Ever since the first mariners sailed off the east coast of Scotland the Bell Rock has claimed many vessels and countless lives. Also known as the Inch Cape Rocks they lie 18 km off the coast at Arbroath. Located near the mouth of the Firth of Forth and its important shipping ports these dangerous rocks cover an area some 440 m long and 90 m wide. Submerged for all but up to four hours a day it is said that a floating bell was moored at the rocks as early as the sixteen century in an effort to warn unwary mariners.

The fierce North Sea storms can blow passing boats onto the rocks. At high tide, these rocks are covered to a depth of 3.5 metres of water. In calm weather and at a moderate tide no breakers betray the presence of the rocks. In years past many ships were lost after having their keels ripped out as they sailed the placid waters over the hidden rocks at moderate tides.

During the first years of the nineteenth century, Captain Joseph Brodie of the Royal Navy and Joseph Couper of Leith proposed the construction of a cast-iron lighthouse supported on four pillars. Even though their plans were rejected they erected on the Bell Rock wooden beacons at their own expense. Three times they erected the beacons and each time the combination of wind and wave destroyed them rapidly.

At the same time Robert Stevenson was an engineer with the Northern Lighthouse Board who had already erected six lighthouses around the Scottish coast. After considering the idea of a cast-iron lighthouse supported on nine legs similar to that suggested by Brodie and Couper he proposed to the Board the construction of a stone lighthouse on the Bell Rock at a cost of no more than £43 000. After discussion a modified version of his proposal was accepted and construction work was begun in 1807.

Granite and sandstone blocks weighing up to 1.5 tonnes were to be carefully carved to precise measurements and shapes on land at Arbroath. They would then be ferried to the construction site on Bell Rock in specially-built vessels. Initially the workmen would only work at low tide when the rock was exposed. This happened twice a day for up to only two hours at a time. In the first year they commuted twice daily to the rock from a ship moored off the rocks but later they were able to live and work in the Beacon House a three-story structure built on stilts beside the lighthouse.

If the lighthouse had have been constructed on land then the whole process would have
been much simpler and safer for many reasons. When constructed on land the stone blocks of a lighthouse could be relied upon to remain in place due to their sheer weight alone. Early in the construction process Stevenson had seen loose rocks known as *travellers* and weighing up to two tonnes thrown up onto the Bell Rock from the sea bed during severe storms. These travellers would stay until disappearing sometimes months or even years later carried away by the power of the immense waves and currents that washed the rocks that accompanied the severe North Sea storms.

Clearly any granite or sandstone blocks used to construct the base of the lighthouse could not be expected to stay in place unless properly anchored. Stevenson’s solution was to use blocks that dovetailed into one another so that it would be impossible for one block alone to move. Much like jigsaw puzzle pieces the blocks locked one another in place. Trenails, joggles and wedges were employed to anchor the blocks down to the rock or the stone course below.

During the summer of 1807 workmen dug out of the rock a flat circular pit, the floor of which would become the foundation of the lighthouse. At one side the floor of the pit was just half a metre below the top of the rock while at the other side the pit floor was 1 metre below the surrounding rock. At the start of each work shift the workers were required to pump the pit free of the water which had filled during the preceding high tide. A partial course of sandstone blocks was laid in the pit to ensure the floor of the foundation pit was flat and horizontal.

The first entire course of granite and sandstone blocks was laid down in the northern summer of 1808. Consisting of 123 hand-carved blocks the first complete course formed a solid circle 12.8 m in diameter (Figure 1). Seventy-one sandstone blocks formed the inner core of the course with fifty-two granite blocks making up the circumference. Over the summers of 1808 and 1809 twenty-six courses of stone were laid until the base of the lighthouse stood 9.5 m above the rock, and some 5.5 m above sea level even at the highest of tides. In these two years 1400 tonnes of masonry had been conveyed to the rock. The lighthouse base tapered in so that its diameter was then 6.0 m.
During 1810 Stevenson oversaw the construction of the remaining masonry work. The first courses built that year housed the doorway and the spiral staircase up to the lowest of the six rooms of the house. These courses and all above were built of sandstone alone. The 90th and last sandstone course was laid on 30 July 1810. Above this rose the Light Room with its 48 panes of glass arranged in an octagonal pattern about the revolving reflectors. Below the Light Room was the Strangers’ Room or Library. Below this was the kitchen and dining area. The third room was the bedroom which could accommodate up to six people in two triple-bunks. The oil used in the burners to generate the light was housed in the second lowest room, The Lighthouse Stores. The lowest of the rooms was the Provisions Store used to store the winch which itself was used to raise stores from the rock below.

The twenty-four reflectors of the light were placed on a rectangular frame which rotated once every eight minutes. Each of the larger sides of the rectangle frame housed seven reflectors in horizontal rows of 2, 3 and 2. The shorter sides housed five reflectors each in rows of 2, 1 and 2. Red panes of glass were placed in front of the two groups of five reflectors to provide an alternating sequence of red and white flashes distinguishing the Bell Rock Lighthouse flashes from other lighthouses along the coast.

On 1 February 1811 the light was lit for the first time and since then no ship has come to grief on Bell Rock. Though automated some 30 years ago the Bell Rock continues to operate as intended 200 years ago. In that time it has survived the worst North Sea storms and even a helicopter crash. It is a fitting monument to Robert Stevenson and the workmen who built it including the three workers who lost their lives during its construction.

In the years following its construction, Stevenson went on to build more lighthouses around the United Kingdom, as did his sons and some of his grandsons, though it is the literary work of his grandson Robert Louis Stevenson that is best remembered.
Some mathematical challenges

The construction of this brilliant piece of 18th century engineering provides us with many examples of the vital role of mathematics in engineering. Let’s consider the geometry of one of the courses laid, course 27, the entrance level to the tower. This was the first level which was constructed entirely of sandstone, the granite now no longer used. As we see in Figure 2, this course consists of 38 sandstone blocks of 14 different designs. The workers would have begun laying the inner ring of interlocking blocks. The exact sequence in which the blocks would have been laid is now unknown but it is likely that the workers would have worked their way outwards after having laid the blocks either side of the doorway.

At the outside of the solid wall a pattern of five blocks were repeated several times. No records exist of the exact design of these blocks. What is shown in Figure 3 is a reconstruction by the author of the rough plans presented in Stevenson’s 1824 diary. Figure 4 shows in more detail the design of Block 3 which is symmetric about DH. Figure 5 presents some information for the repeated pattern of eight blocks from Course 1.

When carving the blocks in the work yard in Arbroath the workmen and engineers would have needed to know some important dimen-

Trenails and joggles

Trenails were used to fix a stone block to the course below. Typically each block was fixed with two trenails. A trenail consists of a wooden rod just under 5 cm diameter and about 40 cm longer than the thickness of the block to be fixed. Both ends of the wood rod are cut and the thin end of a wooden wedge is inserted into the cut. A 5 cm hole is drilled through the stone to be fixed and into the stone or rock below. The trenail is then inserted into the hole and then hammered into place. Hammering down on the trenail causes the wooden wedge to be driven up into the wooden rod. This in turn causes the trenail to spread and bite into the stone block or rock below.

A joggle is a rectangular prism of sandstone measuring some 10 cm by 10 cm by 20 cm. The joggles were used to ensure that one course of stone did not move across the top of the stone course below. Up to 13 joggles were used per course. Recesses were carved into selected stones in the two courses to be joined. The joggles were positioned to act as pins between the two courses. The joggles have not been shown on the plans in Figures 1 and 2 for clarity.
sions to ensure that the blocks would fit exactly together out at the lighthouse. We can use the design information in Figure 4 to pose questions for our students:

- What is the length of sides AG and CE?
- What is the length of sides AB and BC?
- What is the angle ∠HAB?
- What is the angle ∠ABC?
- What is the area of the block?

As an alternative students might be asked to design their own course of interlocking blocks using fewer bricks of simpler designs. Figure 6 presents a simple design of thirteen interlocking bricks having only three different designs. Students could be asked to mathematically describe each block and then to calculate the area of each.

Stephenson’s designs for the Bell Rock lighthouse are exciting examples of how mathematics was and is ingeniously used by engineers.

Further resources

Website
www.bellrock.org.uk — a reference site for Stevenson’s Bell Rock lighthouse. This site contains comprehensive information on the lighthouse and its construction taken from Stevenson’s own work.

Books
Stevenson, Robert, An account of the Bell Rock Light-house, published by Constable & Co, Edinburgh, 1824. The historical images used in this paper were scanned from a microfilm copy of the manuscript.

Video
An episode from the BBC television series, Seven Wonders of the Industrial World, 2003 (broadcast in Australia on the ABC).

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