In this paper the author compares the two games played most frequently in our classrooms, concluding that although both games may prepare the student for success in school only one of the games prepares the student for success in life.

The games we play

There are two fundamentally different games that we play in our classrooms. I call the game that most of us have experienced, the knowledge game. I call the other game, the sense-making game.

It is important that we be able to recognise each game and understand their differing characteristics and purposes because, in my opinion, only one of these games, the sense-making game, prepares students for life.

You can identify which game is being played in a classroom by examining the learning tasks the students and their teacher focus on and the way in which they engage in these tasks.

Prompts for mathematical activity

Elaine Simmt (2002) states,

just as important as solving problems is specifying them in the first place. Hence, placing an emphasis on ‘problem solving’ in school mathematics, that is looking for the solution to a pre-specified problem, misses this key aspect of human cognition observed in my research. The teacher needs to provide students not problems to solve but with prompts for mathematical activity.

In this paper I use the prompt shown in Figure 1.

The sketch in Figure 1 is drawn using The Geometer’s Sketchpad. As point P is dragged, three differently coloured concentric discs change in size, as shown. Readers with Geometer’s Sketchpad can explore the sketch themselves by going to the website, www.mth.msu.edu/~nathsin/sketchmad/sketches/index.html, referred to in Flewelling and Sinclair 2004. Like any situation, this sketch can generate very different tasks, prompt very different kinds of mathematical activity. Below I describe two possible tasks and how they might unfold in the classroom. The task, entitled Triangle-Land, is engaged by students playing the sense-making game. The second task, entitled The Triangle Connection, is engaged by students playing the knowledge game. Though both tasks focus on the connection between the position of point P and the behaviour of three discs, they make very different demands on the student, foster very different dispositions, and ask the student to pursue very different goals.
1. Setting the stage/launching the investigation

You are a scientist who has landed at point P in Triangle Land. You are charged with the task of investigating how the sun near this world behaves. Start your investigation by dragging point P and studying the behaviour you witness.

- Teacher gives students (working independently or in small groups) the opportunity to explore the situation, get familiar with its elements, their behaviour, possible connections/relationships.
- Teacher advises students to record interesting observations/discoveries and be prepared to share them.
- Teacher encourages students to ask questions such as, When does that (interesting thing) happen? Why does it happen? What is this phenomena connected to?

2. Sharing observations/identifying focus problems

- Teacher facilitates student sharing of observations.
- Discussion leads to focusing on the following problems: ‘When does the sun display one colour? Two colours? Three colours?’

3. Conjecturing: Connecting/predicting/testing/proving

- Students continue investigation focused around above questions.
- Students connect particular sun displays with particular locations of dragged point. Geometric significance of these points identified.
- Area of individual discs connected to the areas of triangular regions.
- Predicts sun’s behaviour for various positions of point P.

4. Communicating/demonstrating/evaluating

- Students share findings, providing evidence for claims.
- Demonstrate proofs of assertions
- Evaluate convincingness/elegance of demonstrations

5. Assessing student performance

- Teacher provides new (related) sketch for students to investigate individually.
- Students write report related to investigation. Teacher provides rubric listing elements of a successful report.
- Teacher assesses student performance on task (and report.)

6. Reflecting on/extending the investigation

- Students are encouraged to create their own sketches for others to investigate.
- Are there other points of interest outside Triangle-Land?

Description of the task: Triangle-Land
Description of the task: The Triangle Connection

1. Launching the Task
   • Teacher introduces students to sketch, demonstrating disk size colour change.
   • Teacher directs students to work through the following set of 13 sub-tasks, indicating that doing so will allow them to understand the behaviour witnessed in the sketch.

   TASK
   1. Label the vertices of the triangle A, B, and C.
   2. Join point P to points A, B, and C.
   3. The disk is actually three overlapping disks. Separate them by dragging their centres.
   4. Measure the areas of PAB, PAC, and PBC.
   5. Measure the areas of the circles.
   6. Compare the results in parts 4 and 5 as you drag point P.
   7. Drag point P onto one side of the triangle. Explain why one circle shrinks to a point.
   8. Drag point P onto a vertex of the triangle. Explain why two circles shrink to a point.
   9. Drag point P to a position inside the triangle where the 3 disks have the same area.
   10. Demonstrate that the point found in part 9 is located at the intersection of the medians of triangle ABC.
   11. Drag point P outside the triangle to a point where the 3 disks are again of equal area. Prove that quadrilateral PABC is a parallelogram.
   12. Prove that the areas of PAC, PAB and PBC are equal. [This proves that the three disks are equal in area.]
   13. Sketch where you would expect to find two other points outside the triangle where the disks are again equal in area. Explain your reasoning.

2. Wrapping Up the Activity
   • Teacher ‘takes up work’.
   • Teacher directs students to summarise their findings in their notes.
   • Teacher assesses student performance.
A (brief) comparison of the two tasks

<table>
<thead>
<tr>
<th>Triangle-Land</th>
<th>The Triangle Connection</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1. Goals of task</strong></td>
<td><strong>1. Goals of task</strong></td>
</tr>
<tr>
<td>• (Students) Make sense of situation</td>
<td>• (Students) Follow teacher instructions/answer teacher questions.</td>
</tr>
<tr>
<td>• (Teachers) Develop (and assess) skills/dispositions in anticipation of future sense-making episodes, incidentally reinforcing geometric concepts and procedures.</td>
<td>• (Teachers) Reinforce/assess geometric concepts and procedures.</td>
</tr>
<tr>
<td>• (Both) Success on current task.</td>
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</tr>
<tr>
<td><strong>2. Cognitive demands</strong></td>
<td><strong>2. Cognitive demands</strong></td>
</tr>
<tr>
<td>• Use knowledge in a purposeful, integrated, creative, authentic manner.</td>
<td>• Use knowledge in a narrow, isolated, mechanical, artificial manner.</td>
</tr>
<tr>
<td>• Meta-cognise, monitor progress, reflect on actions/products, consider implications.</td>
<td>• Check accuracy/correctness/completeness of work.</td>
</tr>
<tr>
<td>• Use imagination/intuition.</td>
<td>• Think convergently.</td>
</tr>
<tr>
<td>• Think critically.</td>
<td></td>
</tr>
<tr>
<td><strong>3. Dispositions fostered</strong></td>
<td><strong>3. Dispositions fostered</strong></td>
</tr>
<tr>
<td>• Anticipate challenge/ambiguity/uncertainty.</td>
<td>• Anticipate situations/problems/tasks involving clarity, simplicity, certainty, correctness, closure.</td>
</tr>
<tr>
<td>• Sensitised to notice things, search out patterns/relationships, make connections, be discerning, ask questions, search out/articulate problems, make predictions, want to know why...</td>
<td>• Sensitised to respond to questions, to answer quickly/clearly/accurately, to find/apply needed concepts/procedures (to familiar situations.)</td>
</tr>
<tr>
<td>• Inclination to justify, argue, critique, clarify, convince...</td>
<td>• Inclination to obey/conform/acquiesce/follow.</td>
</tr>
<tr>
<td>• Take more control over learning/how task unfolds, initiate action, persevere...</td>
<td></td>
</tr>
<tr>
<td><strong>4. Beliefs/attitudes fostered</strong></td>
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</tr>
<tr>
<td>• Subject is an important way of thinking/a product of human/personal activity.</td>
<td>• Subject is an important collection of truths/rules/tools developed by others</td>
</tr>
<tr>
<td>• Subject is exciting and pleasurable.</td>
<td>• Subject anxious.</td>
</tr>
<tr>
<td>• ‘I am primary agent for my learning.’</td>
<td>• ‘Teacher primary agent for my learning.’</td>
</tr>
</tbody>
</table>

The Triangle-Land task is a much richer task. I define a ‘rich task’ as one that gives students the opportunity to use (and learn to use) their knowledge in an integrated, creative, authentic, and purposeful fashion to inquire, investigate, experiment, and problem solve and, in so doing, acquire knowledge with understanding, and in the process, develop the beliefs and dispositions of a life-long sense maker.

A comparison of the two games

Herbert Spencer (19th century philosopher and coiner of the phrase ‘survival of the fittest’) once said, ‘The great aim of education is not knowledge but action.’ The knowledge game is essentially about the acquisition of knowledge (stuffing with rules) with time divided between acquiring, reinforcing, and demonstrating acquisition. The sense-making game is about
action, sense-making action (playing with rules.).

The cognitive demands of each game are different. Experience with the knowledge game is not a prerequisite for playing the sense-making game. In fact, playing the knowledge game fosters the development of beliefs, practices and dispositions that act as impediments to playing the sense-making game. The knowledge game handicaps the student for playing the game waiting to be played beyond the classroom.

Whitehead (1929) talks about ‘the aimless accumulation of knowledge, inert and unutilised’. When he talks about the ‘aimless accumulation of knowledge’, he is talking, I believe, about the knowledge game. When he talks about ‘inert and unutilised knowledge’, I believe he is talking about the product of the knowledge game and its lack of usefulness in/transfer to sense making.

The curriculum connection

Most mathematics curricula I am familiar with are structured in ways that promote the knowledge game and discourage the playing of the sense-making game. A curriculum working group of the Canadian Mathematics Educators Study Group, at their May 2003 annual meeting, at Acadia University, Wolfville, Nova Scotia seems to concur. They state in their draft ‘manifesto’ (about Canadian mathematics curricula):

...we believe that the structure of these curricula is an obstacle to student learning of mathematics. Over-specified and fragmented lists of expectations misrepresent what mathematics is and militate against deep and authentic engagement with the subject.-

We find that:

• students coming out of high school mathematics must be able to engage effectively with complex problems; they require the ability to ‘think mathematically’—that is, to investigate the mathematics in a situation, to refine, to expand, and to generalise;
• students’ mathematics concepts must be woven into a connected set of relationships;
• students must be able to independently encounter and make sense out of new mathematics.

Conclusion

Both games, played well, can prepare the student for success in school. But we must ask ourselves the following two questions. ‘What do we want students to be successful at?’ and ‘What do we want students to be prepared for?’ If our answer to the second question is, ‘Life!’ then our answer to the first question must be, ‘Sense making!’

If we really want to graduate life-long sense makers then we will have to ensure that students are with teachers who engage them routinely in rich learning tasks, in classrooms where the sense-making game is played.

References


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