making comparisons and determining the existence of associations. Of course, this complexity in the data brings with it challenges in dealing with the data, to produce the representations and calculations that help identify those relationships and contrasts. In teaching we sometimes leave the study of multivariate data until quite late in schooling because some of the techniques for dealing with such data are deemed too complicated. There are, however, some simple strategies that make such data analysis accessible to younger students. These techniques are probably familiar to us as teachers, especially if we use spreadsheets, and yet often we do not highlight them for our students.

To illustrate this, we will look at the work of some Year 7 students who were asked to consider the data set in Figure 1. The idea for this data set arose from the work of Watson and her colleagues (e.g., Watson, Collis, Callingham & Moritz, 1995). We note that, of course, it is usually better if students collect their own data about a topic of interest, but in this case I wanted to be sure that the data set was not too large and that there were relationships evident.